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4-Prime cordiality of some classes of graphs

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ABSTRACT

Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-prime cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$, $i, j \in \{1, 2, \ldots, k\}$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with a k-prime cordial labeling is called a k-prime cordial graph. In this paper we investigate 4-prime cordial labeling behavior of complete graph, book, flower, mC_n and some more graphs.

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1 Introduction

All graphs in this paper are finite, simple and undirected. Let G be a (p, q) graph where p refers the number of vertices of G and q refers the number of edge of G. The number of vertices of a graph G is called order of G, and the number of edges is called size of G. In 1987, Cahit introduced the concept of cordial labeling of graphs [1]. Sundaram, Ponraj, Somasundaram [6] have introduced the notion of prime cordial labeling. A prime cordial labeling of a graph G with vertex set V is a bijection $f : V(G) \to \{1, 2, \ldots, |V|\}$ such that if each edge uv is assigned the label 1 if $\gcd(f(u), f(v)) = 1$ and 0 if $\gcd(f(u), f(v)) > 1$, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. Also they discussed the prime cordial labeling behavior of various graphs. Recently Ponraj et al. [4], introduced k-prime cordial labeling of graphs. In this paper we investigate 4-prime cordial labeling behavior of complete graph, book, flower, mC_n and some more graphs. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms not defined here follow from Harary [3] and Gallian [2].

2 Preliminaries

Remark 1. *A 2-prime cordial labeling is a product cordial labeling. [7]*

Definition 2.1. *The Join of two graphs $G_1 + G_2$ is obtained from G_1 and G_2 and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.*

Definition 2.2. *The graph $C_n + K_1$ is called a wheel. In a wheel, the vertex of degree n is called the central vertex and the vertices on the cycle C_n are called rim vertices. The helm H_n is the graph obtained from a wheel by attaching a pendent edge at each vertex of the n-cycle. A flower Fl_n as the graph obtained from a helm by joining each pendent vertex to the central vertex of the helm.*

Definition 2.3. *The Cartesian product graph $G_1 \Box G_2$ is defined as follows: Consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \Box G_2$ whenever $[u_1 = v_1$ and $u_2v_2 \in E(G_2)]$ or $[u_2 = v_2$ and $u_1v_1 \in E(G_1)]$.*

Definition 2.4. *The graph $C_n \Box P_2$ is called a prism. Let $V(C_n \Box P_2) = \{u_i, v_i : 1 \le i \le n\}$ and $E(C_n \Box P_2) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_nu_1, v_nv_1\} \cup \{u_iv_i : 1 \le i \le n\}$.*

Definition 2.5. *The book B_m is the graph $S_m \Box P_2$ where S_m is the star with $m+1$ vertices.*

Theorem 2.1. *[4] The cycle C_n, $n \neq 3$ is k-prime cordial where k is even.*

3 Main results

First we look into the complete graph K_n.

Theorem 3.1. *Th complete graph K_n is 4-prime cordial if and only if $n \leq 3$.*

Proof. It is easy to verify that K_1, K_2, K_3 are 4-prime cordial graphs. On the other hand suppose there exists a 4-prime cordial labeling f for $n \geq 4$ then we have the following cases.
Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4t$. In this case $v_f(1) = v_f(2) = v_f(3) = v_f(4) = t$. But $e_f(0) = 3\binom{t}{2} + t^2 = \frac{5t^2-3t}{2}$ and $e_f(1) = \binom{t}{2} + 5t^2 = \frac{11t^2-t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + t$, a contradiction.
Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4t + 1$. We have the following cases.

