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Totally magic cordial labeling of some graphs

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ABSTRACT

A graph G is said to have a totally magic cordial labeling with constant C if there exists a mapping $f : V(G) \cup E(G) \to \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C \pmod 2$ for all $ab \in E(G)$ and $|n_f(0) - n_f(1)| \leq 1$, where $n_f(i)$ $(i = 0,1)$ is the sum of the number of vertices and edges with label i. In this paper, we give a necessary condition for an odd graph to be not totally magic cordial and also prove that some families of graphs admit totally magic cordial labeling.

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1 Introduction

All graphs in this paper are finite, simple and undirected. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$ and we write p for $|V|$ and q for $|E|$. A general reference for graph theoretic notions is [3]. The concept of cordial labeling was introduced by Cahit [1]. A binary vertex labeling $f : V(G) \to \{0,1\}$ induces an edge labeling

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$f^* : E(G) \to \{0,1\}$ defined by $f^*(uv) = |f(u) - f(v)|$. Such labeling is called cordial if the conditions $|v_f(0) - v_f(1)| \le 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \le 1$ are satisfied, where $v_f(i)$ and $e_{f^*}(i)(i = 0,1)$ are the number of vertices and edges with label i, respectively. A graph is called cordial if it admits a cordial labeling.
Kotzig and Rosa introduced the concept of edge-magic total labeling in [6]. A bijection $f : V(G) \cup E(G) \to \{1,2,3,...,p+q\}$ is called an edge-magic total labeling of G if $f(x) + f(xy) + f(y)$ is constant (called the magic constant of f) for every edge xy of G. The graph that admits this labeling is called an edge-magic total graph.
The notion of totally magic cordial (TMC) labeling is due to Cahit [2] as a modification of edge-magic total labeling and cordial labeling. A graph G is said to have totally magic cordial labeling with constant C if there exists a mapping $f : V(G) \cup E(G) \to \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C \pmod 2$ for all $ab \in E(G)$ and $|n_f(0) - n_f(1)| \le 1$, where $n_f(i)$ $(i = 0,1)$ is the sum of the number of vertices and edges with label i. A graph that admits a TMC labeling is called a TMC graph.
In [4], it was proved that the complete graph K_n is TMC if and only if
$\sqrt{4k+1}$ has an integer value when $n = 4k$
$\sqrt{k+1}$ or $\sqrt{k}$ has an integer value when $n = 4k+1$
$\sqrt{4k+5}$ or $\sqrt{4k+1}$ has an integer value when $n = 4k+2$
$\sqrt{k+1}$ has an integer value when $n = 4k+3$. Also it was proved that all trees, cycles $(n \ge 3)$, friendship graph, flower graph and ladder graph $L_n (n \ge 2)$ are TMC.
In [5], totally magic cordial labeling of one-point union of n-copies of cycles, complete graphs and wheels were established.
An odd graph is a graph whose vertices are of odd degree. An odd graph must have an even number of vertices.
We use the following definitions in the subsequent section:

Definition 1.1. *A wheel graph W_n is obtained from a cycle C_n by adding a new vertex and joining it to all the vertices of the cycle by an edge, then the new edges are called spokes of the wheel.*

Definition 1.2. *Ladder graph $L_n (n \ge 2)$ is a product graph $P_2 \times P_n$ with $2n$ vertices and $3n-2$ edges.*

Definition 1.3. *A fan graph F_n is obtained from a path P_n by adding a new vertex and joining it to all the vertices of the path by an edge.*

Definition 1.4. *The graph mW_n is the disjoint union of m copies of W_n.*

Definition 1.5. *The join of graphs G_1 and G_2 is a graph $G_1 + G_2$, with vertex set $V(G_1) \cup V(G_2)$ and edge set consisting of edges of G_1 and G_2 and all the edges joining $V(G_1)$ and $V(G_2)$.*

Definition 1.6. *The Corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.*

2 Main Results

In this section, we give a necessary condition for an odd graph to be not totally magic cordial and also prove that some families of graphs admit totally magic cordial labeling.

