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Vertex Equitable Labeling of Double Alternate Snake Graphs

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ABSTRACT

Let G be a graph with p vertices and q edges and $A = \left\{0, 1, 2, \ldots, \left\lceil \frac{q}{2} \right\rceil\right\}$. A vertex labeling $f : V(G) \to A$ induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv. For $a \in A$, let $v_f(a)$ be the number of vertices v with $f(v) = a$. A graph G is said to be vertex equitable if there exists a vertex labeling f such that for all a and b in $A, |v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, \ldots, q$. In this paper, we prove that $DA(T_n) \odot K_1, DA(T_n) \odot 2K_1 (DA(T_n)$ denote double alternate triangular snake) and $DA(Q_n) \odot K_1, DA(Q_n) \odot 2K_1 (DA(Q_n)$ denote double alternate quadrilateral snake) are vertex equitable graphs.

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1 Introduction

All graphs considered here are simple, finite, connected and undirected. We follow the basic notations and terminology of graph theory as in [1]. There are several types of labeling and a detailed survey of graph labeling is found in [2]. The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan [3]. Let G be a graph with p vertices and q edges and $A = \left\{0, 1, 2, \ldots, \left\lceil \frac{q}{2} \right\rceil \right\}$. A graph G is said to be vertex equitable if there exists a vertex labeling $f : V(G) \to A$ that indcues an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, \ldots, q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$.
The vertex labeling f is known as vertex equitable labeling. In [3] the authors proved that the graphs like path, bistar $B(n, n)$, combs, cycle C_n if $n \equiv 0$ or $3(mod\ 4)$, $K_{2,n}, C_3^{(t)}$ for $t \geq 2$, quadrilateral snake, $K_2 + mK_1, K_{1,n} \cup K_{1,n+k}$ if and only if $1 \leq k \leq 3$, ladder, arbitrary super division of any path and cycle C_n with $n \equiv 0$ or $3(mod\ 4)$ are vertex equitable. Also they proved that the graphs $K_{1,n}$ if $n \geq 4$, any Eulerian graph with n edges where $n \equiv 1$ or $2(mod\ 4)$, the wheel W_n, the complete graph K_n if $n > 3$ and triangular cactus with $q \equiv 0$ or 6 or $9(mod\ 12)$ are not vertex equitable. In addition, they proved that if G is a graph with p vertices and q edges, q is even and $p < \left\lceil \frac{q}{2} \right\rceil + 2$ then G is not vertex equitable graph. Motivated by these results, we [4, 5, 6, 7, 8] proved that T_p-tree, $T \odot \overline{K_n}$ where T is a T_p-tree with even number of vertices, $T\hat{o}P_n$, $T\hat{o}2P_n, T\hat{o}C_n(n \equiv 0, 3(mod\ 4))$, $T\tilde{o}C_n(n \equiv 0, 3(mod\ 4))$, bistar $B(n, n+1)$, square graph of $B_{n,n}$ and splitting graph of $B_{n,n}$, the caterpillar $S(x_1, x_2, \ldots, x_n)$ and $C_n \odot K_1$, P_n^2, tadpoles, $C_m \oplus C_n$, armed crowns, $[P_m; C_n^2]$, $\langle P_m\hat{o}K_{1,n}\rangle$, kC_4-snakes for all $k \geq 1$, generalized kC_n-snake if $n \equiv 0(mod\ 4), n \geq 4$ and the graphs obtained by duplicating an arbitrary vertex and an arbitrary edge of a cycle C_n, total graph of P_n, splitting graph of P_n and fusion of two edges of a cycle C_n are vertex equitable graphs. In this paper we extend our study on vertex equitable labeling and prove that the graphs $DA(T_n) \odot K_1, DA(T_n) \odot 2K_1, DA(Q_n) \odot K_1, DA(Q_n) \odot 2K_1$ are vertex equitable.

