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More On $\lambda_\kappa-$closed sets in generalized topological spaces

R. Jamunarani, *[1], P. Jeyanthi†[2] and M. Velrajan ‡[3]

[1,2]Research Center, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur-628 215, Tamil Nadu, India.

[3]Research Center, Department of Mathematics, Aditanar College of Arts and Science,, Tiruchendur - 628 216, Tamil Nadu, India

ABSTRACT

In this paper, we introduce $\lambda_\kappa-$closed sets and study its properties in generalized topological spaces.

Keyword: Generalized topology, $\mu-$open set, $\mu-$closed set, quasi-topology, strong space, $\Lambda_\kappa-$set, $\lambda_\kappa-$open set,$\lambda_\kappa-$closed set.

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1 Introduction

The theory of generalized topology was introduced by Császár in [1]. The properties of generalized topology, basic operators, generalized neighborhood systems and some con-

*Email:*jamunarani1977@gmail.com*

†E-mail: *jeyajeyanthi@rediffmail.com*

‡

structions for generalized topologies have been studied by the same author in [1, 2, 3, 4, 5, 6]. It is well known that generalized topology in the sense of Császár [1] is a generalization of topology on a nonempty set. On the other hand, many important collections of sets related with topology on a set form a generalized topology. In this paper we define several subsets in a generalized topological spaces and study their properties.
A nonempty family μ of subsets of a set X is said to be a *generalized topology* [2] if $\emptyset \in \mu$ and arbitrary union of elements of μ is again in μ. The pair (X, μ) is called a *generalized topological space* and elements of μ are called $\mu-$*open* sets. $A \subset X$ is $\mu-$*closed* if $X - A$ is $\mu-$open. By a space (X, μ), we always mean a generalized topological space. If $X \in \mu$, (X, μ) is called a *strong* [3] space. Clearly, (X, μ) is strong if and only if $\emptyset$ is $\mu-$closed if and only if $c_\mu(\emptyset) = \emptyset$. In a space (X, μ), if μ is closed under finite intersection, (X, μ) is called a *quasi-topological space* [5]. Clearly, every strong, quasi-topological space is a topological space. For $A \subset X$, $c_\mu(A)$ is the smallest $\mu-$closed set containing A and $i_\mu(A)$ is the largest $\mu-$open set contained in A. Moreover, $X - c_\mu(A) = i_\mu(X - A)$, for every subset A of X. A subset A of a space (X, μ) is said to be $\alpha-$open [4](resp. $\sigma-$open [4], $\pi-$open [4], b-open [7], $\beta-$open [4]) if $A \subset i_\mu c_\mu i_\mu(A)$ (resp. $A \subset c_\mu i_\mu(A)$, $A \subset i_\mu c_\mu(A)$, $A \subset i_\mu c_\mu(A) \cup c_\mu i_\mu(A)$, $A \subset c_\mu i_\mu c_\mu(A)$). A subset A of a space (X, μ) is said to be $\alpha-$closed (resp. $\sigma-$closed, $\pi-$closed, b-closed, $\beta-$closed) if $X - A$ is $\alpha-$open (resp. $\sigma-$open, $\pi-$open, b-open, $\beta-$open). Let (X, μ) be a space and $\zeta = \{\mu, \alpha, \sigma, \pi, b, \beta\}$. For $\kappa \in \zeta$, we consider the space (X, κ), throughout the paper. For $A \subset \mathcal{M}_\kappa = \cup\{B \subset X \mid B \in \mu\}$, the subset $\Lambda_\kappa(A)$ is defined by $\Lambda_\kappa(A) = \cap\{G \mid A \subset G, G \in \kappa\}$. The proof of the following lemma is clear.
Lemma 1.1. Let A, B and $B_\alpha, \alpha \in \triangle$ be subsets of $\mathcal{M}_\kappa$ in a space (X, κ). Then the following properties are hold.
(a) $B \subset \Lambda_\kappa(B)$.
(b) If $A \subset B$ then $\Lambda_\kappa(A) \subset \Lambda_\kappa(B)$.
(c) $\Lambda_\kappa(\Lambda_\kappa(B)) = \Lambda_\kappa(B)$.
(d) If $A \in k$, then $A = \Lambda_\kappa(A)$.
(e) $\Lambda_\kappa(\cup\{B_\alpha \mid \alpha \in \triangle\}) = \cup\{\Lambda_\kappa(B_\alpha) \mid \alpha \in \triangle\}$.
(f) $\Lambda_\kappa(\cap\{B_\alpha \mid \alpha \in \triangle\}) \subset \cap\{\Lambda_\kappa(B_\alpha) \mid \alpha \in \triangle\}$.