Theorem 2.1. *Let $G_1(p_1, q_1)$, $G_2(p_2, q_2)$ be two TMC graphs with $C = 0$. If $p_1 + q_1$ and $p_2 + q_2$ are even and $|p_i - 2m_i| \leq 1$, where m_i is the number of vertex labeled with 0 in G_i, $i = 1, 2$, then $G_1 + G_2$ is TMC.*

Proof. Let f and g be TMC labelings of G_1 and G_2 respectively with $C = 0$. Assume that $p_1 + q_1 = 2m$ and $p_2 + q_2 = 2n$. Then $n_f(0) = n_f(1) = m$ and $n_g(0) = n_g(1) = n$. Define $h : V(G_1 + G_2) \cup E(G_1 + G_2) \to \{0, 1\}$ as follows: For $v \in V(G_1 + G_2)$, $h(v) = \begin{cases} f(v) & \text{if } v \in V(G_1), \\ g(v) & \text{if } v \in V(G_2) \end{cases}$ and for $uv \in E(G_1 + G_2)$,

$$h(uv) = \begin{cases} f(uv) & \text{if} \quad uv \in E(G_1), \\ g(uv) & \text{if} \quad uv \in E(G_2), \\ 0 & \text{if} \quad f(u) = 0 \text{ and } g(v) = 0 \quad \text{or } f(u) = 1 \text{ and } g(v) = 1, \\ 1 & \text{if} \quad f(u) = 0 \text{ and } g(v) = 1 \quad \text{or } f(u) = 1 \text{ and } g(v) = 0. \end{cases}$$

Now $n_h(0) = n_f(0) + n_g(0) + m_1m_2 + (p_1 - m_1)(p_2 - m_2)$ and $n_h(1) = n_f(1) + n_g(1) + m_1(p_2 - m_2) + m_2(p_1 - m_1)$. Therefore, $|n_h(0) - n_h(1)| \leq |n_f(0) - n_f(1)| + |n_g(0) - n_g(1)| + |(p_1 - 2m_1)(p_2 - 2m_2)|$, implies that
$|n_h(0) - n_h(1)| \leq |(p_1 - 2m_1)|\,|(p_2 - 2m_2)|$. Thus, $|n_h(0) - n_h(1)| \leq 1$ whenever $|p_1 - 2m_1| \leq 1$ and $|p_2 - 2m_2| \leq 1$. Therefore, h is a TMC labeling of $G_1 + G_2$ and hence, $G_1 + G_2$ is TMC. □

Corollary 2.2. *If $G_i(p_i, q_i)$, $i = 1, 2, 3, ..., n$ are TMC graphs with $C = 0$ such that $p_i + q_i$, $i = 1, 2, 3, ..., n$ are even,and $|p_i - 2m_i| \leq 1$, where m_i is the number of vertices labeled with 0 in G_i, $i = 1, 2, ..., n$, then $G_1 + G_2 + ... + G_n$ is TMC.*

Theorem 2.3. *If G is an edge magic total graph, then G is TMC.*

Proof. Let f be an edge magic total labeling of a graph G with p vertices and q edges. Define $g : V(G) \cup E(G) \to \{0, 1\}$ by $g(v) \equiv f(v) \pmod 2$ if $v \in V(G)$ and $g(e) \equiv f(e) \pmod 2$ if $e \in E(G)$. Since there are exactly $\lceil \frac{p+q}{2} \rceil$ odd integers and $\lfloor \frac{p+q}{2} \rfloor$ even integers in the set $\{1, 2, 3, ..., p + q\}$ we have, $|n_f(0) - n_f(1)| \leq 1$. Therefore, G is TMC. □

Theorem 2.4. *Let G be an odd graph with $p + q \equiv 2$ (mod 4). Then G is not TMC.*

Proof. Assume that G is TMC with $C = 0$ or 1 and let f be a TMC labeling of G. Thus, for any edge $ab \in E(G)$, $f(a)+f(b)+f(ab) \equiv C \pmod 2$ and $|n_f(0) - n_f(1)| \leq 1$. As there are an even number of edges, summing over all the edges we get, $\sum_{a\in V(G)} deg\,(a)\, f\,(a) + \sum_{ab\in E(G)} f\,(ab) \equiv 0 \pmod 2$. Since degree of each vertex is odd, $n_f(1) = \sum_{a\in V(G)} f\,(a) + \sum_{ab\in E(G)} f\,(ab) \equiv 0 \pmod 2$. Also, since $|n_f(0) - n_f(1)| \leq 1$, we cannot have $n_f(0)+n_f(1) = p + q \equiv 2 \pmod 4$. □