Definition 1.1. *The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.*

Definition 1.2. *A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1}(alternately) to two new vertices v_i and w_i respectively.*

Definition 1.3. *A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to two new vertices v_i, x_i and w_i, y_i respectively and then joining v_i, w_i and x_i, y_i.*

2 Main Results

Theorem 2.1. *The graph $DA(T_n) \odot K_1$ is a vertex equitable graph.*

Proof. Let $G = DA(T_n) \odot K_1$. Let $u_1, u_2, \ldots, u_n$ be the vertices of path P_n.
Case (i) The triangle starts from u_1.
We construct $DA(T_n)$ by joining every u_{2i-1}, u_{2i} to the new vertices v_i, w_i for $1 \le i \le \lfloor \frac{n}{2} \rfloor$. Let $V(G) = V(DA(T_n)) \cup \{u_i' : 1 \le i \le n\} \cup \left\{v_i', w_i' : 1 \le i \le \lfloor \frac{n}{2} \rfloor\right\}$ and $E(G) = E(DA(T_n)) \cup \{u_i u_i' : 1 \le i \le n\} \cup \left\{v_i v_i', w_i w_i' : 1 \le i \le \lfloor \frac{n}{2} \rfloor\right\}$. We consider the following two subcases.
Subcase (i) n is even.
Here $|V(G)| = 4n$ and $|E(G)| = 5n - 1$. Let $A = \left\{0, 1, 2, \ldots, \lceil \frac{5n-1}{2} \rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows. For $1 \le i \le \frac{n}{2}$, $f(u_{2i-1}) = 5(i-1), f(u_{2i}) = 5i, f(u_{2i-1}') = 5i - 4, f(u_{2i}') = 5i - 1, f(v_i) = f(v_i') = 5i - 3$ and $f(w_i) = f(w_i') = 5i - 2$. It can be verified that the induced edge labels of $DA(T_n) \odot K_1$ are $1, 2, \ldots, 5n - 1$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(T_n) \odot K_1$.
Subcase (ii) n is odd.
Here $|V(G)| = 4n - 2$ and $|E(G)| = 5n - 4$. Let $A = \left\{0, 1, 2, \ldots, \lceil \frac{5n-4}{2} \rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows. We label the vertices $u_{2i-1}, u_{2i-1}' \left(1 \le i \le \lceil \frac{n}{2} \rceil\right)$ and $u_{2i}, u_{2i}', v_i, v_i', w_i, w_i' \left(1 \le i \le \frac{n-1}{2}\right)$ as in Subcase (i). It can be verified that the induced edge labels of $DA(T_n) \odot K_1$ are $1, 2, \ldots, 5n - 4$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(T_n) \odot K_1$.
Case (ii) The triangle starts from u_2.
We construct $DA(T_n)$ by joining every u_{2i}, u_{2i+1} to the new vertices v_i, w_i for $1 \le i \le \frac{n-2}{2}$. Let $V(G) = V(DA(T_n)) \cup \{u_i' : 1 \le i \le n\} \cup \left\{v_i', w_i' : 1 \le i \le \lceil \frac{n-2}{2} \rceil\right\}$ and $E(G) = E(DA(T_n)) \cup \{u_i u_i' : 1 \le i \le n\} \cup \left\{v_i v_i', w_i w_i' : 1 \le i \le \lceil \frac{n-2}{2} \rceil\right\}$.
Subcase (i) n is even.
Here $|V(G)| = 4n - 4$ and $|E(G)| = 5n - 7$. Let $A = \left\{0, 1, 2, \ldots, \lceil \frac{5n-7}{2} \rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \le i \le \frac{n}{2}, f(u_{2i-1}) = f(u_{2i}) = 5i - 4, f(u_1') = 0, f(u_{2i}') = 5i - 3$. For $1 \le i \le \frac{n-2}{2}$, $f(u_{2i+1}') = 5i - 1, f(v_i) = 5i, f(v_i') = f(w_i) = 5i - 2$ and $f(w_i') = 5i - 3$. It can be verified that the induced edge labels of $DA(T_n) \odot K_1$ are $1, 2, \ldots, 5n - 7$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(T_n) \odot K_1$.
Subcase (ii) n is odd.

The proof can be omitted since by symmetry, the graph obtained in this subcase is isomorphic to the graph obtained in Subcase (ii) under Case (i).

□

An example for the vertex equitable labeling of $DA(T_8) \odot K_1$ where the two triangles start from u_1 is shown in Figure 1.

