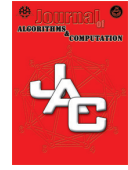




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## Some Properties of $(1, 2)^*$ -Soft b-Connected Spaces

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### ABSTRACT

In this paper we introduce the concept of  $(1, 2)^*$ -sb-separated sets and  $(1, 2)^*$ -soft b-connected spaces and prove some properties related to these topics. Also we discussed the properties of  $(1, 2)^*$ -soft b- compactness in soft bitopological space .

*Keyword:*  $(1, 2)^*$ -sb-separated ,  $(1, 2)^*$ -sb-connected ,  $(1, 2)^*$ -sb-compact

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## 1 Introduction

Soft set theory was first introduced by Molodtsov [5] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. Shabir and Naz [9] initiated the study of the soft topological spaces. In 1963, J.C. Kelly [4], first initiated the concept of bitopological spaces. After then many authors studied some of basic concepts and properties of bitopological space. In 1996, Andrijevic [1] introduced a new class of open sets in a topological space called b- open sets. Metin Akdag and Alkan Ozkan [11] are defined soft b- open sets and soft b- continuous map studied their properties. In the year

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2014, Basavaraj M. Ittanagi [2] initiated the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of parameters. As well known, connectivity occupies very important role in topology. Many authors have presented different kinds of connectivity in general, fuzzy and soft topological spaces. The purpose of this paper is to introduce the concepts of  $(1, 2)^*$ -soft b- connectedness and  $(1, 2)^*$ -soft b-compactness and a detailed study of some of its properties.

## 2 Preliminaries

For basic notations and definitions not given here, the readers can refer [3]-[9]. Throughout this paper,  $X$  is an initial universe,  $E$  is the set of parameters,  $P(X)$  is the power set of  $X$  and  $A$  is a nonempty subset of  $E$ .

**Definition 2.1.** A soft set  $F_A$  on the universe  $X$  is defined by the set of ordered pairs  $F_A = \{(x, f_A(x)) : x \in E\}$ , where  $f_A : E \rightarrow P(X)$  such that  $f_A(x) = \phi$  if  $x \in A$ . Here  $f_A$  is called approximate function of the soft set  $F_A$ . The value of  $f_A(x)$  may be arbitrary, some of them may be empty, some may have non empty intersection. The set of all soft sets over  $X$  will be denoted by  $SS_E(X)$ .

**Definition 2.2.** [3] Let  $F_A \in SS_E(X)$ . The soft power set of  $F_A$  is defined by  $\tilde{P}(A) = \{F_{A_i} : F_{A_i} \subseteq F_A, i \in I \subseteq N\}$  and its cardinality is defined by  $|\tilde{P}(A)| = 2^{\sum_{x \in E} |f_A(x)|}$ , where  $|f_A(x)|$  is the cardinality of  $f_A(x)$ .

**Example 2.3.** [3] Let  $X = \{x_1, x_2\}$  and  $E = \{e_1, e_2\}$  then  $\tilde{X} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$ . The possible soft subsets are

$$\begin{aligned} F_{E_1} &= \{(e_1, \{x_1\})\}, & F_{E_2} &= \{(e_1, \{x_2\})\}, & F_{E_3} &= \{(e_1, \{x_1, x_2\})\}, \\ F_{E_4} &= \{(e_2, \{x_1\})\}, & F_{E_5} &= \{(e_2, \{x_2\})\}, & F_{E_6} &= \{(e_2, \{x_1, x_2\})\}, \\ F_{E_7} &= \{(e_1, \{x_1\}), (e_2, \{x_1\})\}, & F_{E_8} &= \{(e_1, \{x_1\}), (e_2, \{x_2\})\}, \\ F_{E_9} &= \{(e_1, \{x_1\}), (e_2, \{x_1, x_2\})\}, & F_{E_{10}} &= \{(e_1, \{x_2\}), (e_2, \{x_1\})\}, \\ F_{E_{11}} &= \{(e_1, \{x_2\}), (e_2, \{x_2\})\}, & F_{E_{12}} &= \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}, \\ F_{E_{13}} &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}, & F_{E_{14}} &= \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\}, \\ F_{E_{15}} &= \phi, & F_{E_{16}} &= \tilde{X}. \end{aligned}$$

**Definition 2.4.** [9] Let  $\tilde{\tau}$  be the collection of soft sets over  $X$ , then  $\tilde{\tau}$  is said to be a Soft Topology on  $\tilde{X}$  if

- (i)  $\phi, \tilde{X}$  belongs to  $\tilde{\tau}$ .
- (ii) The soft union of any number of soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ .
- (iii) The soft intersection of any two soft sets in  $\tilde{\tau}$  belongs to  $\tilde{\tau}$ .

The triplet  $(\tilde{X}, \tilde{\tau}, E)$  is called a Soft Topological Space over  $X$ .

**Definition 2.5.** [9] Let  $(\tilde{X}, \tilde{\tau}, E)$  be a soft topological space over  $X$ . Then the members of  $\tilde{\tau}$  are said to be soft open sets over  $X$ .