Theorem 2.5. *The fan graph F_n is TMC for $n \geq 2$.*

Proof. Let $V(F_n) = \{u, v_i | 1 \leq i \leq n\}$ and $E(F_n) = \{v_iv_{i+1} | 1 \leq i < n\} \cup \{uv_i | 1 \leq i \leq n\}$. Define $f : V(F_n) \cup E(F_n) \to \{0, 1\}$ as follows:
$f(u) = 0$, $f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod 4, \\ 1 & \text{if } i \equiv 0, 3 \pmod 4, \end{cases}$ $f(v_iv_{i+1}) = \begin{cases} 1 & \text{if } i \text{ is odd,} \\ 0 & \text{if } i \text{ is even} \end{cases}$ and $f(uv_i) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4. \end{cases}$ Clearly, $n_f(0) = n_f(1)$ if n is even and $n_f(0) = n_f(1) + 1$ if n is odd. Hence, the fan graph F_n for $n \geq 2$ is TMC with $C = 1$. □

Theorem 2.6. *The wheel graph $W_n (n \geq 3)$ is TMC if and only if $n \not\equiv 3 (mod\ 4)$.*

Proof. Let $V(W_n) = \{u, v_i | 1 \leq i \leq n\}$ and $E(W_n) = \{v_iv_{i+1} | 1 \leq i < n\} \cup \{uv_i | 1 \leq i \leq n\} \cup \{v_nv_1\}$. Clearly, $p = |V(W_n)| = n + 1$ and $q = |E(W_n)| = 2n$ so that $p + q = 3n + 1$. Necessity follows from Theorem 5 and for sufficiency we assume that $n \not\equiv 3 \pmod 4$. Define $f : V(W_n) \cup E(W_n) \to \{0, 1\}$ as follows:
$f(u) = 0$, $f(v_i) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod 4, \\ 1 & \text{if } i \equiv 0, 3 \pmod 4, \end{cases}$ $f(v_iv_{i+1}) = \begin{cases} 1 & \text{if } i \text{ is odd,} \\ 0 & \text{if } i \text{ is even,} \end{cases}$ $f(uv_i) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4, \end{cases}$ and $f(v_nv_1) = \begin{cases} 0 & \text{if } n \equiv 0 \pmod 4, \\ 1 & \text{if } n \equiv 1, 2 \pmod 4. \end{cases}$ Clearly,

$$\begin{aligned} n_f(0) &= n_f(1) + 1 \quad \text{if} \quad n \equiv 0 \pmod 4, \\ n_f(0) &= n_f(1) \quad \text{if} \quad n \equiv 1 \pmod 4 \\ \text{and} \quad n_f(0) &= n_f(1) - 1 \quad \text{if} \quad n \equiv 2 \pmod 4. \end{aligned}$$

Thus, f is a TMC labeling of W_n with $C = 1$. □

Theorem 2.7. *The graph mW_{4t+3} is TMC if and only if m is even.*

Proof. Let $G = mW_{4t+3}$ and $n = 4t + 3$. Let $V(G) = \{u_j, v_i^j | 1 \leq i \leq n$ and $1 \leq j \leq m\}$ and $E(G) = \{u_jv_i^j | 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{v_i^jv_{i+1}^j | 1 \leq i < n, 1 \leq j \leq m\} \cup \{v_n^jv_1^j | 1 \leq j \leq m\}$. Clearly, $p = |V(G)| = m(n + 1)$ and $q = |E(G)| = 2mn$ so that $p+q = m(3n+1)$. Necessity follows from Theorem 5 and for sufficiency we assume that m is even. Define $f : V(G) \cup E(G) \to \{0, 1\}$ as follows:
Case i. $j \equiv 1 \pmod 2$.

$f(u_j) = f(v_n^j v_1^j) = 0,\ f(v_i^j) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod 4, \\ 1 & \text{if } i \equiv 0, 3 \pmod 4, \end{cases}$

$f(u_j v_i^j) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4 \end{cases}$ and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \text{ is odd}, \\ 0 & \text{if } i \text{ is even}. \end{cases}$

Case ii. $j \equiv 0 \pmod 2$.