① $v_f(1) = t + 1$, $v_f(2) = v_f(3) = v_f(4) = t$

② $v_f(2) = t + 1$, $v_f(1) = v_f(3) = v_f(4) = t$

③ $v_f(3) = t + 1$, $v_f(1) = v_f(2) = v_f(4) = t$

④ $v_f(4) = t + 1$, $v_f(1) = v_f(2) = v_f(3) = t$

In the case ①, $e_f(1) = \binom{t+1}{2} + 3t(t+1) + 2t^2 = \frac{11t^2+7t}{2}$ and $e_f(0) = 3\binom{t}{2} + t^2 = \frac{5t^2-3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 5t$, a contradiction.
Consider the case ②. Here $e_f(1) = \binom{t}{2} + 2t(t+1) + 3t^2 = \frac{11t^2+3t}{2}$ and $e_f(0) = \binom{t+1}{2} + 2\binom{t}{2} + t(t+1) = \frac{5t^2+t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + t$, a contradiction.
For the case ③, $e_f(1) = \binom{t}{2} + 3t(t+1) + 2t^2 = \frac{11t^2+5t}{2}$ and $e_f(0) = \binom{t+1}{2} + 2\binom{t}{2} + t^2 = \frac{5t^2-t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 3t$, a contradiction.
In the case ④, $e_f(1) = \binom{t}{2} + 2t(t+1) + 3t^2 = \frac{11t^2+3t}{2}$ and $e_f(0) = \binom{t+1}{2} + 2\binom{t}{2} + t(t+1) = \frac{5t^2+t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + t$, a contradiction.
Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4t + 2$. Here we consider the following cases.

① $v_f(1) = v_f(2) = t + 1$, $v_f(3) = v_f(4) = t$

② $v_f(1) = v_f(3) = t + 1$, $v_f(2) = v_f(4) = t$

③ $v_f(1) = v_f(4) = t + 1$, $v_f(2) = v_f(3) = t$

④ $v_f(2) = v_f(3) = t + 1$, $v_f(1) = v_f(4) = t$

⑤ $v_f(2) = v_f(4) = t + 1$, $v_f(1) = v_f(3) = t$

⑥ $v_f(3) = v_f(4) = t+1$, $v_f(1) = v_f(2) = t$

In the case ①, $e_f(1) = \binom{t+1}{2} + (t+1)^2 + 3t(t+1) + t^2 = \frac{11t^2+11t}{2} + 1$ and $e_f(0) = \binom{t+1}{2} + 2\binom{t}{2} + t(t+1) = \frac{5t^2+3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 4t + 1$, a contradiction.
Consider the case ②. Here $e_f(1) = \binom{t+1}{2} + (t+1)^2 + 4t(t+1) = \frac{11t^2+13t}{2} + 1$ and $e_f(0) = \binom{t+1}{2} + 2\binom{t}{2} + t^2 = \frac{5t^2-t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 7t + 1$, a contradiction.
Consider the case ③. Here $e_f(1) = \binom{t+1}{2} + (t+1)^2 + 3t(t+1) + t^2 = \frac{11t^2+11t}{2} + 1$ and $e_f(0) = \binom{t+1}{2} + 2\binom{t}{2} + t(t+1) = \frac{5t^2+3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 4t + 1$, a contradiction.
In the case ④, $e_f(1) = \binom{t}{2} + (t+1)^2 + 3t(t+1) + t^2 = \frac{11t^2+9t}{2} + 1$ and $e_f(0) = 2\binom{t+1}{2} + \binom{t}{2} + t(t+1) = \frac{5t^2+3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 3t + 1$, a contradiction.
For the case ⑤, $e_f(1) = \binom{t}{2} + 4t(t+1) + t^2 = \frac{11t^2+7t}{2}$ and $e_f(0) = 2\binom{t+1}{2} + \binom{t}{2} + (t+1)^2 = \frac{5t^2+5t}{2} + 1$. Then $e_f(1) - e_f(0) = 3t^2 + t - 1$, a contradiction.
For the case ⑥, $e_f(1) = \binom{t}{2} + 3t(t+1) + t^2 + (t+1)^2 = \frac{11t^2+9t}{2} + 1$ and $e_f(0) = 2\binom{t+1}{2} + \binom{t}{2} + t(t+1) = \frac{5t^2+3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 3t + 1$, a contradiction.
Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4t+3$. We have the following cases.