**Definition 2.6.** [10] Let  $\tilde{X}$  be a non-empty soft set on the universe  $X$  with a parameter set  $E$  and  $\tilde{\tau}_1, \tilde{\tau}_2$  are two different soft topologies on  $\tilde{X}$ . Then  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is called a soft bitopological space.

**Definition 2.7.** [10] Let  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  be a soft bitopological space and  $F_A \subseteq \tilde{X}$ . Then  $F_A$  is called  $\tilde{\tau}_{1,2}$ -open if  $F_A = F_B \cup F_C$ , where  $F_B \in \tilde{\tau}_1$  and  $F_C \in \tilde{\tau}_2$ . The complement of  $\tilde{\tau}_{1,2}$ -open set is called  $\tilde{\tau}_{1,2}$ -closed.

**Definition 2.8.** [7] Let  $\tilde{X}$  be a soft bitopological space and  $F_A \subseteq \tilde{X}$ . Then  $F_A$  is called  $(1, 2)^*$ -soft b-open set ( briefly  $(1, 2)^*$ -sb-open) if  $F_A \subseteq \tilde{\tau}_{1,2}\text{-int}(\tilde{\tau}_{1,2}\text{-cl}(F_A)) \cup \tilde{\tau}_{1,2}\text{-cl}(\tilde{\tau}_{1,2}\text{-int}(F_A))$ .

**Definition 2.9.** [7] Let  $\tilde{X}$  be a soft bitopological space and  $F_A$  be a soft set over  $\tilde{X}$ .

(i)  $(1, 2)^*$ -soft b-closure ( briefly  $(1, 2)^*$ -sbcl( $F_A$ )) of a set  $F_A$  in  $\tilde{X}$  is defined by

$$(1, 2)^*\text{-sbcl}(F_A) = \tilde{\cap} \left\{ F_E \supseteq F_A : F_E \text{ is } (1, 2)^* \text{ - soft b - closed set in } \tilde{X} \right\}.$$

(ii)  $(1, 2)^*$ -soft b-interior ( briefly  $(1, 2)^*$ -sbint( $F_A$ )) of a set  $F_A$  in  $\tilde{X}$  is defined by  $(1, 2)^*$ -

$$\text{sbint}(F_A) = \tilde{\cup} \left\{ F_B \subseteq F_A : F_B \text{ is } (1, 2)^* \text{ - soft b - open set in } \tilde{X} \right\}.$$

**Definition 2.10.** [8] A soft mapping  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is said to be  $(1, 2)^*$ -soft b-continuous ( briefly  $(1, 2)^*$ -sb-continuous) if the inverse image of each  $\tilde{\tau}_{1,2}$ -open set of  $\tilde{Y}$  is  $(1, 2)^*$ -soft b-open set in  $\tilde{X}$ .

**Definition 2.11.** [8] A soft mapping  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is said to be  $(1, 2)^*$ -soft b-irresolute ( briefly  $(1, 2)^*$ -sb-irresolute) if  $\tilde{f}^{-1}(F_A)$  is a  $(1, 2)^*$ -soft b-closed set in  $\tilde{X}$ , for every  $(1, 2)^*$ -soft b-closed set  $F_A$  in  $\tilde{Y}$ .

**Theorem 2.12.** [8] Every  $(1, 2)^*$ - soft-continuous function is  $(1, 2)^*$ -soft b-continuous.

**Theorem 2.13.** [8] Every  $(1, 2)^*$ - soft b-irresolute mapping is  $(1, 2)^*$ -soft b-continuous.

**Definition 2.14.** [8] A soft mapping  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is said to be  $(1, 2)^*$ -soft b-open map ( briefly  $(1, 2)^*$ -sb-open) if the image of every  $\tilde{\tau}_{1,2}$ -open set of  $\tilde{X}$  is  $(1, 2)^*$ -soft b-open set in  $\tilde{Y}$ .

**Definition 2.15.** [8] A soft mapping  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is said to be  $(1, 2)^*$ -soft b-closed map ( briefly  $(1, 2)^*$ -sb-closed) if the image of every  $\tilde{\tau}_{1,2}$ -closed set of  $\tilde{X}$  is  $(1, 2)^*$ -soft b-closed set in  $\tilde{Y}$ .

### 3 $(1, 2)^*$ -Soft b-connectedness

In this section we introduce the concept of  $(1, 2)^*$ -soft b-separated sets and  $(1, 2)^*$ -soft b-connected space in soft bitopological space. Also we discuss some of the main results and properties.

**Definition 3.1.** Two non empty soft subsets  $F_{E_1}$  and  $F_{E_2}$  of  $SS_E(X)$  are said to be soft disjoint if  $F_{E_1} \widetilde{\cap} F_{E_2} = \phi$ .

