Towards a measure of vulnerability, tenacity of a Graph

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ABSTRACT

If we think of the graph as modeling a network, the vulnerability measure the resistance of the network to disruption of operation after the failure of certain stations or communication links. Many graph theoretical parameters have been used to describe the vulnerability of communication networks, including connectivity, integrity, toughness, binding number and tenacity.
In this paper we discuss tenacity and its properties in vulnerability calculation.

Keyword: connectivity, integrity, toughness, binding number and tenacity.

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Introduction

We consider only finite undirected graphs without loops and multiple edges. Let G be a graph. We denote by V(G), E(G) and $| V(G) |$ the set of vertices, the set of edges and the order of a graph G, respectively. For a subset S of V(G), let G[S] denote the subgraph of G induced by S. The degree of a vertex v in a graph G is denoted by $d_G(v)$. The *end vertex* v of a graph G is a vertex of degree 1 in G, that is $d_G(v) = 1$.
A k-tree of a connected graph is a spanning tree with maximum degree k. Of course, for k=2, this notion reduces to that of a hamiltonian path.

The concept of tenacity of a graph G was introduced in [2], as a useful measure of the "vulnerability" of G. In [6], we compared integrity, connectivity, binding number, toughness, and tenacity for several classes of graphs. The results suggest that tenacity is a

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most suitable measure of stability or vulnerability in that for many graphs it is best able to distinguish between graphs that intuitively should have different levels of vulnerability. In [3,4,7,8,9,10], they studied more about this new invariant. The tenacity of a graph G, T(G), is defined by $T(G) = min\{\frac{|S|+\tau(G-S)}{\omega(G-S)}\}$, where the minimum is taken over all vertex cutsets S of G. We define $\tau(G-S)$ to be the number of the vertices in the largest component of the graph G - S, and $\omega(G-S)$ be the number of components of G - S. A connected graph G is called T-tenacious if $\mid S \mid + \tau(G-S) \geq T\omega(G-S)$ holds for any subset S of vertices of G with $\omega(G-S) > 1$. If G is not complete, then there is a largest T such that G is T-tenacious ; this T is the tenacity of G. On the other hand, a complete graph contains no vertex cutset and so it is T-tenacious for every T. Accordingly, we define $T(K_p) = \infty$ for every p ($p \geq 1$). A set $S \subseteq V(G)$ is said to be a T-set of G if $T(G) = \frac{|S|+\tau(G-S)}{\omega(G-S)}$.
Any undefined terms can be found in the standard references on graph theory,including Bondy and Murty [1].

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