$f(u_j) = 0,\ f(v_n^j v_1^j) = 1,\ f(v_i^j) = \begin{cases} 0 & \text{if } i \not\equiv 0 \pmod 4, \\ 1 & \text{if } i \equiv 0 \pmod 4, \end{cases}$

$f(u_j v_i^j) = \begin{cases} 1 & \text{if } i \not\equiv 0 \pmod 4, \\ 0 & \text{if } i \equiv 0 \pmod 4 \end{cases}$ and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4. \end{cases}$

Clearly, $n_f(0) = n_f(1)$. Hence, mW_{4t+3} is TMC with $C = 1$. □

Theorem 2.8. *The graph mW_n is TMC if $n \not\equiv 3 \pmod 4$ and $m \geq 1$.*

Proof. Let $G = mW_n$. Let $V(G) = \left\{u_j, v_i^j | 1 \leq i \leq n \text{ and } 1 \leq j \leq m\right\}$ and $E(G) = \left\{u_j v_i^j | 1 \leq i \leq n, 1 \leq j \leq m\right\} \cup \left\{v_i^j v_{i+1}^j | 1 \leq i < n, 1 \leq j \leq m\right\}$ $\cup \left\{v_n^j v_1^j | 1 \leq j \leq m\right\}$. Define $f : V(G) \cup E(G) \to \{0, 1\}$ as follows:

Case i. $n \equiv 0 \pmod 4$.

Subcase i. $j \equiv 1 \pmod 2$.

$f(u_j) = 0,\ f(v_n^j v_1^j) = 0,\ f(v_i^j) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod 4, \\ 1 & \text{if } i \equiv 0, 3 \pmod 4, \end{cases}$

$f(u_j v_i^j) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4 \end{cases}$ and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \text{ is odd}, \\ 0 & \text{if } i \text{ is even}. \end{cases}$

Subcase ii. $j \equiv 0 \pmod 2$.

$f(u_j) = 1,\ f(v_n^j v_1^j) = 0,\ f(v_i^j) = f(u_j v_i^j) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4 \end{cases}$

and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \text{ is odd}, \\ 0 & \text{if } i \text{ is even}. \end{cases}$

Thus, $n_f(0) = n_f(1)$ if m is even
and $n_f(0) = n_f(1) + 1$ if m is odd.

Case ii. $n \equiv 1 \pmod 4$.

$f(u_j) = 0,\ f(v_n^j v_1^j) = 1,\ f(v_i^j) = \begin{cases} 0 & \text{if } i \equiv 1, 2 \pmod 4, \\ 1 & \text{if } i \equiv 0, 3 \pmod 4, \end{cases}$

$f(u_j v_i^j) = \begin{cases} 1 & \text{if } i \equiv 1, 2 \pmod 4, \\ 0 & \text{if } i \equiv 0, 3 \pmod 4 \end{cases}$

and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \text{ is odd}, \\ 0 & \text{if } i \text{ is even}. \end{cases}$

Clearly, $n_f(0) = n_f(1)$.

Case iii. $n \equiv 2 \pmod 4$.
Subcase i. $j \equiv 1 \pmod 2$.

$f(u_j) = 0,\ f(v_n^j v_1^j) = 1,\ f(v_i^j) = \begin{cases} 0 & \text{if } i \equiv 1,2 \pmod 4, \\ 1 & \text{if } i \equiv 0,3 \pmod 4, \end{cases}$
$f(u_j v_i^j) = \begin{cases} 1 & \text{if } i \equiv 1,2 \pmod 4, \\ 0 & \text{if } i \equiv 0,3 \pmod 4 \end{cases}$
and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \text{ is odd}, \\ 0 & \text{if } i \text{ is even}. \end{cases}$
Subcase ii. $j \equiv 0 \pmod 2$.

$f(u_j) = f(v_n^j v_1^j) = 1,\ f(v_i^j) = f(u_j v_i^j) = \begin{cases} 0 & \text{if } i \equiv 1,2 \pmod 4, \\ 1 & \text{if } i \equiv 0,3 \pmod 4 \end{cases}$
and $f(v_i^j v_{i+1}^j) = \begin{cases} 1 & \text{if } i \text{ is odd}, \\ 0 & \text{if } i \text{ is even}. \end{cases}$
Clearly,
$n_f(0) = n_f(1)$ if m is even
and $n_f(1) = n_f(0) + 1$ if m is odd.
Hence, mW_n is TMC with $C = 1$. □

Theorem 2.9. *The graph $C_n + \overline{K}_{2m+1}$ is TMC if and only if $n \not\equiv 3 \pmod 4$.*