**Definition 3.2.** Let  $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$  be a soft bitopological space. Two non-empty soft disjoint soft subsets  $F_{E_1}$  and  $F_{E_2}$  of  $SS_E(X)$  are called  $(1, 2)^*$ -soft separated sets over  $X$  if  $(\widetilde{\tau}_{1,2} - cl(F_{E_1})) \widetilde{\cap} F_{E_2} = F_{E_1} \widetilde{\cap} (\widetilde{\tau}_{1,2} - cl(F_{E_2})) = \phi$ .

**Definition 3.3.** Let  $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$  be a soft bitopological space. Two non-empty soft disjoint soft subsets  $F_{E_1}$  and  $F_{E_2}$  of  $SS_E(X)$  are called  $(1, 2)^*$ -soft b-separated ( $(1, 2)^*$ -sb-separated) sets over  $X$  if  $((1, 2)^*\text{-sbcl}(F_{E_1})) \widetilde{\cap} F_{E_2} = F_{E_1} \widetilde{\cap} ((1, 2)^*\text{-sbcl}(F_{E_2})) = \phi$ .

**Remark 3.4.** From the fact that  $(1, 2)^*\text{-sbcl}(F_E) \widetilde{\subset} (\widetilde{\tau}_{1,2} - cl(F_E))$ , for every soft subset  $F_E$  of  $\widetilde{X}$ , every  $(1, 2)^*$ -soft separated set is  $(1, 2)^*$ -soft b-separated. But the converse may not be true.

**Definition 3.5.** A  $(1, 2)^*$ -soft b-separation ( $(1, 2)^*$ -sb-separation) of a soft bitopological space  $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$  is a pair of  $(1, 2)^*$ -soft b-separated sets  $F_{E_1}$  and  $F_{E_2}$  whose soft union is  $\widetilde{X}$ .

**Definition 3.6.** Let  $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$  be a soft bitopological space. Then  $\widetilde{X}$  is called  $(1, 2)^*$ -soft b-connected space if  $\widetilde{X}$  cannot be expressed as the soft union of two  $(1, 2)^*$ -soft b-separated sets.

**Remark 3.7.** (i) In a soft bitopological space soft empty set is trivially  $(1, 2)^*$ -soft b-connected set.

(ii) In a soft bitopological space every soft singleton set is  $(1, 2)^*$ -soft b-connected since it cannot be expressed as a soft union of two non-empty  $(1, 2)^*$ -soft b-separated sets.

**Theorem 3.8.** Let  $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$  be a soft bitopological space. Then the following statements are equivalent:

- (i)  $\widetilde{X}$  is a  $(1, 2)^*$ -soft b-connected space.
- (ii)  $\widetilde{X}$  and  $\phi$  are the only  $(1, 2)^*$ -soft b-clopen sets in  $\widetilde{X}$ .
- (iii)  $\widetilde{X}$  cannot be expressed as the union of two disjoint non-empty  $(1, 2)^*$ -soft b-open sets.
- (iv)  $\widetilde{X}$  cannot be expressed as the union of two disjoint non-empty  $(1, 2)^*$ -soft b-closed sets.

*Proof.* (i)  $\rightarrow$  (ii) Let  $\widetilde{X}$  be  $(1, 2)^*$ -soft b-connected space. Let  $F_E$  be non-empty proper subset of  $\widetilde{X}$  that is  $(1, 2)^*$ -soft b-clopen. Then  $\widetilde{X} \setminus F_E$  is a non-empty  $(1, 2)^*$ -soft b-clopen set and  $\widetilde{X} = F_E \widetilde{\cup} (\widetilde{X} \setminus F_E)$ . This is a contradiction to  $\widetilde{X}$  is a  $(1, 2)^*$ -soft b-connected

space. Therefore  $\tilde{X}$  and  $\phi$  are the only  $(1, 2)^*$ -soft b-clopen sets in  $\tilde{X}$ .

(ii)  $\rightarrow$  (iii) Assume that  $\tilde{X}$  and  $\phi$  are the only  $(1, 2)^*$ -soft b-clopen sets in  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . suppose (iii) is false. Then  $\tilde{X} = F_{E_1} \tilde{\cup} F_{E_2}$  where  $F_{E_1}$  and  $F_{E_2}$  are disjoint non-empty  $(1, 2)^*$ -soft b-open sets. Then  $F_{E_2} = \tilde{X} \setminus F_{E_1}$  is  $(1, 2)^*$ -soft b- closed and non-empty. Thus  $F_{E_2}$  is a non-empty proper  $(1, 2)^*$ -soft b-clopen sets in  $\tilde{X}$ , which contradicts (ii).

(iii)  $\rightarrow$  (iv) Assume  $\tilde{X}$  cannot be expressed as the union of two disjoint non-empty  $(1, 2)^*$ -soft b-open sets. Suppose (iv) false. Then  $\tilde{X} = F_{E_1} \tilde{\cup} F_{E_2}$  where  $F_{E_1}$  and  $F_{E_2}$  are disjoint non-empty  $(1, 2)^*$ -soft b-closed sets. Then  $F_{E_1} = \tilde{X} \setminus F_{E_2}$  and  $F_{E_2} = \tilde{X} \setminus F_{E_1}$  are disjoint non-empty  $(1, 2)^*$ -soft b-open sets in  $\tilde{X}$ . Thus  $\tilde{X}$  is the soft union of two soft disjoint non-empty  $(1, 2)^*$ -soft b-open sets . This contradicts (iii).

