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Edge pair sum labeling of some cycle related graphs

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ABSTRACT

Let G be a (p,q) graph. An injective map $f : E(G) \to \{\pm 1, \pm 2, \cdots, \pm q\}$ is said to be an edge pair sum labeling if the induced vertex function $f^* : V(G) \to Z - \{0\}$ defined by $f^*(v) = \sum_{e \epsilon E_v} f(e)$ is one-one where E_v denotes the set of edges in G that are incident with a vertex v and $f^*(V(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p}{2}}\right\}$ or $\left\{\pm k_1, \pm k_2, \cdots, \pm k_{\frac{p-1}{2}}\right\} \bigcup \left\{\pm k_{\frac{p+1}{2}}\right\}$ according as p is even or odd. A graph with an edge pair sum labeling is called an edge pair sum graph. In this paper we prove that the graphs $GL(n)$, double triangular snake $D(T_n)$, W_n, Fl_n, $\langle C_m, K_{1,n}\rangle$ and $\langle C_m * K_{1,n}\rangle$ admit edge pair sum labeling.

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1 Introduction

Throughout this paper we consider finite, simple and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. G is also called a (p, q) graph. We follow the basic notations

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and terminologies of graph theory as in [2]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and for a dynamic survey of various graph labeling problems along with extensive bibliography we refer to Gallian [1]. Ponraj and Parthipan introduced the concept of pair sum labeling in [12]. An injective map $f : V(G) \to \{\pm 1, \pm 2, \ldots, \pm p\}$ is said to be a pair sum labeling of a graph $G(p, q)$ if the induced edge function $f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u)+f(v)$ is one-one and $f_e(E(G))$ is either of the form $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q}{2}}\right\}$ or $\left\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q-1}{2}}\right\} \cup \left\{\pm k_{\frac{q+1}{2}}\right\}$ according as q is even or odd. A graph with a pair sum labeling, it is called a pair sum graph. Analogous to pair sum labeling we defined a new labeling called edge pair sum labeling [3] and further studied in [4-10]. In this paper we prove that the graphs $GL(n)$, double triangular snake $D(T_n)$, W_n, Fl_n, $\langle C_m, K_{1,n}\rangle$ and $\langle C_m * K_{1,n}\rangle$ admit edge pair sum labeling.

We use the following definitions in the subsequent sequel.

Definition 1. *A complete bipartite graph is a simple bipartite graph such that two vertices are adjacent if and only if they are in different partite sets. If partite sets V_1 and V_2 are having m and n vertices respectively then the related complete bipartite graph is denoted by $K_{m,n}$ and V_1 is called m-vertices part and V_2 is called n-vertices part of $K_{m,n}$.*

Definition 2. *The double triangular snake $D(T_n)$ is the graph obtained from the path $u_1, u_2, u_3, ..., u_n$ by joining u_i and u_{i+1} with two new vertices v_i and w_i for $1 \leq i \leq (n-1)$.*

Definition 3. *The wheel graph W_n is the joining of the graphs C_n and K_1 that is, $W_n = C_n + K_1$. Here the vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n while the vertex corresponds to K_1 is called apex vertex.*

Definition 4. *A helm H_n $n \geq 3$ is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the wheels's rim.*

Definition 5. *The flower graph Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm.*

Definition 6. *The graph $\langle C_m, K_{1,n}\rangle$ is the graph obtained from C_m and $K_{1,n}$ by identifying any one of the vertices of C_m with the central vertex of $K_{1,n}$ [11].*

Definition 7. *The graph $\langle C_m * K_{1,n}\rangle$ is the graph obtained from C_m and $K_{1,n}$ by identifying any one of the vertices of C_m with a pendant vertex of $K_{1,n}$ (that is a non-central vertex of $K_{1,n}$)[11].*

2 Preliminary Results

The following results have been proved in [3].

- Every path P_n is an edge pair sum graph for $n \geq 3$.
- Every cycle C_n $(n \geq 3)$ is an edge pair sum graph.
- The star graph $K_{1,n}$ is an edge pair sum graph if and only if n is even.
- The complete graph K_4 is not an edge pair sum graph.

3 Main Results

Theorem 1. *The complete bipartite graph $K_{2,n}$ is an edge pair sum graph.*

Proof. Define $V(K_{2,n}) = \{u, v, u_i : 1 \leq i \leq 2n\}$ and $E(K_{2,n}) = \{e_i = uu_i, e_i^{'} = vu_i : 1 \leq i \leq 2n\}$ are the vertices and edges of the graph $K_{2,n}$. Define the edge labeling $f : E(K_{2,n}) \rightarrow \{\pm 1, \pm 2, \pm 3, ..., \pm 4n\}$. For $1 \leq i \leq 2n$ $f(e_i) = i$ and $f(e_i^{'}) = -(2n - i + 1)$. Then the induced vertex labeling is as follows: For $1 \leq i \leq n$ $f^*(u_i) = -(2n - 2i + 1)$ and $f^*(u_{n+i}) = 2i - 1$, $f^*(u) = n(2n + 1) = -f^*(v)$. Then $f^*(V(K_{2,n})) = \{\pm 1, \pm 3, \pm 5, ..., \pm(2n - 1), \pm(2n^2 + n)\}$. Hence f is an edge pair sum labeling for all n. The example for the edge pair sum graph labeling of $K_{2,2}$ is shown in Figure 1. □

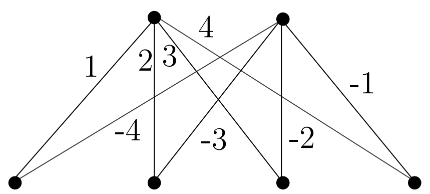

Figure 1: Edge pair sum labeling for the graph $K_{2,2}$

Theorem 2. *The double triangular snake $D(T_n)$ is an edge pair sum graph.*

Proof. Let $G(V, E) = D(T_n)$. Then $V(G) = \{u_i : 1 \leq i \leq n, v_i, w_i : 1 \leq i \leq (n - 1)\}$ and $E(G) = \{e_{2i-1} = u_iv_i, e_{2i} = u_{1+i}v_i, e_{2i-1}^{'} = u_iw_i, e_{2i}^{'} = u_{1+i}w_i, e_i^{''} = u_iu_{1+i} : 1 \leq i \leq (n - 1)\}$ are the vertices and edges of the graph G. Define the edge labeling $f : E(G) \rightarrow \{\pm 1, \pm 2, \pm 3, ..., \pm(5n - 5)\}$ by considering the following two cases.