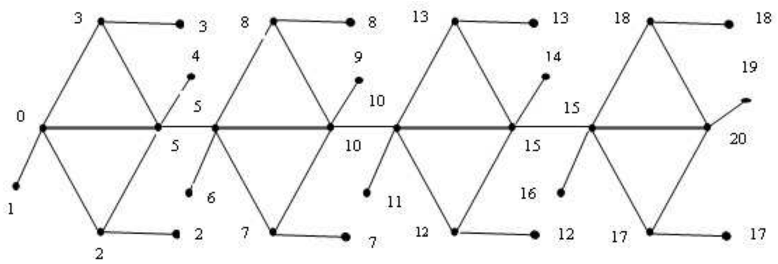

Figure 1

Theorem 2.2. *The graph $DA(T_n) \odot 2K_1$ is a vertex equitable graph.*

Proof. Let $G = DA(T_n) \odot 2K_1$. Let $u_1, u_2, \ldots, u_n$ be the vertices of path P_n.
Case (i) The triangle starts from u_1.
We construct $DA(T_n)$ by joining every u_{2i-1}, u_{2i} to the new vertices v_i, w_i for $1 \le i \le \lfloor \frac{n}{2} \rfloor$.
Let $V(G) = V(DA(T_n)) \cup \{u_i', u_i'' : 1 \le i \le n\} \cup \{v_i', v_i'', w_i', w_i'' : \ , \ 1 \le i \le \lfloor \frac{n}{2} \rfloor\}$ and $E(G) = E(DA(T_n)) \cup \{u_i u_i', u_i u_i'' : 1 \le i \le n\} \cup \{v_i v_i', v_i v_i'', \ \ w_i w_i', w_i w_i'' : 1 \le i \le \lfloor \frac{n}{2} \rfloor\}$.
We consider the following two subcases.
Subcase (i) n is even.
Here $|V(G)| = 6n$ and $|E(G)| = 7n - 1$. Let $A = \{0, 1, 2, \ldots, \lceil \frac{7n-1}{2} \rceil\}$. Define a vertex labeling $f : V(G) \to A$ as follows. For $1 \le i \le \frac{n}{2}, f(u_{2i-1}) = 7(i-1), f(u_{2i}) = 7i, f(u_{2i-1}') = 7i - 6, f(u_{2i}') = f(w_i') = 7i - 2, f(u_{2i-1}'') = f(v_i') = 7i - 5, f(u_{2i}'') = 7i - 1, f(v_i) = f(v_i'') = 7i - 4, f(w_i) = f(w_i') = 7i - 3$. It can be verified that the induced edge labels of $DA(T_n) \odot 2K_1$ are $1, 2, \ldots, 7n - 1$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(T_n) \odot 2K_1$.
Subcase (ii) n is odd.
Here $|V(G)| = 6n - 3$ and $|E(G)| = 7n - 5$. Let $A = \{0, 1, 2, \ldots, \lceil \frac{7n-5}{2} \rceil\}$. Define a vertex labeling $f : V(G) \to A$ as follows. We label the vertices $u_{2i-1}, u_{2i-1}', u_{2i-1}''$ $\left(1 \le i \le \lceil \frac{n}{2} \rceil\right)$ and $u_{2i}, u_{2i}', u_{2i}'', v_i, v_i', v_i'', w_i, w_i', w_i''$ $\left(1 \le i \le \frac{n-1}{2}\right)$ as in Subcase (i). It can be verified that the induced edge labels of $DA(T_n) \odot 2K_1$ are $1, 2, \ldots, 7n - 5$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(T_n) \odot 2K_1$.
Case (ii) The triangle starts from u_2.

We construct $DA(T_n)$ by joining every u_{2i}, u_{2i+1} to the vertices v_i, w_i for $1 \leq i \leq \lceil \frac{n-2}{2} \rceil$. Let $V(G) = V(DA(T_n)) \cup \{u_i', u_i'' : 1 \leq i \leq n\} \cup \{v_i', v_i'', w_i', w_i'' : 1 \leq i \leq \lceil \frac{n-2}{2} \rceil\}$ and $E(G) = E(DA(T_n)) \cup \{u_iu_i', u_iu_i'' : 1 \leq i \leq n\} \cup \{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'' : 1 \leq i \leq \lceil \frac{n-2}{2} \rceil\}$.

Subcase (i) n is even.

Here $|V(G)| = 6n-6$ and $|E(G)| = 7n-9$. Let $A = \left\{0, 1, 2, \ldots, \lceil \frac{7n-9}{2} \rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \leq i \leq \frac{n}{2}, f(u_{2i-1}) = f(u''_{2i-1}) = 7i - 6, f(u_{2i}) = 7i - 5, f(u'_{2i-1}) = 7(i - 1), f(u'_{2i}) = 7i - 5, f(u''_{2i}) = 7i - 4$. For $1 \leq i \leq \frac{n-2}{2}, f(v_i) = 7i-3, f(v_i') = 7i-4, f(v_i'') = f(w_i') = 7i-2, f(w_i) = 7i-1, f(w_i'') = 7i$. It can be verified that the induced edge labels of $DA(T_n) \odot 2K_1$ are $1, 2, \ldots, 7n-9$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(T_n) \odot 2K_1$.