2 More on $\lambda_\kappa-$closed sets

In a space (X, κ), a subset B of $\mathcal{M}_\kappa$ is called a $\Lambda_\kappa-$*set* if $B = \Lambda_\kappa(B)$. We state the following theorem without proof.
Theorem 2.1. For subsets A and A_α, $\alpha \in \triangle$ of $\mathcal{M}_\kappa$ in a space (X, κ), the following hold.
(a) $\Lambda_\kappa(A)$ is a $\Lambda_\kappa-$set.
(b) If $A \in \kappa$, then A is a $\Lambda_\kappa-$set.

(c) If A_α is a $\Lambda_\kappa-$set for each $\alpha \in \triangle$, then $\cap\{A_\alpha \mid \alpha \in \triangle\}$ is a $\Lambda_\kappa-$set.
(d) If A_α is a $\Lambda_\kappa - set$ for each $\alpha \in \triangle$, then $\cup\{A_\alpha \mid \alpha \in \triangle\}$ is a $\Lambda_\kappa-$set.
A subset A of $\mathcal{M}_\kappa$ in a space (X, κ) is said to be a $\lambda_\kappa-$*closed* set if $A = T \cap C$, where T is a $\Lambda_\kappa-$set and C is a $\kappa-$closed set. The complement of a $\lambda_\kappa-$closed set is called a $\lambda_\kappa-$*open set*. We denote the collection of all $\lambda_\kappa-$open (resp., $\lambda_\kappa-$closed) set of X by $\lambda_\kappa O(X)$ (resp., $\lambda_\kappa C(X)$). The following theorem gives the characterization of $\lambda_\kappa-$closed sets.
Lemma 2.2. Let $A \subset \mathcal{M}_\kappa$ be a subset in a space (X, κ). Then the following are equivalent.
(a) A is a $\lambda_\kappa-$closed set.
(b)$A = T \cap c_\kappa(A)$, where T is a $\Lambda_\kappa-$set.
(c) $A = \Lambda_\kappa(A) \cap c_\kappa(A)$.
Let (X, κ) be a space. A point $x \in \mathcal{M}_\kappa$ is called a $\lambda_\kappa-$*cluster* point of A if for every $\lambda_\kappa-$open set U of $\mathcal{M}_\kappa$ containing x we have $A \cap U \neq \emptyset$. The set of all $\lambda_\kappa-$cluster points of A is called the $\lambda_\kappa-$*closure* of A and is denoted by $c_{\lambda_\kappa}(A)$.
Lemma 2.3 gives some properties of c_{λ_κ}, the easy proof of which is omitted.
Lemma 2.3. *Let (X, κ) be a space and $A, B \subset \mathcal{M}_\kappa$. Then the following properties hold.*
(a) $A \subset c_{\lambda_\kappa}(A)$.
(b) $c_{\lambda_\kappa}(A) = \cap\{F \mid A \subset F$ and F is $\lambda_\kappa - closed\}$.
(c) If $A \subset B$, then $c_{\lambda_\kappa}(A) \subset c_{\lambda_\kappa}(B)$.
(d) A is a $\lambda_\kappa-$closed set if and only if $A = c_{\lambda_\kappa}(A)$.
(e) $c_{\lambda_\kappa}(A)$ is a $\lambda_\kappa-$closed set.
Let (X, κ) be a space and $A \subset \mathcal{M}_\kappa$. A point $x \in \mathcal{M}_\kappa$ is said to be a $\kappa-$*limit point* of A if for each $\kappa-$open set U containing x, $U \cap \{A - \{x\}\} \neq \emptyset$. The set of all $\kappa-$ limit points of A is called a $\kappa-$derived set of A and is denoted by $D_\kappa(A)$.
Let (X, κ) be a space and $A \subset \mathcal{M}_\kappa$. A point $x \in \mathcal{M}_\kappa$ is said to be a $\lambda_\kappa-$*limit point* of A if for each $\lambda_\kappa-$open set U containing x, $U \cap \{A - \{x\}\} \neq \emptyset$. The set of all $\lambda_\kappa-$ limit points of A is called a $\lambda_\kappa-$derived set of A and is denoted by $D_{\lambda_\kappa}(A)$.
Theorem 2.4 gives some properties of $\lambda_\kappa-$derived sets and Theorem 2.5 gives the characterization of $\lambda_\kappa-$derived sets.
Theorem 2.4. Let (X, κ) be a space and $A, B \subset \mathcal{M}_\kappa$. Then the following hold.
(a) $D_{\lambda_\kappa}(A) \subset D_\kappa(A)$. (b) If $A \subset B$, then $D_{\lambda_\kappa}(A) \subset D_{\lambda_\kappa}(B)$.
(c) $D_{\lambda_\kappa}(A) \cup D_{\lambda_\kappa}(B) \subset D_{\lambda_\kappa}(A \cup B)$ and $D_{\lambda_\kappa}(A \cap B) \subset D_{\lambda_\kappa}(A) \cap D_{\lambda_\kappa}(B)$.
(d)$D_{\lambda_\kappa} D_{\lambda_\kappa}(A) - A \subset D_{\lambda_\kappa}(A)$.
(e) $D_{\lambda_\kappa}(A \cup D_{\lambda_\kappa}(A)) \subset A \cup D_{\lambda_\kappa}(A)$.
Proof. (a) Since every $\kappa-$open set is a $\lambda_\kappa-$open set, it follows.
(b) Let $x \in D_{\lambda_\kappa}(A)$. Let U be any $\lambda_\kappa-$open set containing x. Then $U \cap \{A - \{x\}\} \neq \emptyset$ and so $V \cap \{B - \{x\}\} \neq \emptyset$, since $A \subset B$. Therefore, $x \in D_{\lambda_\kappa}(B)$.
(c) Since $A \cap B \subset A, B$ we have $D_{\lambda_\kappa}(A \cap B) \subset D_{\lambda_\kappa}(A) \cap D_{\lambda_\kappa}(B)$. Since $A, B \subset A \cup B$,