Case(i). n is even.

Subcase (a). $n = 4$.

Define $f(e_1^{''}) = -2$, $f(e_2^{''}) = -1$, $f(e_3^{''}) = 3$, for $1 \leq i \leq (n - 1)$ $f(e_{2i-1}) = n - 2 + 2i = -f(e_{2i-1}^{'})$ and $f(e_{2i}) = n - 1 + 2i = -f(e_{2i}^{'})$. The induced vertex labeling is as follows: $f^*(u_1) = -2 = -f^*(u_3)$, $f^*(u_2) = -3 = -f^*(u_4)$ and for $1 \leq i \leq (n-1)$ $f^*(v_i) = 2n-3+4i = -f^*(w_i)$. Then $f^*(V(G)) = \{\pm 2, \pm 3, \pm(2n+1), \pm(2n+5), \pm(2n+9), ..., \pm(6n-7)\}$. Hence f is an edge pair sum labeling for $n = 4$.

Subcase (b). $n > 4$.

For $1 \le i \le (n-1)$ $f(e_{2i-1}) = n-2+2i = -f(e'_{2i-1})$ and $f(e_{2i}) = n-1+2i = -f(e'_{2i})$, for $1 \le i \le \frac{n}{2}-2$ $f(e''_i) = n+1-2i$, $f(e_{\frac{n}{2}-1}) = -2$, $f(e_{\frac{n}{2}}) = -1$, $f(e_{\frac{n}{2}+1}) = 3$ and for $\frac{n}{2}+2 \le i \le (n-1)$ $f(e''_i) = n-1-2i$. The induced vertex labeling is as follows: for $1 \le i \le (n-1)$ $f^*(v_i) = 2n-3+4i = -f^*(w_i)$, $f^*(u_1) = n-1 = -f^*(u_n)$, for $2 \le i \le \frac{n}{2}-2$ $f^*(u_i) = 2(n+2-2i)$, $f^*(u_{\frac{n}{2}-1}) = 3 = -f^*(u_{\frac{n}{2}})$, $f^*(u_{\frac{n}{2}+1}) = 2 = -f^*(u_{\frac{n}{2}+2})$ and for $(\frac{n}{2}+3) \le i \le (n-1)$ $f^*(u_i) = 2(n-2i)$. Then we get $f^*(V(G)) = \{\pm2, \pm3, \pm12, \pm16, \pm20, ..., \pm(2n-4), \pm(n-1), \pm(2n+1), \pm(2n+5), \pm(2n+9), ..., \pm(6n-7)\}$. Hence f is an edge pair sum labeling. The example for the edge pair sum graph labeling of $D(T_6)$ is shown in Figure 2.

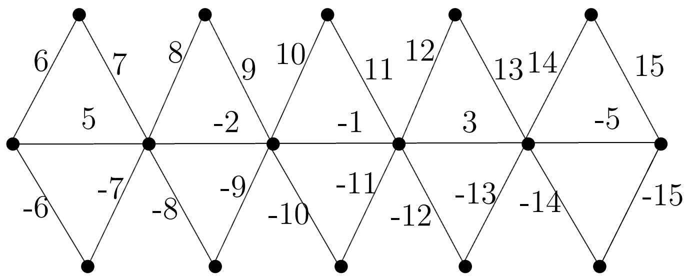

Figure 2: Edge pair sum labeling for the graph $D(T_6)$

Case(ii). n is odd.
Subcase (a). $n = 3$.
Define $f(e''_1) = -2$, $f(e''_2) = 1$, for $1 \le i \le (n-1)$ $f(e_{2i-1}) = 2i+1 = -f(e'_{2i-1})$ and $f(e_{2i}) = 2i+2 = -f(e'_{2i})$. The induced vertex labeling is as follows: $f^*(u_1) = -2$, $f^*(u_2) = -1 = -f^*(u_3)$ and for $1 \le i \le (n-1)$ $f^*(v_i) = 4i+3 = -f^*(w_i)$. Then $f^*(V(G)) = \{\pm1, \pm7, \pm11, \pm15, ..., \pm(4n-1)\} \bigcup \{-2\}$. Hence f is an edge pair sum labeling for $n = 3$.
Subcase (b). $n > 3$.
For $1 \le i \le (n-1)$ $f(e_{2i-1}) = n-1+2i = -f(e'_{2i-1})$ and $f(e_{2i}) = n+2i = -f(e'_{2i})$, for $1 \le i \le \frac{n-3}{2}$ $f(e''_i) = -(n+2-2i)$, $f(e''_{\frac{n+1}{2}}) = 1$, $f(e''_{\frac{n-1}{2}}) = 2$ and for $\frac{n+3}{2} \le i \le (n-1)$ $f(e''_i) = -n+2+2i$. The induced vertex labeling is as follows: for $1 \le i \le (n-1)$ $f^*(v_i) = 2n-1+4i = -f^*(w_i)$, $f^*(u_1) = -n = -f^*(u_n)$, for $2 \le i \le \frac{n-3}{2}$ $f^*(u_i) = 2(-n-3+2i)$, $f^*(u_{\frac{n-1}{2}}) = -3 = -f^*(u_{\frac{n+1}{2}})$, $f^*(u_{\frac{n+3}{2}}) = 6$ and for $(\frac{n+5}{2}) \le i \le (n-1)$ $f^*(u_i) = -2(n-1-2i)$. Then we get $f^*(V(G)) = \{\pm3, \pm12, \pm16, \pm20, ..., \pm(2n-2), \pm n, \pm(2n+3), \pm(2n+7), \pm(2n+11), ..., \pm(6n-5)\} \bigcup \{6\}$. Hence f is an edge pair sum labeling. The example for the edge pair sum graph labeling of $D(T_5)$ is shown in Figure 3. □