(iv)  $\rightarrow$  (i) Suppose  $\tilde{X}$  is not  $(1, 2)^*$ -soft b-connected space. Then  $\tilde{X} = F_{E_1} \tilde{\cup} F_{E_2}$  where  $F_{E_1}$  and  $F_{E_2}$  are disjoint non-empty  $(1, 2)^*$ -soft b-open sets. Then  $F_{E_1} = \tilde{X} \setminus F_{E_2}$  and  $F_{E_2} = \tilde{X} \setminus F_{E_1}$  are disjoint non-empty  $(1, 2)^*$ -soft b-closed sets in  $\tilde{X}$ . This is a contradiction to (iv).  $\square$

**Proposition 3.9.** Every  $(1, 2)^*$ -soft b-connected space is  $(1, 2)^*$ -soft connected.

*Proof.* Let  $F_E$  be a  $(1, 2)^*$ -soft b-connected set in the soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ . Then there does not exist a  $(1, 2)^*$ -soft b-separation of  $F_E$  . Since every  $\tilde{\tau}_{1,2}$ -open set is a  $(1, 2)^*$ -soft b-open set, there does not exist a  $(1, 2)^*$ -soft separation of  $F_E$  . Hence  $F_E$  is a  $(1, 2)^*$ -soft connected set in in the soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$ .  $\square$

The converse is not true as shown in the following example.

**Example 3.10.**  $(1, 2)^*$ -soft connectedness does not imply  $(1, 2)^*$ -soft b-connectedness.

Let  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  be a soft bitopological space, where  $\tilde{X}$  and its soft subsets are considered as in Example 2.3 .

Let  $\tilde{\tau}_1 = \{ \tilde{X}, \phi, F_{E_1}, F_{E_7}, F_{E_{13}} \}$  and  $\tilde{\tau}_2 = \{ \tilde{X}, \phi, F_{E_4}, F_{E_{10}} \}$ . Then  $\tilde{\tau}_{1,2}$ -open sets are  $\{ \tilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_{10}}, F_{E_{13}} \}$  and  $\tilde{\tau}_{1,2}$ -closed sets are  $\{ \tilde{X}, \phi, F_{E_{12}}, F_{E_{14}}, F_{E_{11}}, F_{E_8}, F_{E_5} \}$ .

It is clear that  $\tilde{X}$  is  $(1, 2)^*$ -soft-connected since the only  $(1, 2)^*$ -soft-clopen sets are  $\phi$  and  $\tilde{X}$ . Also  $(1, 2)^*$ -soft b-open sets are  $\{ \tilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_6}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{10}}, F_{E_{12}}, F_{E_{13}} \}$

and  $(1, 2)^*$ -soft b-closed sets are  $\{ \tilde{X}, \phi, F_{E_{12}}, F_{E_{14}}, F_{E_3}, F_{E_{11}}, F_{E_{10}}, F_{E_2}, F_{E_8}, F_{E_1}, F_{E_5} \}$ .

Let  $\tilde{X} = F_{E_1} \tilde{\cup} F_{E_{12}}$  , then  $(1, 2)^*$ -sbcl( $F_{E_1}$ ) =  $F_{E_1}$  ,  $(1, 2)^*$ -sbcl( $F_{E_{12}}$ ) =  $F_{E_2}$  and  $(1, 2)^*$ -sbcl( $F_{E_1}$ )  $\tilde{\cap}$   $F_{E_{12}}$  =  $\phi$  ,  $(1, 2)^*$ -sbcl( $F_{E_{12}}$ )  $\tilde{\cap}$   $F_{E_1}$  =  $\phi$  . Hence  $\tilde{X}$  can be expressed as a union of two  $(1, 2)^*$ -soft b-separated sets  $F_{E_1}$  and  $F_{E_{12}}$ . Therefore  $\tilde{X}$  is not  $(1, 2)^*$ -soft b-connected.