we have $D_{\lambda_\kappa}(A) \cup D_{\lambda_\kappa}(B) \subset D_{\lambda_\kappa}(A \cup B)$.
(d) Let $x \in D_{\lambda_\kappa} D_{\lambda_\kappa}(A) - A$ and U be a $\lambda_\kappa-$open set containing x. Then $U \cap (D_{\lambda_\kappa}(A) - \{x\}) \neq \emptyset$. Let $y \in U \cap (D_{\lambda_\kappa}(A) - \{x\})$. Since $y \in D_{\lambda_\kappa}(A)$ and $x \neq y \in U$, $U \cap (A - \{y\}) \neq \emptyset$. Let $z \in U \cap (A - \{y\})$. Then $z \in U \cap (A - \{y\})$ implies that $z \in U$ and $z \in A - \{y\}$ and so $z \neq y$. Since $x \notin A$, $z \in U \cap (A - \{x\})$ and so $U \cap (A - \{x\}) \neq \emptyset$. Therefore, $x \in D_{\lambda_\kappa}(A)$.
(e) Let $x \in D_{\lambda_\kappa}(A \cup D_{\lambda_\kappa}(A))$. If $x \in A$, the result is clear. Suppose $x \notin A$. Since $x \in D_{\lambda_\kappa}(A \cup D_{\lambda_\kappa}(A)) - A$, then for $\lambda_\kappa-$open set U containing x, $U \cap ((A \cup D_{\lambda_\kappa}(A)) - \{x\}) \neq \emptyset$. Thus $U \cap (A - \{x\}) \neq \emptyset$ or $U \cap (D_{\lambda_\kappa}(A) - \{x\}) \neq \emptyset$. Now it follows from (d) that $U \cap (A - \{x\}) \neq \emptyset$. Hence, $x \in D_{\lambda_\kappa}(A)$. Therefore, in all the cases $D_{\lambda_\kappa}(A \cup D_{\lambda_\kappa}(A)) \subset A \cup D_{\lambda_\kappa}(A)$.
Theorem 2.5. Let (X, κ) be space and $A \subset X$. Then $c_{\lambda_\kappa}(A) = A \cup D_{\lambda_\kappa}(A)$.
Proof. Since $D_{\lambda_\kappa}(A) \subset c_{\lambda_\kappa}(A)$, $A \cup D_{\lambda_\kappa}(A) \subset c_{\lambda_\kappa}(A)$. On the other hand, let $x \in c_{\lambda_\kappa}(A)$. If $x \in A$, the proof is complete. If $x \notin A$, then each $\lambda_\kappa-$open set U containing x intersects A at a point distinct from x. Therefore, $x \in D_{\lambda_\kappa}(A)$. Thus, $c_{\lambda_\kappa}(A) \subset A \cup D_{\lambda_\kappa}(A)$ and so $c_{\lambda_\kappa}(A) = A \cup D_{\lambda_\kappa}(A)$ which completes the proof.
Let (X, κ) be a space and $A \subset X$. Then $i_{\lambda_\kappa}(A)$ is the union of all $\lambda_\kappa-$open set contained in A.
Theorem 2.6 gives some properties of i_{λ_κ}.
Theorem 2.6. Let (X, κ) be a space and $A, B \subset X$. Then the following hold.
(a) A is a $\lambda_\kappa-$open set if and only if $A = i_{\lambda_\kappa}(A)$.
(b) $i_{\lambda_\kappa}(i_{\lambda_\kappa}(A)) = i_{\lambda_\kappa}(A)$.
(c) $i_{\lambda_\kappa}(A) = A - D_{\lambda_\kappa}(X - A)$.
(d) $X - i_{\lambda_\kappa}(A) = c_{\lambda_\kappa}(X - A)$.
(e) $X - c_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(X - A)$.
(f) $A \subset B$ then $i_{\lambda_\kappa}(A) \subset i_{\lambda_\kappa}(B)$.
(g) $i_{\lambda_\kappa}(A) \cup i_{\lambda_\kappa}(B) \subset i_{\lambda_\kappa}(A \cup B)$ and $i_{\lambda_\kappa}(A) \cap i_{\lambda_\kappa}(B) \supset i_{\lambda_\kappa}(A \cap B)$.
Proof. (c) If $x \in A - D_{\lambda_\kappa}(X - A)$, then $x \notin D_{\lambda_\kappa}(X - A)$ and so, there exists a $\lambda_\kappa-$open set U containing x such that $U \cap (X - A) = \emptyset$. Then $x \in U \subset A$ and hence $x \in i_{\lambda_\kappa}(A)$. That is, $A - D_{\lambda_\kappa}(X - A) \subset i_{\lambda_\kappa}(A)$. On the other hand, if $x \in i_{\lambda_\kappa}(A)$, then $x \notin D_{\lambda_\kappa}(X - A)$, since $i_{\lambda_\kappa}(A)$ is a $\lambda_\kappa-$open set and $i_{\lambda_\kappa}(A) \cap (X - A) = \emptyset$. Hence, $i_{\lambda_\kappa}(A) = A - D_{\lambda_\kappa}(X - A)$.
(d) $X - i_{\lambda_\kappa}(A) = X - (A - D_{\lambda_\kappa}(X - A)) = (X - A) \cup D_{\lambda_\kappa}(X - A) = c_{\lambda_\kappa}(X - A)$.
Let (X, κ) be a space and $A \subset X$. Then $b_\kappa(A) = A - i_\kappa(A)$ is said to be $\kappa-$*border* of A.
Let (X, κ) be a space and $A \subset X$. Then $b_{\lambda_\kappa}(A) = A - i_{\lambda_\kappa}(A)$ is said to be $\lambda_\kappa-$*border* of A.
Theorem 2.7 gives some properties of b_{λ_κ}.
Theorem 2.7. Let (X, κ) be a space and $A \subset X$. Then the following hold.
(a) $b_{\lambda_\kappa}(A) \subset b_\kappa(A)$.
(b) $A = i_{\lambda_\kappa}(A) \cup b_{\lambda_\kappa}(A)$.
(c) $i_{\lambda_\kappa}(A) \cap b_{\lambda_\kappa}(A) = \emptyset$.