Subcase (ii) n is odd.

The proof can be omitted since by symmetry, the graph obtained in this subcase is isomorphic to the graph obtained in Subcase (ii) under Case (i). □

An example for the vertex equitable labeling of $DA(T_5) \odot 2K_1$ where the two triangles start from u_1 is shown in Figure 2.

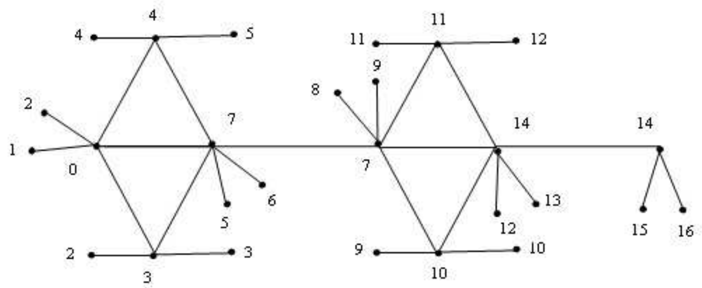

Figure 2

Theorem 2.3. *The graph $DA(Q_n) \odot K_1$ is a vertex equitable graph.*

Proof. Let $G = DA(Q_n) \odot K_1$. Let $u_1, u_2, \ldots, u_n$ be the vertices of path P_n.

Case (i) The quadrilateral starts from u_1.

We construct $DA(Q_n)$ by joining every u_{2i-1} to v_i, x_i and u_{2i} is adjacent to w_i, y_i and v_i is adjacent to w_i and x_i is adjacent to y_i for $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$. Let
$V(G) = V(DA(Q_n)) \cup \{u_i' : 1 \leq i \leq n\} \cup \left\{v_i', w_i', x_i', y_i' : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor\right\}$ and
$E(G) = E(DA(Q_n)) \cup \left\{v_iv_i', w_iw_i', x_ix_i', y_iy_i' : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor\right\} \cup \{u_iu_i' : 1 \leq i \leq n\}$. We consider the following two subcases.

Subcase (i) n is even.

Here $|V(G)| = 6n$ and $|E(G)| = 7n - 1$. Let $A = \left\{0, 1, 2, \ldots, \lceil \frac{7n-1}{2} \rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows.

For $1 \leq i \leq \frac{n}{2}, f(u_{2i-1}) = 7(i-1), f(u_{2i}) = 7i, f(u'_{2i-1}) = 7i-5, f(u'_{2i}) = f(w'_i) = 7i-1, f(v_i) = f(x'_i) = 7i-6, f(x_i) = 7i-4, f(v'_i) = f(y_i) = 7i-2$ and $f(w_i) = f(y'_i) = 7i-3$. It can be verified that the induced edge labels of $DA(Q_n) \odot K_1$ are $1, 2, \ldots, 7n-1$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(Q_n) \odot K_1$.

Subcase (ii) n is odd.

Here $|V(G)| = 6n-4$ and $|E(G)| = 7n-6$. Let $A = \left\{0, 1, 2, \ldots, \left\lceil \frac{7n-6}{2} \right\rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows. We label the vertices u_{2i-1} $\left(1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil\right), \left(1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right)$ and $u_{2i}, u'_{2i-1}, u'_{2i}, v_i, v'_i, w_i, w'_i, x_i, x'_i, y_i, y'_i$, $\left(1 \leq i \leq \frac{n-1}{2}\right)$ as in Subcase(i) and define $f(u'_n) = \left\lceil \frac{7n-6}{2} \right\rceil$. It can be verified that the induced edge labels of $DA(Q_n) \odot K_1$ are $1, 2, \ldots, 7n-6$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(Q_n) \odot K_1$.

Case (ii) The quadrilateral starts from u_2.