**Example 3.11.**  $(1, 2)^*$ -soft b-connectivity is not a hereditary property.

Consider the soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2)$  , where  $\tilde{X}$  and its soft subsets are considered as in Example 2.3 . Let  $\tilde{\tau}_1 = \{ \tilde{X}, \phi, F_{E_1}, F_{E_7} \}$  and  $\tilde{\tau}_2 = \{ \tilde{X}, \phi, F_{E_3} \}$ . Then  $\tilde{\tau}_{1,2}$ -open

set =  $\{\tilde{X}, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_{13}}\}$  .Also  $(1, 2)^*$ -soft b-open sets are  $\{\tilde{X}, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{13}}, F_{E_{14}}\}$  and  $(1, 2)^*$ -soft b-closed sets are  $\{\tilde{X}, \phi, F_{E_{12}}, F_{E_6}, F_{E_{11}}, F_{E_{10}}, F_{E_2}, F_{E_5}, F_{E_4}\}$  .It is clear that  $\tilde{X}$  is  $(1, 2)^*$ -soft b-connected, since the only  $(1, 2)^*$ -soft b-clopen sets are  $\phi$  and  $\tilde{X}$ .  
 Let  $Y = \{x_1\} \subseteq X$  and  $E = \{e_1, e_2\}$ . Then  $\tilde{Y} = \{(e_1, \{x_1\}), (e_2, \{x_1\})\} = F_{E_7}$ .  
 Let  $\tilde{\sigma}_1 = \{\tilde{Y}, \phi, F_{E_1}\}$  and  $\tilde{\sigma}_2 = \{\tilde{Y}, \phi, F_{E_4}\}$ . Then  $\tilde{\sigma}_{1,2}$ -open set =  $\{\tilde{Y}, \phi, F_{E_1}, F_{E_4}\}$ . Also  $(1, 2)^*$ -soft b-clopen sets are  $\{\tilde{Y}, \phi, F_{E_1}, F_{E_4}\}$  .Clearly  $\tilde{Y}$  is not  $(1, 2)^*$ -soft b-connected, since  $F_{E_1}, F_{E_4}$  are  $(1, 2)^*$ -soft b-clopen sets other than  $\phi$  and  $\tilde{Y}$ .

**Proposition 3.12.** Let  $F_E$  be  $(1, 2)^*$ -soft b-connected set and  $F_{E_1}$  and  $F_{E_2}$  are  $(1, 2)^*$ -soft b-seperated sets. If  $F_E \subseteq F_{E_1} \cup F_{E_2}$  then either  $F_E \subseteq F_{E_1}$  or  $F_E \subseteq F_{E_2}$  .

*Proof.* Let  $F_E$  be  $(1, 2)^*$ -soft b-connected set and  $F_{E_1}$  and  $F_{E_2}$  are  $(1, 2)^*$ -soft b-seperated sets such that  $F_E \subseteq F_{E_1} \cup F_{E_2}$ . Let  $F_E \not\subseteq F_{E_1}$  and  $F_E \not\subseteq F_{E_2}$ . Suppose  $G_E = F_{E_1} \cap F_E \neq \phi$  and  $H_E = F_{E_2} \cap F_E \neq \phi$  then  $F_E = G_E \cup H_E$ . Since  $G_E \subseteq F_{E_1}$ ,  $((1, 2)^*\text{-sbcl}(G_E)) \subseteq ((1, 2)^*\text{-sbcl}(F_{E_1}))$  . Also  $((1, 2)^*\text{-sbcl}(F_{E_1})) \cap F_{E_2} = \phi$  then  $((1, 2)^*\text{-sbcl}(G_E)) \cap H_E = \phi$ . Since  $H_E \subseteq F_{E_2}$ ,  $((1, 2)^*\text{-sbcl}(H_E)) \subseteq ((1, 2)^*\text{-sbcl}(F_{E_2}))$  . Also  $((1, 2)^*\text{-sbcl}(F_{E_2})) \cap F_{E_1} = \phi$  then  $((1, 2)^*\text{-sbcl}(H_E)) \cap G_E = \phi$ . But  $F_E = G_E \cup H_E$  , therefore  $F_E$  is not  $(1, 2)^*$ -soft b-connected space.This is a contradiction.Then either  $F_E \subseteq F_{E_1}$  or  $F_E \subseteq F_{E_2}$ .  $\square$

**Theorem 3.13.** If  $F_E$  be  $(1, 2)^*$ -soft b-connected set and  $F_E \subseteq G_E \subseteq ((1, 2)^*\text{-sbcl}(F_E))$  then  $G_E$  is  $(1, 2)^*$ -soft b-connected.

*Proof.* Suppose  $G_E$  is not  $(1, 2)^*$ -soft b-connected then there exists two non-empty soft sets  $F_{E_1}$  and  $F_{E_2}$  such that  $((1, 2)^*\text{-sbcl}(F_{E_1})) \cap F_{E_2} = F_{E_1} \cap ((1, 2)^*\text{-sbcl}(F_{E_2})) = \phi$  and  $G_E = F_{E_1} \cup F_{E_2}$  .Since  $F_E \subseteq G_E$  then either  $F_E \subseteq F_{E_1}$  or  $F_E \subseteq F_{E_2}$  .Suppose  $F_E \subseteq F_{E_1}$  then  $((1, 2)^*\text{-sbcl}(F_E)) \subseteq ((1, 2)^*\text{-sbcl}(F_{E_1}))$  , thus  $((1, 2)^*\text{-sbcl}(F_E)) \cap F_{E_2} = F_E \cap ((1, 2)^*\text{-sbcl}(F_{E_2})) = \phi$  . But  $F_{E_2} \subseteq G_E \subseteq ((1, 2)^*\text{-sbcl}(F_E))$  ,thus  $((1, 2)^*\text{-sbcl}(F_E)) \cap F_{E_2} = F_{E_2}$  . Therefore  $F_{E_2} = \phi$  , which is a contradiction . If  $F_E \subseteq F_{E_2}$  , then by the same way we can prove that  $F_{E_1} = \phi$  .This is a contradiction.Thus  $G_E$  is  $(1, 2)^*$ -soft b-connected.  $\square$

**Theorem 3.14.** If  $F_E$  be  $(1, 2)^*$ -soft b-connected set then  $(1, 2)^*\text{-sbcl}(F_E)$  is  $(1, 2)^*$ -soft b-connected .