① $v_f(1) = t$, $v_f(2) = v_f(3) = v_f(4) = t+1$

② $v_f(2) = t$, $v_f(1) = v_f(3) = v_f(4) = t+1$

③ $v_f(3) = t$, $v_f(1) = v_f(2) = v_f(4) = t+1$

④ $v_f(4) = t$, $v_f(1) = v_f(2) = v_f(3) = t+1$

In the case ①, $e_f(1) = \binom{t}{2} + 3t(t+1) + 2(t+1)^2 = \frac{11t^2+13t}{2} + 2$ and $e_f(0) = 3\binom{t+1}{2} + (t+1)^2 = \frac{5t^2+7t}{2} + 1$. Then $e_f(1) - e_f(0) = 3t^2 + 3t + 1$, a contradiction.
Consider the case ②. Here $e_f(1) = \binom{t+1}{2} + 2t(t+1) + 3(t+1)^2 = \frac{11t^2+1t}{2} + 3$ and $e_f(0) = 2\binom{t+1}{2} + \binom{t}{2} + t(t+1) = \frac{5t^2+3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 7t + 3$, a contradiction.
For the case ③, $e_f(1) = \binom{t+1}{2} + 3t(t+1) + 2(t+1)^2 = \frac{11t^2+15t}{2} + 2$ and $e_f(0) = 2\binom{t+1}{2} + \binom{t}{2} + (t+1)^2 = \frac{5t^2+5t}{2} + 1$. Then $e_f(1) - e_f(0) = 3t^2 + 5t + 1$, a contradiction.
In the case ④, $e_f(1) = \binom{t+1}{2} + 2t(t+1) + 3(t+1)^2 = \frac{11t^2+17t}{2} + 3$ and $e_f(0) = 2\binom{t+1}{2} + 2\binom{t}{2} + t(t+1) = \frac{5t^2+3t}{2}$. Then $e_f(1) - e_f(0) = 3t^2 + 7t + 3$, a contradiction. □

mC_n denote the m copies of the cycle C_n.

Theorem 3.2. *The graph mC_n is 4-prime cordial for all values of m and $n \geq 3$.*

Proof. We consider the following cases.
Case 1. m is even.
Subcase 1a. n is even.
Assign the label 2, 4 to the first two vertices of the first cycle, then assign $2, 4$ to the next two vertices of the first two cycles and so on. In this process the last vertex of the

cycle received the label 2. Next we move to second cycle and assign the label as in first cycle. Proceeding like this until we assign the label to the $\frac{m}{2}^{th}$ cycle. We now move to the $\frac{m+2}{2}^{th}$ cycle. Assign the labels $1, 3$ to the first two vertices of the $\frac{m+2}{2}^{th}$ cycle and assign $1, 3$ to the next two vertices and so on. Clearly 3 is the label of the last vertex of the $\frac{m+2}{2}^{th}, \ldots, m^{th}$ cycle vertices.

Subcase 1b. n is odd.

As in subcase 1a, assign the label to the $n-1$ vertices of the first cycle. Then assign the label 2 to the last vertex of the first cycle. Now we move to the second cycle. Assign the labels $4, 2$ to the first two vertices of the second cycle and $4, 2$ to the next two vertices and so on. This process is stopped at the $(n-1)^{th}$ vertex. Then assign 4 to the n^{th} vertex of the second cycle. Next we move to the third cycle and assign the label to the vertices as in first cycle. Similarly assign the label to the 4^{th} cycle as in second cycle. That is the i^{th} cycle vertices is labeled as in $(i-2)^{th}$ cycle. Proceeding like this until we reach the $\frac{m}{2}^{th}$ cycle. We now consider the $\frac{m+2}{2}^{th}$ cycle. Assign the label to the first, second, $\ldots$ $(n-1)^{th}$ values of the $\frac{m+2}{2}^{th}$ cycle as in subcase 1a. Then assign 3 to the last vertex of the $\frac{m+2}{2}^{th}$ cycle. Next we move to the $\frac{m+4}{2}^{th}$ cycle and assign assign the labels $3, 1$ to the first two vertices and assign $3, 1$ to the next two vertices and so on. In this process the $(n-1)^{th}$ vertex received the label 1. Then assign the label 3 to the last vertex of the $\frac{m+4}{2}^{th}$ cycle. Next assign the label to the vertices of the $\frac{m+4}{2}^{th}$ cycle. Next assign the label to the vertices of $\frac{m+6}{2}^{th}$ cycle as in $\frac{m}{2}^{th}$ cycle and $\frac{m+8}{2}^{th}$ cycle as in $\frac{m+4}{2}^{th}$ cycle. That is the i^{th} cycle is labeled as in $(i-2)^{th}$ cycle.