(d) A is a $\lambda_\kappa-$open set if and only if $b_{\lambda_\kappa}(A) = \emptyset$.
(e) $b_{\lambda_\kappa}(i_{\lambda_\kappa}(A)) = \emptyset$.
(f) $i_{\lambda_\kappa}(b_{\lambda_\kappa}(A)) = \emptyset$.
(g) $b_{\lambda_\kappa}(b_{\lambda_\kappa}(A)) = b_{\lambda_\kappa}(A)$.
(h) $b_{\lambda_\kappa}(A) = A \cap c_{\lambda_\kappa}(X - A)$.
(i) $b_{\lambda_\kappa}(A) = D_{\lambda_\kappa}(X - A)$.
Proof. (f) If $x \in i_{\lambda_\kappa}(b_{\lambda_\kappa}(A))$, then $x \in b_{\lambda_\kappa}(A)$. On the other hand, since $b_{\lambda_\kappa}(A) \subset A$, $x \in i_{\lambda_\kappa}(b_{\lambda_\kappa}(A)) \subset i_{\lambda_\kappa}(A)$. Hence $x \in i_{\lambda_\kappa}(A) \cap b_{\lambda_\kappa}(A)$ which contradicts (c). Thus, $i_{\lambda_\kappa}(b_{\lambda_\kappa}(A)) = \emptyset$.
(h) $b_{\lambda_\kappa}(A) = A - i_{\lambda_\kappa}(A) = A - (X - c_{\lambda_\kappa}(X - A)) = A \cap c_{\lambda_\kappa}(X - A)$.
(i) $b_{\lambda_\kappa}(A) = A - i_{\lambda_\kappa}(A) = A - (A - D_{\lambda_\kappa}(X - A)) = D_{\lambda_\kappa}(X - A)$.
Let (X, κ) be a space and $A \subset X$. Then $F_\kappa(A) = c_\kappa(A) - i_\kappa(A)$ is said to be the $\kappa-$*frontier* of A.
Let (X, κ) be a space and $A \subset X$. Then $F_{\lambda_\kappa}(A) = c_{\lambda_\kappa(A)} - i_{\lambda_\kappa}(A)$ is said to be the $\lambda_\kappa-$*frontier* of A.
Theorem 2.8 gives some properties of F_{λ_κ}.
Theorem 2.18 Let (X, κ) be a space and $A \subset X$. Then the following hold.
(a) $F_{\lambda_\kappa}(A) \subset F_\kappa(A)$.
(b) $c_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(A) \cup F_{\lambda_\kappa}(A)$.
(c) $i_{\lambda_\kappa}(A) \cap F_{\lambda_\kappa}(A) = \emptyset$.
(d) $b_{\lambda_\kappa}(A) \subset F_{\lambda_\kappa}(A)$.
(e) $F_{\lambda_\kappa}(A) = b_{\lambda_\kappa}(A) \cup D_{\lambda_\kappa}(A)$.
(f) A is a $\lambda_\kappa-$open set if and only if $F_{\lambda_\kappa}(A) = D_{\lambda_\kappa}(A)$.
(g) $F_{\lambda_\kappa}(A) = c_{\lambda_\kappa}(A) \cap c_{\lambda_\kappa}(X - A)$.(h) $F_{\lambda_\kappa}(A) = F_{\lambda_\kappa}(X - A)$.