*Proof.* Suppose  $F_E$  is  $(1, 2)^*$ -soft b-connected and  $(1, 2)^*\text{-sbcl}(F_E)$  is not  $(1, 2)^*$ -soft b-connected. Then there exists two  $(1, 2)^*$ -soft b-seperated sets  $F_{E_1}$  and  $F_{E_2}$  such that  $(1, 2)^*\text{-sbcl}(F_E) = F_{E_1} \cup F_{E_2}$  .But  $F_E \subseteq (1, 2)^*\text{-sbcl}(F_E)$  then  $F_E \subseteq F_{E_1} \cup F_{E_2}$  and since  $F_E$  is  $(1, 2)^*$ -soft b-connected set then either  $F_E \subseteq F_{E_1}$  or  $F_E \subseteq F_{E_2}$ .

(i) If  $F_E \subseteq F_{E_1}$  then  $((1, 2)^*\text{-sbcl}(F_E)) \subseteq ((1, 2)^*\text{-sbcl}(F_{E_1}))$ . But  $(1, 2)^*\text{-sbcl}(F_{E_1}) \cap F_{E_2} = \phi$ , hence  $(1, 2)^*\text{-soft bcl}(F_E) \cap F_{E_2} = \phi$ . Since  $F_{E_2} \subseteq (1, 2)^*\text{-sbcl}(F_E)$ , then  $(1, 2)^*\text{-soft bcl}(F_E) \cap F_{E_2} = F_{E_2}$  hence  $F_{E_2} = \phi$  which is a contradiction.

(ii) If  $F_E \subseteq F_{E_2}$  then the same way we can prove that  $F_{E_1} = \phi$  which is a contradiction. Therefore  $(1, 2)^*\text{-sbcl}(F_E)$  is  $(1, 2)^*$ -soft b-connected.  $\square$

**Theorem 3.15.** The soft union  $F_E$  of any family  $\{F_{E_i} : i \in I\}$  of  $(1, 2)^*$ -soft b-connected sets having a nonempty soft intersection is  $(1, 2)^*$ -soft b-connected set.

*Proof.* Let  $F_E$  be a soft union of any family of  $(1, 2)^*$ -soft b-connected sets having a non-empty soft intersection. Suppose that  $F_E = F_{E_1} \tilde{\cup} F_{E_2}$ , where  $F_{E_1}$  and  $F_{E_2}$  form a  $(1, 2)^*$ -soft b-separation of  $F_E$ . By hypothesis, we may choose a soft point  $x_e \tilde{\in} \tilde{\cap}_{i \in I} F_{E_i}$ . Then  $x_e \tilde{\in} F_{E_i}$  for all  $i \in I$ . If  $x_e \tilde{\in} F_E$ , then either  $x_e \tilde{\in} F_{E_1}$  or  $x_e \tilde{\in} F_{E_2}$  but not both. Since,  $F_{E_1}$  and  $F_{E_2}$  are soft disjoint, we must have  $F_{E_i} \tilde{\subseteq} F_{E_1}$ , since  $F_{E_i}$  is  $(1, 2)^*$ -soft b-connected and it is true for all  $i \in I$ , and so  $F_E \tilde{\subseteq} F_{E_1}$ . From this we obtain that  $F_{E_2} = \phi$ , which is a contradiction. Thus, there does not exist a  $(1, 2)^*$ -soft b-separation of  $F_E$ . Therefore,  $F_E$  is  $(1, 2)^*$ -soft b-connected set.  $\square$

**Theorem 3.16.** (i) If  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is a  $(1, 2)^*$ -soft b-continuous surjection and  $\tilde{X}$  is  $(1, 2)^*$ -soft b-connected then  $\tilde{Y}$  is  $(1, 2)^*$ -soft connected.

(ii) If  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is a  $(1, 2)^*$ -soft b-irresolute surjection and  $\tilde{X}$  is  $(1, 2)^*$ -soft b-connected then  $\tilde{Y}$  is  $(1, 2)^*$ -soft b-connected.