Case 2. m is odd.

As in case 1, assign the label to the first, second, $\ldots$, $(m-1)^{th}$ cycle vertices. We now consider the m^{th} cycle. By theorem 2.1, any cycle has a 4-prime cordial labeling. Let g be such a labeling. Assign the label to the m^{th} cycle as in g. It is easy to verify that this vertex labeling is a 4-prime cordial labeling. □

Next we consider the One point union of m paths.

Theorem 3.3. *One point union of m paths P_n is 4-prime cordial.*

Proof. Let the common vertex be u.

Case 1. n is odd.

Subcase 1a. m is odd.

Assign the label 2 to the common vertex u. Next we move to the first row. Assign the labels $2, 4$ to the first two vertices of the first row. Then assign $2, 4$ to the next two vertices and continue in this pattern. Note that in this process the last vertex of the first row received the label 2. We now move to the second row. Assign the labels $4, 2$ to the first two vertices and $4, 2$ to the next two vertices and so on. Clearly, the last vertex of the second row received the label 4. Next we move to the third row and assign the label $2, 4$

to the first two vertices of the third row, and $2,4$ to the next two vertices and so on. Note that 2 is the label of the last vertex. Next consider the fourth row and assign the labels to the first two vertices by $4,2$ and so on. Proceed like this until we have labeled $\frac{n-2}{2}$ row vertices.
Next consider the $\frac{n+1}{2}^{th}$ row. Assign the labels $1,3$ to the first two vertices and $1,3$ to the next two vertices and so on. Clearly 1 is the label of the last vertex. Next we move to the $\frac{n+3}{2}^{th}$ row. Assign the labels $3,1$ to the first two vertices and $3,1$ to the next two vertices and so on. The label of the last vertex is 3. Proceed like this till the n^{th} row.
Subcase 1b. m is even.
As in subcase 1a, assign the labels to the first two vertices of the first row by $2,4$ and next two vertices by $2,4$ and so on. In this case the last vertex received the label 4. We now move to next row and assign the label to the first two vertices by $2,4$ and next two vertices by $2,4$ and so on. Proceed like this till $\frac{n-1}{2}^{th}$ row.
Next we move to $\frac{n+1}{2}^{th}$ row. Assign the labels to the first row vertices by $1,3$ and next two vertices by $1,3$ and so on. The last vertex receives the label 3. Next we move on to the $\frac{n+3}{2}^{th}$ row. Assign the labels $3,1$ to th first two vertices and next two vertices by $3,1$ and so on. The last vertex receives the label 1. Proceed in this fashion to the next row and assign the labels $1,3$ to the first two vertices and next two vertices by $1,3$ and so on. Proceeding in this fashion to the n^{th} row, clearly the vertices of the n^{th} row are labeled as $1,3,1,3,1,\ldots,1,3$.
Note that the labeling pattern in this case is such that the label of the last vertex of the $\frac{n+1}{2}^{th}$ row and the label of the first vertex of $\frac{n+3}{2}^{th}$ row are the same. Also, the n^{th} row vertices are labeled in the same pattern as in $\frac{n+1}{2}^{th}$ row vertices.
Case 2. n is even.
Assign the labels to the vertices of the first, second, $\ldots$ $\frac{n-2}{2}^{th}$ rows as in case 1. Next we move to the $\frac{n+2}{2}^{th}$ row. Assign the labels to the $\frac{n+2}{2}^{th}$ row and subsequent rows as in case 1. We consider the $\frac{n}{2}^{th}$ row. Assign the labels to the first $\left\lfloor \frac{m}{2} \right\rfloor$ vertices as in the $\frac{n-2}{2}^{th}$ row corresponding vertices and assign the $\left\lfloor \frac{m}{2} \right\rfloor$, $\left\lfloor \frac{m}{2} \right\rfloor + 1$, $\ldots$, m^{th} vertex of the $\frac{n}{2}^{th}$ row as in the corresponding vertices of the n^{th} row. Obviously this vertex labeling is a 4-prime cordial labeling. □

Example 3.1. *A 4-prime cordial labeling of one point union of* 8 *paths* P_6 *is given in figure 1.*

Now we investigate the 4-prime cordiality of $K_2 + mK_1$.