Theorem 3. *The wheel graph W_n is an edge pair sum graph.*

Proof. Let $V(W_n) = \{v, u_i : 1 \le i \le n\}$ and $E(W_n) = \{e'_i = vu_i : 1 \le i \le n, e_1 = u_nu_1, e_{1+i} = u_iu_{1+i} : 1 \le i \le (n-1)\}$ are the vertices and edges of the graph W_n. Define

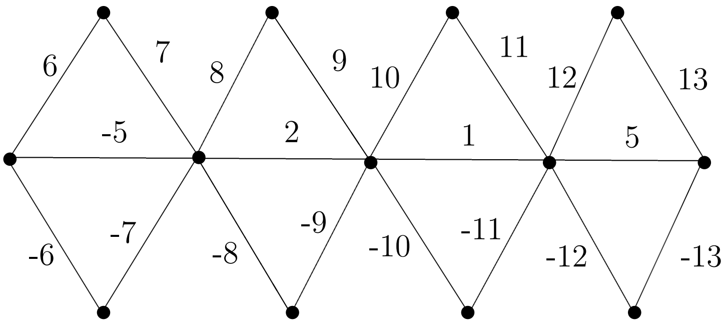

Figure 3: Edge pair sum labeling for the graph $D(T_5)$

the edge labeling $f : E(W_n) \rightarrow \{\pm1, \pm2, \pm3, ..., \pm2n\}$ by considering the following two cases.

Case(i). n is even.

For $1 \leq i \leq \frac{n}{2}$ $f(e_i) = i$ and $f(e_{\frac{n}{2}+i}) = -(\frac{n}{2}+1-i)$, for $1 \leq i \leq (\frac{n}{2}-1)$ $f(e_i^{'}) = n+2+2i$ and $f(e^{'}_{\frac{n}{2}+i}) = -2(n+1-i)$, $f(e^{'}_{\frac{n}{2}}) = -(n+2)$ and $f(e^{'}_n) = (\frac{n}{2}+1)$. Then the induced vertex labeling is as follows: for $1 \leq i \leq (\frac{n}{2}-1)$ $f^*(u_i) = n+3+4i$ and $f^*(u_{\frac{n}{2}+i}) = -(3n+3-4i)$, $f^*(u_{\frac{n}{2}}) = -(n+2)$ and $f^*(u_n) = (\frac{n}{2}+1) = -f^*(v)$. Therefore we get $f^*(V(W_n)) = \{\pm(\frac{n}{2}+1), \pm(n+7), \pm(n+11), \pm(n+15), ..., \pm(3n-1)\} \bigcup \{-(n+2)\}$. Hence f is an edge pair sum labeling. The examples for the edge pair sum graph labeling of W_4 and W_8 are shown in Figure 4.

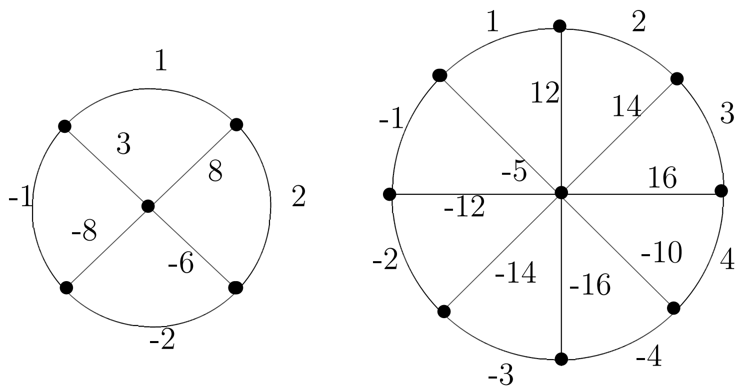

Figure 4: Edge pair sum labeling for the graph W_4 and W_8

Case(ii). n is odd.