Proof. Let $u_1, u_2, ..., u_n$ be the vertices of C_n and $v_1, v_2, ..., v_{2m+1}$ be the vertices of $\overline{K}_{2m+1}$. Let $G = C_n + \overline{K}_{2m+1}$. Clearly, $p = |V(G)| = n + 2m + 1$ and $q = |E(G)| = 2n(m + 1)$ so that $p + q = 2n + (n + 1)(2m + 1)$. Necessity follows from Theorem 5. For sufficiency, assume that $n \not\equiv 3 \pmod 4$. Define $f : V(G) \cup E(G) \to \{0, 1\}$ as follows:
$f(u_i) = \begin{cases} 0 & \text{if } i \equiv 0 \pmod 4, \\ 1 & \text{if } i \not\equiv 0 \pmod 4, \end{cases}$ $f(v_j) = \begin{cases} 0 & \text{if } 1 \le j \le m+1, \\ 1 & \text{if } m+1 < j \le 2m+1, \end{cases}$
$f(u_i u_{i+1}) = \begin{cases} 0 & \text{if } i \equiv 0,3 \pmod 4, \\ 1 & \text{elsewhere}, \end{cases}$ $f(u_n u_1) = \begin{cases} 0 & \text{if } n \equiv 0 \pmod 4, \\ 1 & \text{elsewhere}, \end{cases}$
and $f(u_i v_j) = \begin{cases} 0 & \text{if } i \equiv 0 \pmod 4 \text{ and } m+1 < j \le 2m+1 \\ & \text{or } i \not\equiv 0 \pmod 4 \text{ and } 1 \le j \le m+1, \\ 1 & \text{if } i \equiv 0 \pmod 4 \text{ and } 1 \le j \le m+1 \\ & \text{or } i \not\equiv 0 \pmod 4 \text{ and } m+1 < j \le 2m+1. \end{cases}$
Thus,
$n_f(0) = n_f(1) + 1$ if $n \equiv 0 \pmod 4$,
$n_f(0) = n_f(1)$ if $n \equiv 1 \pmod 4$
and $n_f(0) = n_f(1) - 1$ if $n \equiv 2 \pmod 4$.
Hence, G is TMC with $C = 1$. □

Theorem 2.10. *The graph $C_{2n+1} \odot \overline{K}_m$ is TMC if and only if m is odd.*

Proof. Let $G = C_{2n+1} \odot \overline{K}_m$. Necessity follows from Theorem 5. For sufficiency, assume that m is odd. If we assign 0 to all the edges of G and 1 to all the vertices of G then we get $C = 0$. If we assign 1 to all the edges of G and 0 to all the vertices of G then we get $C = 1$. In either case, $|n_f(0) - n_f(1)| = 0$. Clearly, G is TMC. □

Theorem 2.11. *The disjoint union of $K_{1,m}$ and $K_{1,n}$ is TMC if and only if m or n is even.*

Proof. Let $G = K_{1,m} \cup K_{1,n}$. Let c_1 and c_2 be the central vertices of $K_{1,m}$ and $K_{1,n}$ respectively. Let $u_1, u_2, ..., u_m$ be the pendant vertices of $K_{1,m}$ and $v_1, v_2, ..., v_n$ be those of $K_{1,n}$. Clearly, $p = |V(G)| = m + n + 2$ and $q = |E(G)| = m + n$ so that $p + q = 2(m + n + 1)$. Necessity follows from Theorem 1. For sufficiency, assume m is even. Define $f : V(G) \cup E(G) \to \{0, 1\}$ as follows:
Case i. n is even.

$f(c_1) = 0$, $f(c_2) = 1$, $f(u_i) = 1$, $f(c_1u_i) = 0$ for $1 \le i \le m$ and $f(v_j) = f(c_2v_j) = \begin{cases} 0 \text{ if } & 1 \le j \le \frac{n}{2}, \\ 1 \text{ if } & \frac{n}{2} < j \le n. \end{cases}$

Case ii. n is odd.

$$f(c_1) = f(c_2) = 1, f(u_i) = f(c_1u_i) = \begin{cases} 1 \text{ if } & 1 \le i \le \frac{m}{2}, \\ 0 \text{ if } & \frac{m}{2} < i \le m. \end{cases}$$

$$f(v_j) = f(c_2v_j) = \begin{cases} 1 \text{ if } & 1 \le j \le \frac{n-1}{2}, \\ 0 \text{ if } & \frac{n-1}{2} < j \le n. \end{cases}$$

Clearly, $n_f(0) = n_f(1)$. Thus, G is TMC with $C = 1$. □

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