Theorem 3.4. $K_2 + mK_1$ *is* 4*-prime cordial if and only if* $m \equiv 0,1,3 \pmod 4$.

Proof. Let $V(K_2+mK_1) = \{u, v, u_i : 1 \le i \le m\}$ and $E(K_2+mK_1) = \{uv, uu_i, vu_i : 1 \le i \le m\}$. Clearly K_2+mK_1 has $m+2$ verices and $2m+1$ edges. We divide the proof into four cases.

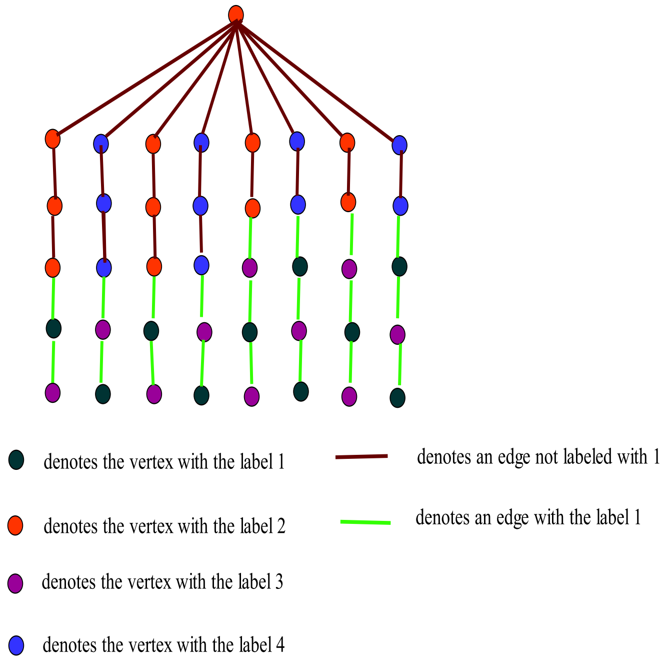

Figure 1:

Case 1. $m \equiv 0 \pmod 4$.
Let $m = 4t$. We define a labeling $f : V(K_2 + mK_1) \to \{1, 2, 3, 4\}$ by $f(u) = 2$, $f(v) = 4$ and

$$\begin{array}{rcl} f(u_i) & = & 2, \quad 1 \le i \le t \\ f(u_{t+i}) & = & 3, \quad 1 \le i \le t \\ f(u_{2t+i}) & = & 4, \quad 1 \le i \le t \\ f(u_{3t+i}) & = & 1, \quad 1 \le i \le t. \end{array}$$

In this case $v_f(1) = v_f(3) = t$, $v_f(2) = v_f(4) = t + 1$ and $e_f(0) = 4t + 1$, $e_f(1) = 4t$.
Case 2. $m \equiv 1 \pmod 4$.
Let $m = 4t + 1$. Assign the labels to the vertices $u, v, u_i (1 \le i \le 4t)$ as in case 1. Then put the label 3 to u_{4t+1}. Let g be the above vertex labeling. Here $v_g(1) = t$, $v_g(2) = v_g(3) = v_g(4) = t + 1$ and $e_g(0) = 4t + 1$, $e_g(1) = 4t + 2$.
Case 3. $m \equiv 2 \pmod 4$.
Let $m = 4t + 2$. Suppose there exists a 4-prime cordial labeling ϕ.
Subcase 3.1. $\phi(u) = \phi(v) = 1$.
Here $e_\phi(0) = 0$ and $e_\phi(1) = 8t + 5$. Then $e_\phi(1) - e_\phi(0) = 8t + 5$, a contradiction.
Subcase 3.2. $\phi(u) = 1$, $\phi(v) = 2$.
Here $e_\phi(0) = 2t + 1$ and $e_\phi(1) = 6t + 4$. Then $e_\phi(1) - e_\phi(0) = 4t + 3$, a contradiction.
Subcase 3.3. $\phi(u) = 1$, $\phi(v) = 3$.
Here $e_\phi(0) = t$ and $e_\phi(1) = 7t + 5$. Then $e_\phi(1) - e_\phi(0) = 6t + 5$, a contradiction.