Subcase (a). $n = 1, 2(mod3)$.
Define $f(e_1) = 1$, $f(e_2) = -2$, for $1 \leq i \leq (\frac{n-3}{2})$ $f(e_{2+i}) = 2 + i = -f(e_{\frac{n+3}{2}+i})$, $f(e_{\frac{n+3}{2}}) = -1$, $f(e_1^{'}) = n - 1 = -f(e_2^{'})$, for $1 \leq i \leq (\frac{n-5}{2})$ $f(e_{2+i}^{'}) = n - 1 + 2i = -f(e^{'}_{\frac{n+3}{2}+i})$, $f(e^{'}_{\frac{n+1}{2}}) = 2n - 5 = -f(e^{'}_n)$ and $f(e^{'}_{\frac{n+3}{2}}) = 2$. Then the induced vertex labeling is as follows: $f^*(u_1) = n - 2 = -f^*(u_2)$, for $1 \leq i \leq (\frac{n-5}{2})$ $f^*(u_{2+i}) = n + 4 + 4i = -f^*(u_{\frac{n+3}{2}+i})$, $f^*(u_{\frac{n+1}{2}}) = \frac{5n-11}{2} = -f^*(u_n)$ and $f^*(u_{\frac{n+3}{2}}) = -2 = f^*(v)$. Then
$f^*(V(W_n)) = \{\pm 2, \pm(\frac{5n-11}{2}), \pm(n-2), \pm(n+8), \pm(n+12), \pm(n+16), ..., \pm(3n-6)\}$.
Hence f is an edge pair sum labeling.
Subcase (b). $n = 0(mod3)$.
Define $f(e_1) = 3$, $f(e_2) = 4$, $f(e_3) = -5$, for $1 \leq i \leq (\frac{n-5}{2})$ $f(e_{3+i}) = 5 + i = -f(e_{\frac{n+5}{2}+i})$, $f(e_{\frac{n+3}{2}}) = -3$, $f(e_{\frac{n+5}{2}}) = -4$, $f(e_1^{'}) = -(n+1)$, $f(e_2^{'}) = -1 = -f(e_3^{'})$, for $1 \leq i \leq (\frac{n-3}{2})$ $f(e_{3+i}^{'}) = \frac{n+5}{2} + i$, $f(e^{'}_{\frac{n+5}{2}}) = 5$ and for $1 \leq i \leq (\frac{n-5}{2})$ $f(e^{'}_{\frac{n+5}{2}+i}) = -(\frac{n+5}{2} + i)$. Then the induced vertex labeling is as follows: $f^*(u_1) = -(n-6) = -f^*(u_{\frac{n+3}{2}})$, $f^*(u_2) = -2 = -f^*(u_3)$, for $1 \leq i \leq (\frac{n-7}{2})$ $f^*(u_{3+i}) = \frac{n+5}{2} + 11 + 3i = -f^*(u_{\frac{n+5}{2}+i})$, $f^*(u_{\frac{n+1}{2}}) = \frac{3n-1}{2} = -f^*(u_n)$ and $f^*(u_{\frac{n+5}{2}}) = -5 = f^*(v)$. Then $f^*(V(W_n)) = \{\pm 2, \pm 5, \pm(n-6), \pm(\frac{3n-1}{2}), \pm(\frac{n+33}{2}), \pm(\frac{n+39}{2}), \pm(\frac{n+45}{2}), ..., \pm(2n+3)\}$. Hence f is an edge pair sum labeling. The examples for the edge pair sum graph labeling of W_7 and W_9 are shown in Figure 5. □

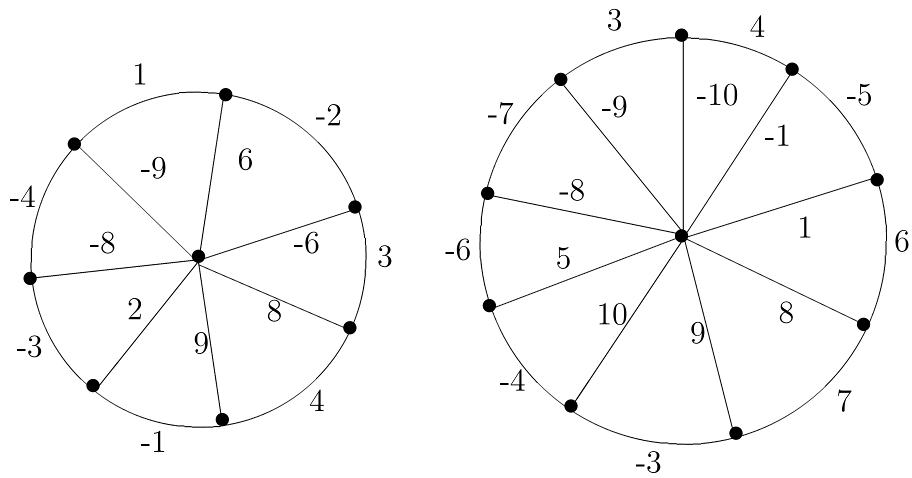

Figure 5: Edge pair sum labeling for the graph W_7 and W_9

Theorem 4. *The flower graph Fl_n is an edge pair sum graph.*

Proof. Let $V(Fl_n) = \{w, u_i, v_i : 1 \leq i \leq n\}$ and $E(Fl_n) = \{e_1 = u_n u_1, e_{1+i} = u_i u_{1+i} : 1 \leq i \leq (n-1), e_i^{'} = wu_i, e_i^{''} = u_i v_i, e_i^{'''} = wv_i : 1 \leq i \leq n\}$ are the vertices and edges of the graph Fl_n. Define the edge labeling $f : E(Fl_n) \to \{\pm 1, \pm 2, \pm 3, ..., \pm 4n\}$ by considering the following two cases.

Case(i). n is odd.
For $1 \leq i \leq (\frac{n+1}{2})$ $f(e_{2i-1}) = -(4i-2)$, for $1 \leq i \leq (\frac{n-1}{2})$ $f(e_{2i}) = -(2n+4i)$, for $1 \leq i \leq n$ $f(e_i^{'}) = -(2i-1) = -f(e_i^{''})$ and $f(e_i^{'''}) = -(2n-1+2i)$. Then the induced vertex labeling is as follows: for $1 \leq i \leq (n-1)$ $f^*(u_i) = -(2n+2+4i)$, for $1 \leq i \leq n$ $f^*(v_i) = 2n-2+4i$, $f^*(u_n) = -(2n+2)$ and $f^*(w) = 2n^2$. Then we get $f^*(V(Fl_n)) = \{\pm(2n+2), \pm(2n+6), \pm(2n+10), \pm(2n+14), ..., \pm(6n-2)\} \bigcup \{2n^2\}$. Hence f is an edge pair sum labeling for n is odd. The example for the edge pair sum graph labeling of Fl_5 is shown in Figure 6.