*Proof.* (i) Suppose  $\tilde{Y}$  is not  $(1, 2)^*$ -soft connected. Let  $\tilde{Y} = F_E \tilde{\cup} G_E$  where  $F_E$  and  $G_E$  are disjoint non-empty  $\tilde{\tau}_{1,2}$ -open set in  $\tilde{Y}$ . Since  $\tilde{f}$  is  $(1, 2)^*$ -soft b-continuous and onto  $\tilde{X} = \tilde{f}^{-1}(F_E) \tilde{\cup} \tilde{f}^{-1}(G_E)$  where  $\tilde{f}^{-1}(F_E)$  and  $\tilde{f}^{-1}(G_E)$  are disjoint non-empty  $(1, 2)^*$ -soft b-open in  $\tilde{X}$ . This contradicts the fact that  $\tilde{X}$  is  $(1, 2)^*$ -soft b-connected. Hence  $\tilde{Y}$  is  $(1, 2)^*$ -soft connected.

(ii) Suppose  $\tilde{Y}$  is not  $(1, 2)^*$ -soft b-connected. Let  $\tilde{Y} = F_E \tilde{\cup} G_E$  where  $F_E$  and  $G_E$  are disjoint non-empty  $(1, 2)^*$ -soft b-open set in  $\tilde{Y}$ . Since  $\tilde{f}$  is  $(1, 2)^*$ -soft b-irresolute and onto  $\tilde{X} = \tilde{f}^{-1}(F_E) \tilde{\cup} \tilde{f}^{-1}(G_E)$  where  $\tilde{f}^{-1}(F_E)$  and  $\tilde{f}^{-1}(G_E)$  are disjoint non-empty  $(1, 2)^*$ -soft b-open in  $\tilde{X}$ . This contradicts the fact that  $\tilde{X}$  is  $(1, 2)^*$ -soft b-connected. Hence  $\tilde{Y}$  is  $(1, 2)^*$ -soft b-connected.  $\square$

## 4 $(1, 2)^*$ -Soft b-Compactness

In this section  $(1, 2)^*$ -soft b-compactness is defined and some of the characterizations are proved.

**Definition 4.1.** A collection  $\{F_{E_i} : i \in \Lambda\}$  of  $(1, 2)^*$ -soft b-open sets in soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is called a  $(1, 2)^*$ -soft b-open cover of a subset  $F_E$  if  $F_E \tilde{\subseteq} \tilde{\bigcup} \{F_{E_i} : i \in \Lambda\}$ .

**Definition 4.2.** A soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is called a  $(1, 2)^*$ -soft b-compact if every  $(1, 2)^*$ -soft b-open cover of  $\tilde{X}$  has a finite subcover.

**Definition 4.3.** A subset  $F_E$  of a soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is said to be  $(1, 2)^*$ -soft b-compact relative to  $\tilde{X}$ , if for every collection  $\{F_{E_i} : i \in \Lambda\}$  of  $(1, 2)^*$ -soft b-open subsets of  $\tilde{X}$  such that  $F_E \tilde{\subset} \tilde{\bigcup} \{F_{E_i} : i \in \Lambda\}$  there exists a finite subset  $\Lambda_0$  of  $\Lambda$  such that  $F_E \tilde{\subset} \tilde{\bigcup} \{F_{E_i} : i \in \Lambda_0\}$ .

**Definition 4.4.** A subset  $F_E$  of a soft bitopological space  $(\tilde{X}, \tilde{\tau}_1, \tilde{\tau}_2, E)$  is said to be  $(1, 2)^*$ -soft b-compact if  $F_E$  is  $(1, 2)^*$ -soft b-compact as a subspace of  $\tilde{X}$ .

**Theorem 4.5.** Every  $(1, 2)^*$ -soft b-closed subset of a  $(1, 2)^*$ -soft b-compact space  $\tilde{X}$  is  $(1, 2)^*$ -soft b-compact relative to  $\tilde{X}$ .

*Proof.* Let  $F_E$  be a  $(1, 2)^*$ -soft b-closed subset of  $\tilde{X}$ . Then  $F_E^{\tilde{C}}$  is  $(1, 2)^*$ -soft b-open set in  $\tilde{X}$ . Let  $S = \{G_{E_i} : i \in \Lambda\}$  be a cover of  $F_E^{\tilde{C}}$  by  $(1, 2)^*$ -soft b-open subsets in  $\tilde{X}$ . Then  $S \tilde{\cup} F_E^{\tilde{C}}$  is a  $(1, 2)^*$ -soft b-open cover for  $\tilde{X}$ . Since  $\tilde{X}$  is a  $(1, 2)^*$ -soft b-compact, it has a finite sub cover say  $\tilde{X} = G_{E_1} \tilde{\cup} G_{E_2} \tilde{\cup} \dots \tilde{\cup} G_{E_n} \tilde{\cup} F_E^{\tilde{C}}$ ,  $G_{E_i} \tilde{\in} S$ . But  $F_E$  and  $F_E^{\tilde{C}}$  are disjoint. Hence  $F_E \tilde{\subset} G_{E_1} \tilde{\cup} G_{E_2} \tilde{\cup} \dots \tilde{\cup} G_{E_n} \tilde{\in} S$ . Thus we have shown that any  $(1, 2)^*$ -soft b-open cover of  $\tilde{S}$  of  $F_E$  contains a finite sub cover. Therefore  $F_E$  is  $(1, 2)^*$ -soft b-compact relative to  $\tilde{X}$ .  $\square$

**Theorem 4.6.** A  $(1, 2)^*$ -soft b-continuous image of a  $(1, 2)^*$ -soft b-compact space is  $(1, 2)^*$ -soft compact.