Subcase 3.4. $\phi(u) = 1$, $\phi(v) = 4$.
Similar to subcase 3.2.
Subcase 3.5. $\phi(u) = \phi(v) = 2$.
In this case $e_\phi(0) = 4t + 1$ and $e_\phi(1) = 4t + 4$. Then $e_\phi(1) - e_\phi(0) = 3$, a contradiction.
Subcase 3.6. $\phi(u) = \phi(v) = 3$.
In this case $e_\phi(0) = 2t-1$ and $e_\phi(1) = 6t+6$. Then $e_\phi(1)-e_\phi(0) = 4t+5$, a contradiction.
Subcase 3.7. $\phi(u) = \phi(v) = 4$.
Similar to subcase 3.5.
Subcase 3.8. $\phi(u) = 2$, $\phi(v) = 3$.
In this case $e_\phi(0) = 3t+1$ and $e_\phi(1) = 5t+4$. Then $e_\phi(1)-e_\phi(0) = 2t+3$, a contradiction.
Subcase 3.9. $\phi(u) = 2$, $\phi(v) = 4$.
Similar to subcase 3.5.
Subcase 3.10. $\phi(u) = 3$, $\phi(v) = 4$.
Similar to subcase 3.8.
Case 4. $m \equiv 3 \pmod 4$.
Let $m = 4t + 3$. Assign the labels to the vertices $u, v, u_i\,(1 \leq i \leq 4t + 1)$ as in case 2. Then put the label $2, 1$ to u_{4t+2}, u_{4t+3} respectively. If ψ be the above vertex labeling then $v_\psi(1) = v_\psi(3) = v_\psi(4) = t + 1$, $v_\psi(2) = t + 2$ and $e_\psi(0) = 4t + 3$, $e_\psi(1) = 4t + 4$. □

Next investigation is about the prism $C_n \square P_2$.

Theorem 3.5. *The prism $C_n \square P_2$ is 4-prime cordial if and only if $n \neq 4$.*

Proof. First we observe that $C_n \square P_2$ has $2n$ vertices and $3n$ edges. We consider the folowing cases.
Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4t$. It is easy to see that $C_4 \square P_2$ is not 4-prime as it has number of edges not labeled with 1 is not more than 5. So we may assume that $t > 1$. First we consider the vertices $u_1, u_2, \ldots, u_n$. Put the integer 2 to the first 24 vertics, namely, $u_1, u_2, \ldots, u_{2t}$. For the vertices u_{2t+1}, u_{2t+2}, u_{2t+3}, we allocate the integer 3. The remaining vertices from u_{2t+4} to u_{4t} are labeled by $1, 3$ alternatively so that u_{4t} receive the label 1. Now we move to the vertices $v_1, v_2, \ldots, v_n$. Assign the label 4 to the vertices $v_1, v_2, \ldots, v_{2t}$. Then put the label 1 to the vertices v_{2t+1}, v_{2t+2}, v_{2t+3}. The remaining vertices v_i where $2t + 4 \leq i \leq 4t$ are labeled with 3 or 1 according as i is even or odd. One can easily check that each label $1, 2, 3, 4$ used to the vertices of the prism is exactly $\frac{n}{2}$. Also the number of edges labeled with 1 and not labeled with 1 are each $\frac{3n}{2}$.
Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4t + 1$. Assign the labels to the vertices u_i, v_i, $(1 \leq i \leq 4t)$ as in case 1. Then relabel the vertices u_{2t+1}, u_{2t+2}, v_{2t+1}, v_{2t+2} by $2, 1, 4, 3$ respectively. Finally we use the labels $3, 1$ to assign the vertices u_{2t+1}, v_{2t+1} respectively. If f denote the above labeling then $v_f(1) = v_f(3) = 2t$, $v_f(2) = v_f(4) = 2t + 1$ and $e_f(0) = 6t + 1$, $e_f(1) = 6t + 2$.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4t + 2$. Assign the labels to the vertices u_i, v_i, $(1 \le i \le 4t)$ as in case 1. Then relabel the vertices u_{2t+2}, u_{2t+4}, v_{2t+2}, v_{2t+4}, u_i, v_i, $(2t+5 \le i \le 4t+1)$ with 1 if they are already labeled by 3 or relabel with 3 if they are already labeled with 1. If we denote g as this labeling then $v_g(1) = v_g(2) = v_g(3) = v_g(4) = 2t + 1$ and $e_g(0) = e_g(1) = 6t + 3$.
Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4t+3$. Assign the labels to the vertices u_i, v_i, $(1 \le i \le 2t+1)$ as in case 3. Then put the integers $3, 4$ to the vertices u_{2t+2}, v_{2t+4} respectively. Then assign the label 1 to the vertices u_i, $(2t + 3 \le i \le 4t + 3)$. If i is odd and put the label 3 if i is even. For he vertices v_i, $(2t + 3 \le i \le 4t + 3)$ assign 3 or 1 according as i is odd or even. If ϕ denotes the above labeling then $v_\phi(1) = v_\phi(3) = 2t$, $v_\phi(2) = v_\phi(4) = 2t + 1$ and $e_\phi(0) = 6t + 4$, $e_\phi(1) = 6t + 5$. □