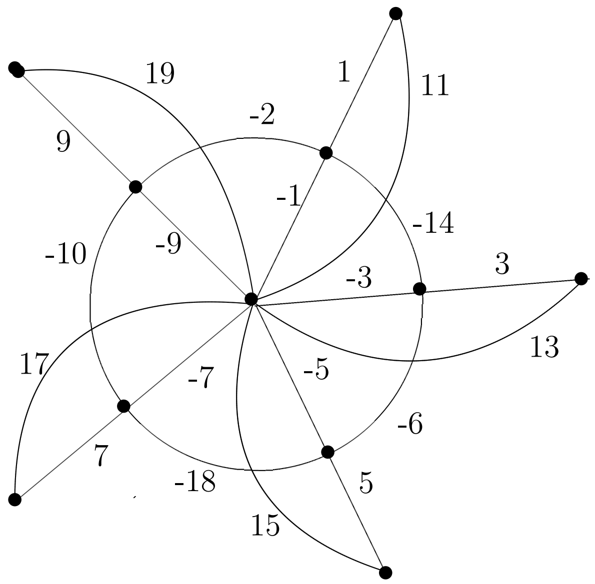

Figure 6: Edge pair sum labeling for the graph Fl_5

Case(ii). n is even.
Subcase (a). $n = 4$.
For $1 \leq i \leq \frac{n}{2}$ $f(e_i) = i = -f(e_{\frac{n}{2}+i})$, $f(e_i^{'}) = 4n-i+1 = -f(e_i^{''})$, $f(e_{\frac{n}{2}+i}^{'}) = -(\frac{7n+2}{2}-i) = -f(e_{\frac{n}{2}+i}^{''})$, $f(e_i^{'''}) = -(\frac{n}{2}-1+2i)$ and $f(e_{\frac{n}{2}+i}^{'''}) = n+2i-1$. Then the induced vertex labeling is as follows: for $1 \leq i \leq \frac{n-2}{2}$ $f^*(u_i) = 2i+1 = -f^*(u_{\frac{n}{2}+i})$, $f^*(u_{\frac{n}{2}}) = -1 = -f^*(u_n)$, for $1 \leq i \leq \frac{n}{2}$ $f^*(v_i) = -\frac{1}{2}(9n+2i) = -f^*(v_{\frac{n}{2}+i})$ and $f^*(w) = \frac{n^2}{2}$. Then $f^*(V(Fl_n)) = \{\pm 1, \pm 3, \pm 5, \pm 7, ..., \pm(n-1), \pm(\frac{9n+2}{2}), \pm(\frac{9n+4}{2}), \pm(\frac{9n+6}{2}), ..., \pm 5n\} \bigcup \{\frac{n^2}{2}\}$.
Hence f is an edge pair sum labeling.
Subcase (b). $n = 2(mod 4)$.
For $1 \leq i \leq \frac{n}{2}$ $f(e_i) = i = -f(e_{\frac{n}{2}+i})$, $f(e_i^{'}) = 4n-i+1 = -f(e_i^{''})$, $f(e_{\frac{n}{2}+i}^{'}) = -(\frac{7n+2}{2}-i) = -f(e_{\frac{n}{2}+i}^{''})$, $f(e_i^{'''}) = -(\frac{n}{2}-1+2i)$ and $f(e_{\frac{n}{2}+i}^{'''}) = n+2i-1$. Then the induced vertex labeling is as follows: for $1 \leq i \leq \frac{n-2}{2}$ $f^*(u_i) = 2i+1 = -f^*(u_{\frac{n}{2}+i})$, $f^*(u_{\frac{n}{2}}) = \frac{n-2}{2} = -f^*(u_n)$,

for $1 \le i \le \frac{n}{2}$ $f^*(v_i) = -\frac{1}{2}(9n + 2i) = -f^*(v_{\frac{n}{2}+i})$ and $f^*(w) = \frac{n^2}{2}$. Then $f^*(V(Fl_n)) = \{\pm(\frac{n-2}{2}), \pm3, \pm5, \pm7, ..., \pm(n-1), \pm(\frac{9n+2}{2}), \pm(\frac{9n+4}{2}), \pm(\frac{9n+6}{2}), ..., \pm5n\} \bigcup \{\frac{n^2}{2}\}$. Hence f is an edge pair sum labeling.

Subcase (c). $n = 0(mod 4)$.

For $1 \le i \le \frac{n}{2}$ $f(e_i) = i$, $f(e_{\frac{n}{2}+1}) = -2$, $f(e_{\frac{n}{2}+2}) = -1$, for $1 \le i \le \frac{n-4}{2}$ $f(e_{\frac{n}{2}+2+i}) = f(e_{\frac{n}{2}+i}) - 2$, for $1 \le i \le \frac{n}{2}$ $f(e_i^{'}) = 4n - i + 1 = -f(e_i^{''})$, $f(e^{'}_{\frac{n}{2}+i}) = -(\frac{7n+2}{2} - i) = -f(e^{''}_{\frac{n}{2}})$, $f(e_i^{'''}) = -(\frac{n}{2} + 2i)$ and $f(e^{'''}_{\frac{n}{2}+i}) = n + 2i$. Then the induced vertex labeling is as follows: for $1 \le i \le (\frac{n}{2}-1)$ $f^*(u_i) = 2i+1 = -f^*(u_{\frac{n}{2}}+i)$, $f^*(u_{\frac{n}{2}}) = \frac{n}{2}-2 = -f^*(u_n)$, for $1 \le i \le \frac{n}{2}$ $f^*(v_i) = -\frac{1}{2}(9n + 2 + 2i) = -f^*(v_{\frac{n}{2}+i})$ and $f^*(w) = \frac{n^2}{2}$. Then $f^*(V(Fl_n)) = \{\pm(\frac{n}{2} - 2), \pm3, \pm5, \pm7, ..., \pm(n-1), \pm(\frac{9n+4}{2}), \pm(\frac{9n+6}{2}), \pm(\frac{9n+10}{2}), ..., \pm(5n+1)\} \bigcup \{\frac{n^2}{2}\}$. Hence f is an edge pair sum labeling. The example for the edge pair sum graph labeling of Fl_6 is shown in Figure 7. □