We construct $DA(Q_n)$ by joining every u_{2i} to v_i, x_i and u_{2i+1} is adjacent to w_i, y_i and v_i is adjacent to w_i and x_i is adjacent to y_i for $1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil$. Let $V(G) = V(DA(Q_n)) \cup \{u'_i : 1 \leq i \leq n\} \cup \left\{v'_i, w'_i, x'_i, y'_i : 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil\right\}$ and

$E(G) = E(DA(Q_n)) \cup \left\{v_iv'_i, w_iw'_i, x_ix'_i, y_iy'_i : 1 \leq i \leq \left\lceil \frac{n-2}{2} \right\rceil\right\} \cup \{u_iu'_i : 1 \leq i \leq n\}$.

Subcase (i) n is even.

Here $|V(G)| = 6n-8$ and $|E(G)| = 7n-11$. Let $A = \left\{0, 1, 2, \ldots, \left\lceil \frac{7n-11}{2} \right\rceil\right\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \leq i \leq \frac{n}{2}, f(u_{2i-1}) = f(u_{2i}) = 7i-6, f(u'_1) = 0, f(u'_n) = \left\lceil \frac{7n-11}{2} \right\rceil$. For $1 \leq i \leq \frac{n-2}{2}, f(u'_{2i}) = f(v'_i) = 7i-4, f(v_i) = f(u'_{2i+1}) = 7i-5, f(w_i) = f(w'_i) = 7i-3, f(x_i) = 7i-1, f(x'_i) = 7i-2$ and $f(y_i) = f(y'_i) = 7i$. It can be verified that the induced edge labels of $DA(Q_n) \odot K_1$ are $1, 2, \ldots, 7n-11$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(Q_n) \odot K_1$.

Subcase (ii) n is odd.

The proof can be omitted since by symmetry, the graph obtained in this subcase is isomorphic to the graph obtained in Subcase (ii) under Case (i). □

An example for the vertex equitable labeling of $DA(Q_6) \odot K_1$ where the two quadrilateral start from u_2 is shown in Figure 3.