*Proof.* Let  $\{F_{E_i} : i \in \Lambda\}$  be an  $\tilde{\tau}_{1,2}$ -open cover of  $\tilde{Y}$ . Then  $\{f^{-1}(F_{E_i}) : i \in \Lambda\}$  is a  $(1, 2)^*$ -soft b-open cover of  $\tilde{X}$ . Since  $\tilde{X}$  is  $(1, 2)^*$ -soft b-compact, it has a finite subcover say,  $\{f^{-1}(F_{E_1}), f^{-1}(F_{E_2}), \dots, f^{-1}(F_{E_n})\}$ . Since  $\tilde{f}$  is onto,  $\{F_{E_1}, F_{E_2}, \dots, F_{E_n}\}$  is a  $\tilde{\tau}_{1,2}$ -open cover of  $\tilde{Y}$  and hence  $\tilde{Y}$  is  $(1, 2)^*$ -soft compact.  $\square$

**Theorem 4.7.** If a map  $\tilde{f} : \tilde{X} \rightarrow \tilde{Y}$  is  $(1, 2)^*$ -soft b-irresolute and a subset  $F_E$  of  $\tilde{X}$  is  $(1, 2)^*$ -soft b-compact relative to  $\tilde{X}$  then the image  $\tilde{f}(F_E)$  is  $(1, 2)^*$ -soft b-compact relative to  $\tilde{Y}$ .

*Proof.* Let  $\{F_{E_i} : i \in \Lambda\}$  be a collection of  $(1, 2)^*$ -soft b-open sets in  $\tilde{Y}$  such that  $\tilde{f}(F_E) \tilde{\subset} \tilde{\bigcup} \{F_{E_i} : i \in \Lambda\}$ . Then  $F_E \tilde{\subset} \tilde{\bigcup} \{f^{-1}(F_{E_i}) : i \in \Lambda\}$  where  $f^{-1}(F_{E_i})$  is a  $(1, 2)^*$ -soft b-open in  $\tilde{X}$  for each  $i$ . Since  $F_E$  of  $\tilde{X}$  is  $(1, 2)^*$ -soft b-compact relative to  $\tilde{X}$ , there exists a finite sub collection  $\{F_{E_1}, F_{E_2}, \dots, F_{E_n}\}$  such that  $F_E \tilde{\subset} \tilde{\bigcup} \{f^{-1}(F_{E_i}) : i = 1 \text{ to } n\}$  that is,  $\tilde{f}(F_E) \tilde{\subset} \tilde{\bigcup} \{F_{E_i} : i = 1 \text{ to } n\}$ . Hence  $\tilde{f}(F_E)$  is  $(1, 2)^*$ -soft b-compact relative to  $\tilde{Y}$ .  $\square$

## 5 Conclusion

$(1, 2)^*$ -soft b-connected sets and  $(1, 2)^*$ -soft b-separated sets in the soft bitopological space have been introduced. Also we have introduced the  $(1, 2)^*$ -soft b-compactness and we



hope that the findings in this paper will help researchers to enhance and promote the further study on Soft bitopology to carry out a general framework for their applications in practical life.

## References

- [1] Andrijevic, D. *On b-open sets*, Mat. Vesnik. **48** (1996), 59-64.
- [2] Basavaraj M. Ittanagi , *Soft Bitopological Spaces*, Comp.and Math.with App.107, No.(2014).
- [3] Cagman N. , Karatas.S , Enginoglu.S : *Soft topology* , Comp.and Math.with App. 62(1), 351-358 (2011).
- [4] Kelly J.C, *Bitopological Spaces*,proc.London Math.Soc 13(3) 71-83(1963).
- [5] Molodtsov D.A, : *Soft set theory- first results*, Comp.and Math.with App.37(4-5),19-31(1999).
- [6] Peygan.E,Samadi.B and Tayebi.A, *On Soft Connectedness*, arXiv:1202. 1668v1 [math.GN],8 Feb 2012.
- [7] Revathi.N, Bageerathi.K, *On Soft B- open Sets in Soft Bitopological Spaces*, International Journal of Applied Research 2015; 1 (11): 615-623.
- [8] Revathi.N, Bageerathi.K, *(1, 2)\*-soft b-continuous and (1, 2)\*-soft b-closed map*, communicated.
- [9] Shabir M., and Naz M. : *On Soft topological spaces*. Comp.and Math.with App.61, 1786 1799(2011).
- [10] Senel G., Cagman N.: *Soft Closed sets on Soft bitopological Spaces*. Journal of New Results in Science, pp 57-66(2014).