Finally we look into the flower and book graphs.

Theorem 3.6. *Flowers Fl_n are 4-prime cordial for all n.*

Proof. Let $V(Fl_n) = \{u, u_i, v_i : 1 \le i \le n\}$ and $E(Fl_n) = \{u_iu_{i+1} : 1 \le i \le n-1\} \cup \{u_nu_1\} \cup \{uu_i, u_iv_i, vv_i : 1 \le i \le n\}$. Clearly Fl_n has $2n + 1$ vertices and $4n$ edges.
Case 1. $n \equiv 0 \pmod 4$.
Let $n = 4t$. Define a labeling $f : V(Fl_n) \to \{1, 2, 3, 4\}$ by $f(u) = 2$, $f(u_{2t+1}) = 3$, $f(u_{2t+2}) = 1$ and

$$\begin{array}{lcl} f(u_i) & = & 2, \;\; 1 \le i \le 2t \\ f(u_{2t+2i-1}) & = & 3, \;\; 1 \le i \le t \\ f(u_{2t+2i}) & = & 1, \;\; 1 \le i \le t \\ f(v_i) & = & 4, \;\; 1 \le i \le 2t \\ f(v_{2t+2i+1}) & = & 1, \;\; 1 \le i \le t-1 \\ f(v_{2t+2i+2}) & = & 3, \;\; 1 \le i \le t-1. \end{array}$$

Here $v_f(1) = v_f(3) = v_f(4) = 2t$, $v_f(2) = 2t + 1$ and $e_f(0) = e_f(1) = 8t$.
Case 2. $n \equiv 1 \pmod 4$.
Let $n = 4t + 1$. Define a function $g : V(Fl_n) \to \{1, 2, 3, 4\}$ by $g(u) = 2$, $g(u_{2t+1}) = 4$, $g(v_{2t+1}) = g(v_{2t+2}) = 3$, $g(v_{2t+3}) = 1$ and

$$\begin{array}{lcl} g(u_i) & = & 2, \;\; 1 \le i \le 2t \\ g(u_{2t+2i}) & = & 3, \;\; 1 \le i \le t \\ g(u_{2t+2i+1}) & = & 1, \;\; 1 \le i \le t \\ g(v_i) & = & 4, \;\; 1 \le i \le 2t \\ g(v_{2t+2i+2}) & = & 1, \;\; 1 \le i \le t-1 \\ g(v_{2t+2i+3}) & = & 3, \;\; 1 \le i \le t-1. \end{array}$$

Here $v_g(1) = 2t$, $v_g(2) = v_g(3) = v_g(4) = 2t + 1$ and $e_g(0) = e_g(1) = 8t + 2$.