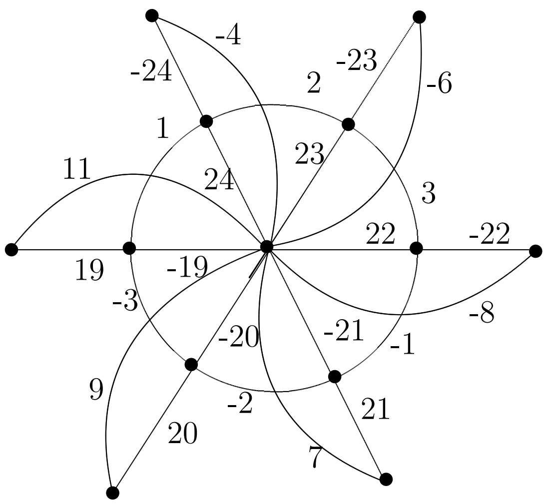

Figure 7: Edge pair sum labeling for the graph Fl_6

Theorem 5. *The graph $\langle C_m, K_{1,n}\rangle$ is an edge pair sum graph for $m \ge 4$ and n is odd.*

Proof. Let $V(\langle C_m, K_{1,n}\rangle)$=$\{u_i : 1 \le i \le m, v_i : 1 \le i \le n\}$ and $E(\langle C_m, K_{1,n}\rangle)$=$\{e_i = u_iu_{i+1} : 1 \le i \le (m-1), e_m = u_mu_1, e_i^{'} = u_iv_i : 1 \le i \le n\}$ are the vertices and edges of the graph $\langle C_m, K_{1,n}\rangle$. Define the edge labeling $f : E(\langle C_m, K_{1,n}\rangle) \rightarrow \{\pm1, \pm2, \pm3, ..., \pm(m+n)\}$ by considering the following four cases.

Case(i). $m = 4$.

Define $f(e_1) = 2 = -f(e_3)$, $f(e_2) = -1 = -f(e_4)$, $f(e_1^1) = 3$ and for $1 \le i \le \frac{n-1}{2}$ $f(e^{'}_{1+i}) = 6 + i = -f(e^{'}_{\frac{n+1}{2}+i})$. The induced vertex labeling is as follows: $f^*(u_1) = 6$, $f^*(u_2) = 1 = -f^*(u_4)$, $f^*(u_3) = -3 = -f^*(v_1)$ and for $1 \le i \le \frac{n-1}{2}$ $f^*(v_{1+i}) = 6 + i = -f^*(v_{\frac{n+1}{2}+i})$. Then $f^*(V(\langle C_m, K_{1,n}\rangle)) = \{\pm1, \pm3, \pm7, \pm8, \pm9, ..., \pm(\frac{n+11}{2})\} \bigcup \{6\}$. Hence f is an edge pair sum labeling.

Case(ii). $m = 5$.

Define $f(e_1) = -1 = -f(e_3)$, $f(e_2) = -3 = -f(e_5)$, $f(e_4) = -2 = f(e_1^1)$ and for $1 \leq i \leq \frac{n-1}{2}$ $f(e^{'}_{1+i}) = 4+i = -f(e^{'}_{\frac{n+1}{2}+i})$. The induced vertex labeling is as follows: $f^*(u_1) = 4 = -f^*(u_2)$, $f^*(u_3) = -2 = -f^*(v_1)$, $f^*(u_4) = -1 = -f^*(u_5)$ and for $1 \leq i \leq \frac{n-1}{2}$ $f^*(v_{1+i}) = 4 + i = -f^*(v_{\frac{n+1}{2}+i})$. Then $f^*(V(\langle C_m, K_{1,n}\rangle)) = \{\pm 1, \pm 2, \pm 4, \pm 5, \pm 6, \pm 7, ..., \pm(\frac{n+7}{2})\}$. Hence f is an edge pair sum labeling. The examples for the edge pair sum graph labeling of $< C_4, K_{1,3} >$ and $< C_5, K_{1,5} >$ are shown in Figure 8.

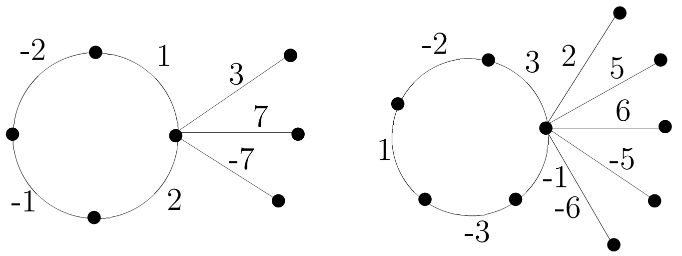

Figure 8: Edge pair sum labeling for the graph $< C_4, K_{1,3} >$ and $< C_5, K_{1,5} >$