(i) $F_{\lambda_\kappa}(A)$ is a $\lambda_\kappa-$closed set.
(j) $F_{\lambda_\kappa}(F_{\lambda_\kappa}(A)) \subset F_{\lambda_\kappa}(A)$.
(k) $F_{\lambda_\kappa}(i_{\lambda_\kappa}(A)) \subset F_{\lambda_\kappa}(A)$.
(l) $F_{\lambda_\kappa}(c_{\lambda_\kappa}(A)) \subset F_{\lambda_\kappa}(A)$.
(m) $i_{\lambda_\kappa}(A) = A - F_{\lambda_\kappa}(A)$.
Proof. (b) $i_{\lambda_\kappa}(A) \cup F_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(A) \cup (c_{\lambda_\kappa}(A) - i_{\lambda_\kappa}(A)) = c_{\lambda_\kappa}(A)$.
(c) $i_{\lambda_\kappa}(A) \cap F_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(A) \cap (c_{\lambda_\kappa}(A) - i_{\lambda_\kappa}(A)) = \emptyset$.
(e) Since $i_{\lambda_\kappa}(A) \cup F_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(A) \cup b_{\lambda_\kappa}(A) \cup D_{\lambda_\kappa}(A)$, $F_{\lambda_\kappa}(A) = b_{\lambda_\kappa}(A) \cup D_{\lambda_\kappa}(A)$.
(g) $F_{\lambda_\kappa}(A) = c_{\lambda_\kappa}(A) - i_{\lambda_\kappa}(A) = c_{\lambda_\kappa}(A) \cap c_{\lambda_\kappa}(X - A)$.
(i)$c_{\lambda_\kappa}(F_{\lambda_\kappa}(A)) = c_{\lambda_\kappa}(c_{\lambda_\kappa}(A) \cap c_{\lambda_\kappa}(X - A)) \subset c_{\lambda_\kappa}(c_{\lambda_\kappa}(A)) \cap c_{\lambda_\kappa}(c_{\lambda_\kappa}(X - A)) = F_{\lambda_\kappa}(A)$. Hence $F_{\lambda_\kappa}(A)$ is a $\lambda_\kappa-$closed set.
(j) $F_{\lambda_\kappa}(F_{\lambda_\kappa}(A)) = c_{\lambda_\kappa}(F_{\lambda_\kappa}(A) \cap c_{\lambda_\kappa}(X - F_{\lambda_\kappa}(A)) \subset c_{\lambda_\kappa}(F_{\lambda_\kappa}(A)) = F_{\lambda_\kappa}(A)$.
(l) $F_{\lambda_\kappa}(c_{\lambda_\kappa}(A)) = c_{\lambda_\kappa}((c_{\lambda_\kappa}(A)) - i_{\lambda_\kappa}(c_{\lambda_\kappa}(A)) = c_{\lambda_\kappa}(A) - i_{\lambda_\kappa}(c_{\lambda_\kappa}(A)) \subset c_{\lambda_\kappa}(A) - i_{\lambda_\kappa}(A) = F_{\lambda_\kappa}(A)$.
(m) $A - F_{\lambda_\kappa}(A) = A - (c_{\lambda_\kappa}(A) - i_{\lambda_\kappa}(A)) = i_{\lambda_\kappa}(A)$.