Case 3. $n \equiv 2 \pmod 4$.
Let $n = 4t + 2$. Define a function $\phi : V(Fl_n) \to \{1, 2, 3, 4\}$ by $\phi(u) = 2$, $\phi(u_{2t+2}) = 3$, $\phi(v_{2t+3}) = 1$ and

$$\begin{array}{lcl} \phi(u_i) & = & 2, \ \ 1 \le i \le 2t+1 \\ \phi(u_{2t+2i}) & = & 3, \ \ 1 \le i \le t+1 \\ \phi(u_{2t+2i+1}) & = & 1, \ \ 1 \le i \le t \\ \phi(v_i) & = & 4, \ \ 1 \le i \le 2t+1 \\ \phi(v_{2t+2i+2}) & = & 1, \ \ 1 \le i \le t \\ \phi(v_{2t+2i+3}) & = & 3, \ \ 1 \le i \le t-1. \end{array}$$

Here $v_\phi(2) = 2t + 2$, $v_\phi(1) = v_\phi(3) = v_\phi(4) = 2t + 1$ and $e_\phi(0) = e_\phi(1) = 8t + 4$.
Case 4. $n \equiv 3 \pmod 4$.
Let $n = 4t + 3$. For Fl_3, put the labels $2, 3, 4$ to the vertices u_1, u_2, u_3 respectively and 2 for the vertex u. For the vertices v_1, v_2, v_3 respectively, assign the labels $4, 1, 3$. Obviously this labeling is a 4-prime cordial labeling of Fl_3. Let $t > 0$. We define a map $\psi : V(Fl_n) \to \{1, 2, 3, 4\}$ by $\psi(u) = 2$, $\psi(u_{2t+2}) = 4$, $\psi(v_{2t+2}) = \psi(v_{2t+3}) = 3$, $\psi(v_{2t+4}) = 1$ and

$$\begin{array}{lcl} \psi(u_i) & = & 2, \ \ 1 \le i \le 2t+1 \\ \psi(u_{2t+2i+1}) & = & 3, \ \ 1 \le i \le t+1 \\ \psi(u_{2t+2i+2}) & = & 1, \ \ 1 \le i \le t \\ \psi(v_i) & = & 4, \ \ 1 \le i \le 2t+1 \\ \psi(v_{2t+2i+3}) & = & 1, \ \ 1 \le i \le t \\ \psi(v_{2t+2i+4}) & = & 3, \ \ 1 \le i \le t-1. \end{array}$$

Here $v_\psi(1) = 2t + 1$, $v_\psi(2) = v_\psi(3) = v_\psi(4) = 2t + 2$ and $e_\psi(0) = e_\psi(1) = 8t + 6$.
Hence Fl_n is 4-prime cordial for all n. □

Theorem 3.7. *The book B_m is 4-prime cordial if and only if $m > 1$.*

Proof. Let $V(B_m) = \{u, v, u_i, v_i : 1 \le i \le m\}$ and $E(B_m) = \{uv, uu_i, u_iv_i, v_i : 1 \le i \le m\}$. Clearly, the number of vertices and edges in B_m are $2m + 2$, $3m + 1$ repectively. It is easy to see that $B_1 \cong C_4$ is not a 4-prime cordial graph.
Case 1. m is even.
Let $m = 2t$. Define a map $f : V(B_m) \to \{1, 2, 3, 4\}$ by $f(u) = 2$, $f(v) = 4$ and

$$\begin{array}{lcl} f(u_i) & = & 2, \ \ 1 \le i \le t \\ f(u_{t+i}) & = & 3, \ \ 1 \le i \le t \\ f(v_i) & = & 4, \ \ 1 \le i \le t \\ f(v_{t+i}) & = & 1, \ \ 1 \le i \le t. \end{array}$$

Here $v_f(1) = v_f(3) = t$, $v_f(2) = v_f(4) = t + 1$ and $e_f(0) = 3t + 1$, $e_f(1) = 3t$.
Case 2. m is odd.

Let $m = 2t+1$. Define a function $g : V(B_m) \rightarrow \{1,2,3,4\}$ by $g(u) = 2$, $g(v) = 4$, $g(u_{2t+1}) = 1$, $g(v_{t+1}) = 3$ and

$$\begin{array}{lcl} g(u_i) & = & 2, \quad 1 \leq i \leq t \\ g(u_{t+i}) & = & 3, \quad 1 \leq i \leq t \\ g(v_i) & = & 4, \quad 1 \leq i \leq t \\ g(v_{t+1+i}) & = & 1, \quad 1 \leq i \leq t. \end{array}$$

Here $v_g(1) = v_g(2) = v_g(3) = v_g(4) = t+1$ and $e_g(0) = e_g(1) = 3t+2$. □

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