Case(iii). m is even.
Subcase (a). $m = 2(mod 4)$.
Define $f(e_1) = \frac{m}{2} = -f(e_{\frac{m+2}{2}})$, for $1 \leq i \leq \frac{m-2}{2}$ $f(e_{1+i}) = -i = -f(e_{\frac{m+2}{2}+i})$, $f(e^{'}_1) = m-1$ and for $1 \leq i \leq \frac{n-1}{2}$ $f(e^{'}_{1+i}) = m - 1 + 2i = -f(e^{'}_{\frac{n+1}{2}+i})$. The induced vertex labeling is as follows: $f^*(u_1) = 2(m-1)$, $f^*(u_2) = \frac{m-2}{2} = -f^*(u_{\frac{m+4}{2}})$, for $1 \leq i \leq \frac{m-4}{2}$ $f^*(u_{2+i}) = -2i-1 = -f^*(u_{\frac{m+4}{2}+i})$, $f^*(u_{\frac{m+2}{2}}) = -m+1 = -f^*(v_1)$ and for $1 \leq i \leq \frac{n-1}{2}$ $f^*(v_{1+i}) = m-1+2i = -f^*(v_{\frac{n+1}{2}+i})$. Then $f^*(V(\langle C_m, K_{1,n}\rangle))=\{\pm(m-1), \pm(\frac{m-2}{2}), \pm 3, \pm 5, \pm 7, ..., \pm(m-3), \pm(m+1), \pm(m+3), \pm(m+5), ..., \pm(m+n-2)\} \bigcup \{2(m-1)\}$. Hence f is an edge pair sum labeling.
Subcase (b). $m = 0(mod 4)$.
Define $f(e_1) = \frac{m}{2} = -f(e_{\frac{m}{2}})$, $f(e_2) = -2$, $f(e_3) = -1$, for $1 \leq i \leq \frac{m-8}{2}$ $f(e_{3+i}) = f(e_{1+i})-2$, $f(e_{\frac{m+2}{2}}) = -(\frac{m}{2}-1) = -f(e_m)$, for $1 \leq i \leq \frac{m-4}{2}$ $f(e_{\frac{m+2}{2}+i}) = i$, $f(e^{'}_1) = m-1$ and for $1 \leq i \leq \frac{n-1}{2}$ $f(e^{'}_{1+i}) = m-1+2i = -f(e^{'}_{\frac{n+1}{2}+i})$. The induced vertex labeling is as follows: $f^*(u_1) = 2m-2$, $f^*(u_2) = \frac{m-4}{2} = -f^*(u_{\frac{m+4}{2}})$, for $1 \leq i \leq \frac{m-4}{2}$ $f^*(u_{2+i}) = -(2i+1) = -f^*(u_{\frac{m+4}{2}+i})$, $f^*(u_{\frac{m+2}{2}}) = -m+1 = -f^*(v_1)$ and for $1 \leq i \leq \frac{n-1}{2}$ $f^*(v_{1+i}) = m-1+2i = -f^*(v_{\frac{n+1}{2}+i})$. Then $f^*(V(\langle C_m, K_{1,n}\rangle))=\{\pm(m-1), \pm(\frac{m-4}{2}), \pm 3, \pm 5, \pm 7, ..., \pm(m-3), \pm(m+1), \pm(m+3), \pm(m+5), ..., \pm(m+n-2)\} \bigcup \{2(m-1)\}$. Hence f is an edge pair sum labeling. The examples for the edge pair sum graph labeling of $< C_6, K_{1,3} >$ and $< C_8, K_{1,3} >$ are shown in Figure 9.
Case(iv). m is odd.
Subcase (a). $m = 1, 3(mod 4)$.
For $1 \leq i \leq \frac{m-3}{2}$ $f(e_i) = -2 - i = -f(e_{\frac{m+1}{2}+i})$, $f(e_{\frac{m-1}{2}}) = 1 = -f(e_m)$, $f(e_{\frac{m+1}{2}}) = -2 =$

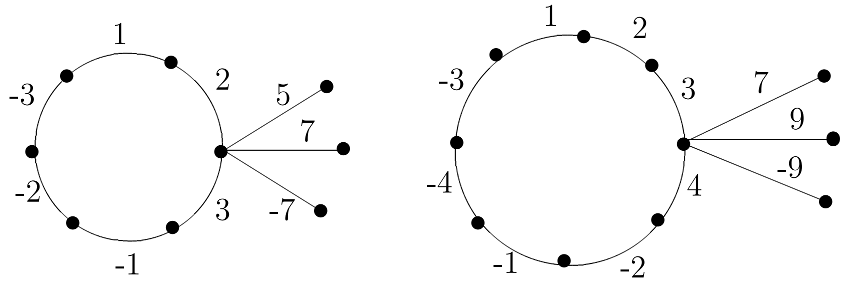

Figure 9: Edge pair sum labeling for the graph $< C_6, K_{1,3} >$ and $< C_8, K_{1,3} >$