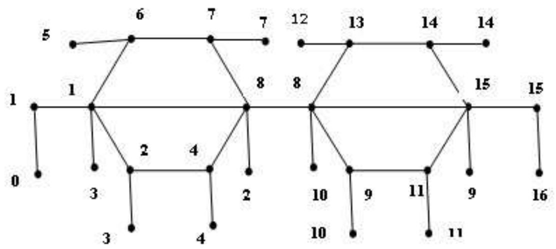

Figure 3

Theorem 2.4. *The graph $DA(Q_n) \odot 2K_1$ is a vertex equitable graph.*

Proof. Let $G = DA(Q_n) \odot 2K_1$. Let $u_1, u_2, \ldots, u_n$ be the vertices of path P_n.
Case (i) The quadrilateral starts from u_1.
We construct $DA(Q_n)$ by joining every u_{2i-1} to v_i, x_i and u_{2i} is adjacent to w_i, y_i and v_i is adjacent to w_i and x_i is adjacent to y_i for $1 \le i \le \lfloor \frac{n}{2} \rfloor$. Let $V(G) = V(DA(Q_n)) \cup \{u_i', u_i'' : 1 \le i \le n\} \cup \{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'' : 1 \le i \le \lfloor \frac{n}{2} \rfloor\}$ and $E(G) = E(DA(Q_n)) \cup \{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'', x_ix_i', x_ix_i'', y_iy_i', y_iy_i'' : , 1 \le i \le \lfloor \frac{n}{2} \rfloor\} \cup \{u_iu_i', u_iu_i'' : 1 \le i \le n\}$. We consider the following two subcases.
Subcase (i) n is even.
Here $|V(G)| = 9n$ and $|E(G)| = 10n - 1$. Let $A = \{0, 1, 2, \ldots, \lceil \frac{10n-1}{2} \rceil\}$. Define a vertex labeling $f : V(G) \to A$ as follows. For $1 \le i \le \frac{n}{2}, f(u_{2i-1}) = 10(i-1), f(u_{2i}) = 10i, f(u_{2i-1}') = f(w_i') = 10i - 9, f(u_{2i}') = 10i - 3, f(u_{2i-1}'') = 10i - 8, f(u_{2i}'') = f(w_i'') = 10i - 1, f(w_i) = 10i - 4, f(v_i) = f(v_i'') = 10i - 7, f(v_i') = 10i - 8, f(y_i) = 10i - 2, f(y_i') = 10i - 5, f(y_i'') = 10i - 4, f(x_i) = f(x_i') = 10i - 6, f(x_i'') = 10i - 3$.
It can be verified that the induced edge labels of $DA(Q_n) \odot 2K_1$ are $1, 2, \ldots, 10n - 1$ and $|v_f(i) - v_f(j)| \le 1$ for al $i, j \in A$. Hence f is a vertex equitable labeling of $DA(Q_n) \odot 2K_1$.
Subcase (ii) n is odd.
Here $|V(G)| = 9n-6$ and $|E(G)| = 10n-8$. Let $A = \{0, 1, 2, \ldots, \lceil \frac{10n-8}{2} \rceil\}$. Define a vertex labeling $f : V(G) \to A$ as follows. We label the vertices $u_{2i-1}, u_{2i-1}', u_{2i-1}'' \left(1 \le i \le \lceil \frac{n}{2} \rceil\right)$ and $u_{2i}, u_{2i}', v_i, v_i', w_i, w_i', x_i, x_i', y_i, y_i' \left(1 \le i \le \frac{n-1}{2}\right)$ as in Subcase (i). It can be verified that the induced edge labels of $DA(Q_n) \odot 2K_1$ are $1, 2, \ldots, 10n - 8$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(Q_n) \odot 2K_1$.
Case (ii) The quadrilateral starts from u_2.
We construct $DA(Q_n)$ by joining every u_{2i} to v_i, x_i and u_{2i+1} is adjacent to w_i, y_i and v_i is adjacent to w_i and x_i is adjacent to y_i for $1 \le i \le \lceil \frac{n-2}{2} \rceil$. Let $V(G) = V(DA(Q_n)) \cup \{u_i', u_i'' : 1 \le i \le n\} \cup \{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'' : 1 \le i \le \lceil \frac{n-2}{2} \rceil\}$ and $E(G) = E(DA(Q_n)) \cup \{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'', x_ix_i', x_ix_i'', \ y_iy_i', y_iy_i'' : 1 \le i \le \lceil \frac{n-2}{2} \rceil\} \cup \{u_iu_i', u_iu_i'' : 1 \le i \le n\}$.
Subcase (i) n is even.
Here $|V(G)| = 9n - 12$ and $|E(G)| = 10n - 15$. Let $A = \{0, 1, 2, \ldots, \lceil \frac{10n-15}{2} \rceil\}$. Define a vertex labeling $f : V(G) \to A$ as follows: For $1 \le i \le \frac{n}{2}$ $f(u_{2i-1}) = f(u_{2i-1}'') = 10i - 9, f(u_{2i}) = f(u_{2i}') = 10i - 8, f(u_{2i}'') = 10i - 7, f(u_1') = 0$. For $1 \le i \le \frac{n-2}{2}, f(x_i') = 10i - 7, f(w_i) = f(w_i') = 10i - 2, f(u_{2i+1}'') = 10i - 1, f(v_i) = f(y_i') = 10i - 6, f(y_i'') = 10i - 3, f(x_i) = f(v_i'') = 10i - 5, f(y_i) = f(w_i'') = 10i, f(v_i') = f(x_i'') = 10i - 4$. It can be verified that the induced edge labels of $DA(Q_n) \odot 2K_1$ are $1, 2, \ldots, 10n - 15$ and $|v_f(i) - v_f(j)| \le 1$ for all $i, j \in A$. Hence f is a vertex equitable labeling of $DA(Q_n) \odot 2K_1$.
Subcase (ii) n is odd.
The proof can be omitted since by symmetry, the graph obtained in this subcase is isomorphic to the graph obtained in Subcase (ii) under Case (i). □

An example for the vertex equitable labeling of $DA(Q_5) \odot 2K_1$ where the two quadrilateral start from u_2 is shown in Figure 4.

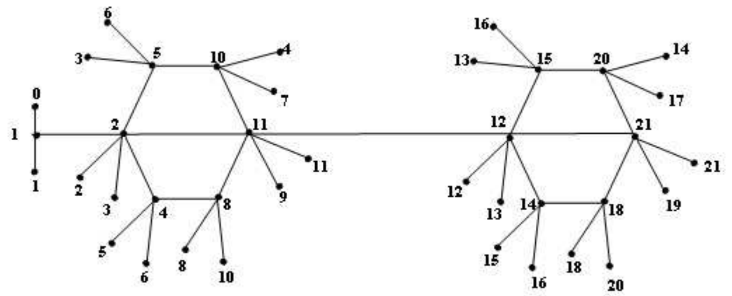

Figure 4

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