Let (X, κ) be a space and $A \subset X$. Then $E_\kappa(A) = i_\kappa(X - A)$ is said to be $\kappa-$exterior of A.
Let (X, κ) be a space and $A \subset X$. Then $E_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(X - A)$ is said to be $\lambda_\kappa-$exterior of A.
Theorem 2.9 gives some properties of E_{λ_κ}.
Theorem 2.9. Let (X, κ) be a space and $A \subset X$. Then the following hold.
(a) $E_\kappa(A) \subset E_{\lambda_\kappa}(A)$ where $E_\kappa(A)$ denotes the exterior of A.
(b) $E_{\lambda_\kappa}(A)$ is $\lambda_\kappa-$open.
(c) $E_{\lambda_\kappa}(A) = i_{\lambda_\kappa}(X - A) = X - c_{\lambda_\kappa}(A)$.
(d) $E_{\lambda_\kappa}(E_{\lambda_\kappa}(A)) = i_{\lambda_\kappa}(c_{\lambda_\kappa}(A))$.
(e) If $A \subset B$,then $E_{\lambda_\kappa}(A) \supset E_{\lambda_\kappa}(B)$.
(f) $E_{\lambda_\kappa}(A \cup B) \subset E_{\lambda_\kappa}(A) \cup E_{\lambda_\kappa}(B)$.
(g)$E_{\lambda_\kappa}(A \cup B) \supset E_{\lambda_\kappa}(A) \cap E_{\lambda_\kappa}(B)$.
(h) $E_{\lambda_\kappa}(X) = \emptyset$.
(i) $E_{\lambda_\kappa}(\emptyset) = X$.
(j) $E_{\lambda_\kappa}(A) = E_{\lambda_\kappa}(X - E_{\lambda_\kappa}(A))$.
(k) $i_{\lambda_\kappa}(A) \subset E_{\lambda_\kappa}(E_{\lambda_\kappa}(A))$.
(l) $X = i_{\lambda_\kappa}(A) \cup E_{\lambda_\kappa}(A) \cup F_{\lambda_\kappa}(A)$.
Proof. (d)$E_{\lambda_\kappa}(E_{\lambda_\kappa}(A)) = E_{\lambda_\kappa}(X - c_{\lambda_\kappa}(A)) = i_{\lambda_\kappa}(X - (X - x_{\lambda_\kappa}(A)) = i_\lambda(c_\lambda(A))$.
(j)$E_{\lambda_\kappa}(X - E_{\lambda_\kappa}(A)) = E_{\lambda_\kappa}(X - i_{\lambda_\kappa}(X - A)) = i_{\lambda_\kappa}(X - (X - i_{\lambda_\kappa}(X - A))) = i_{\lambda_\kappa}(i_{\lambda_\kappa}(X - A)) = i_{\lambda_\kappa}(X - A) = E_\lambda(A)$.
(k) $i_{\lambda_\kappa}(A) \subset i_{\lambda_\kappa}(c_{\lambda_\kappa}(A)) = i_{\lambda_\kappa}(X - i_{\lambda_\kappa}(X - A)) = i_{\lambda_\kappa}(X - E_{\lambda_\kappa}(A)) = E_{\lambda_\kappa}(E_{\lambda_\kappa}(A))$.

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