$-f(e_1^{'})$ and for $1 \leq i \leq \frac{n-1}{2}$ $f(e_{1+i}^{'}) = m+2i = -f(e_{\frac{n+1}{2}+i}^{'})$. The induced vertex labeling is as follows: $f^*(u_1) = -2 = -f^*(v_1)$, for $1 \leq i \leq \frac{m-5}{2}$ $f^*(u_{1+i}) = -(5+2i) = -f^*(u_{\frac{m+3}{2}+i})$, $f^*(u_{\frac{m-1}{2}}) = -(\frac{m-1}{2}) = -f^*(u_m)$, $f^*(u_{\frac{m+1}{2}}) = -1 = -f^*(u_{\frac{m+3}{2}})$ and for $1 \leq i \leq \frac{n-1}{2}$ $f^*(v_{1+i}) = m+2i = -f^*(v_{\frac{n+1}{2}+i})$. Then $f^*(V(\langle C_m, K_{1,n}\rangle))$=$\{\pm 1, \pm 2, \pm(\frac{m-1}{2}), \pm 7, \pm 9, \pm 11, ..., \pm m,$ $\pm\,(m+2), \pm(m+4), \pm(m+6), ..., \pm(m+n-1)\}$.
Hence f is an edge pair sum labeling.
Subcase (b). $m = 0(mod 3)$.
Define $f(e_1) = 2 = -f(e_{\frac{m+3}{2}})$, $f(e_2) = -3 = f(e_1^{'})$, for $1 \leq i \leq \frac{m-5}{2}$ $f(e_{3+i}) = 3+i = -f(e_{\frac{m+5}{2}+i})$, $f(e_{\frac{m+1}{2}}) = -1 = -f(e_m)$ and for $1 \leq i \leq \frac{n-1}{2}$ $f(e_{1+i}^{'}) = m+2i = -f(e_{\frac{n+1}{2}+i}^{'})$. The induced vertex labeling is as follows: $f^*(u_1) = 6 = -f^*(u_{\frac{m+5}{2}})$, $f^*(u_2) = -1 = -f^*(u_3)$, for $1 \leq i \leq \frac{m-7}{2}$ $f^*(u_{3+i}) = -(7+2i) = -f^*(u_{\frac{m+5}{2}+i})$, $f^*(u_{\frac{m+1}{2}}) = \frac{m-1}{2} = -f^*(u_m)$, $f^*(u_{\frac{m+3}{2}}) = -3 = -f^*(v_1)$ and for $1 \leq i \leq \frac{n-1}{2}$ $f^*(v_{1+i}) = m+2i = -f^*(v_{\frac{n+1}{2}+i})$. Then $f^*(V(\langle C_m, K_{1,n}\rangle))$=$\{\pm 1, \pm 3, \pm 6, \pm(\frac{m-1}{2}), \pm 9, \pm 11, \pm 13, ...,$ $\pm\, m, \pm(m+2), \pm(m+4), \pm(m+6), ..., \pm(m+n-1)\}$. Hence f is an edge pair sum labeling. The example for the edge pair sum graph labeling of $< C_9, K_{1,3} >$ is shown in Figure 10. □

Theorem 6. *The graph $\langle C_m, K_{1,n}\rangle$ is an edge pair sum graph for $m \geq 4$ and n is even.*

Proof. In [3] we have proved that C_m is and edge pair sum graph for $m \geq 3$. Let f be an edge pair sum labeling of C_m.
Then $f^*(V(C_m)) = \{\pm K_1, \pm K_2, ..., \pm K_{\frac{p}{2}}\}$ if p is even.
$f^*(V(C_m)) = \{\pm K_1, \pm K_2, ..., \pm K_{\frac{p-1}{2}}\} \bigcup \{K_{\frac{p}{2}}\}$ if p is odd.
Let the vertex and edge sets are as follows: $V(\langle C_m, K_{1,n}\rangle) = V(C_m) \bigcup \{v_i : 1 \leq i \leq n\}$ and $E(\langle C_m, K_{1,n}\rangle) = E(C_m) \bigcup \{e_i^{'} : 1 \leq i \leq n\}$
Define the edge labeling $h : E(\langle C_m, K_{1,n}\rangle) \to \{\pm 1, \pm 2, \pm 3, ..., \pm(q+n)\}$.
$h(e) = f(e)$ if $e \in E(C_m)$
$h(e_i^{'}) = q + 2i : 1 \leq i \leq \frac{n}{2}$

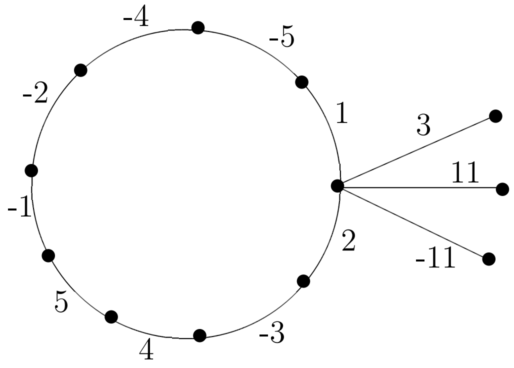

Figure 10: Edge pair sum labeling for the graph $< C_9, K_{1,3} >$

$h(e'_{\frac{n}{2}+i}) = -(q+2i) : 1 \leq i \leq \frac{n}{2}$
The induced vertex labeling is as follows:
$h^*(v_i) = q + 2i : 1 \leq i \leq \frac{n}{2}$
$h^*(v_{\frac{n}{2}+i}) = -(q+2i) : 1 \leq i \leq \frac{n}{2}$
Then $h^*(V(\langle C_m, K_{1,n}\rangle)) = \{\pm K_1, \pm K_2, ..., \pm K_{\frac{p}{2}}\}$ if p is even.
$h^*(V(\langle C_m, K_{1,n}\rangle)) = \{\pm K_1, \pm K_2, ..., \pm K_{\frac{p-1}{2}}\} \bigcup \{K_{\frac{p}{2}}\}$ if p is odd.
Hence h is an edge pair sum labeling. □

Corollary 7. *The graph $\langle C_m * K_{1,n}\rangle$ is an edge pair sum graph for $m \geq 4$ and n is odd.*

We use the previous edge labeling for this corollary. The example for the edge pair sum graph labeling of $< C_4 * K_{1,3} >$ is shown in Figure 11. □

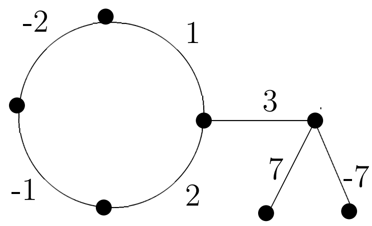

Figure 11: Edge pair sum labeling for the graph $< C_4 * K_{1,3} >$

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