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Remainder Cordial Labeling of Graphs

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ABSTRACT

In this paper we introduce remainder cordial labeling of graphs. Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, ..., p\}$ be a $1-1$ map. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ or $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function$f$ is called a remainder cordial labeling of G if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labeled with even integers and odd integers. A graph G with a remainder cordial labeling is called a remainder cordial graph. We investigate the remainder cordial behavior of path, cycle, star, bistar, crown, comb, wheel, complete bipartite $K_{2,n}$ graph. Finally we propose a conjecture on complete graph K_n.

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1 Introduction

In this paper we consider only finite and simple graphs. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their join $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1 \text{ and } v \in V_2\}$. The graph $W_n = C_n + K_1$ is called a wheel. In a wheel, a vertex of degree 3 is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with the rim and the other incident with the central vertex are called spokes. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. Cahit [1] introduced the concept of cordial labeling of graphs. Ponraj et al. [4] introduced quotient cordial labeling of graphs. In [4, 5], Ponraj et al. investigate the quotient cordial labeling behavior of path, cycle, star, bistar, complete graph, subdivided star $S(K_{1,n})$, subdivided bistar $S(B_{n,n})$ and union of some star related graphs. Motivated by this labeling, we introduce remainder cordial labeling of graphs. Also, in this paper we investigate the remainder cordial labeling behavior of path, cycle, star, bistar, crown, comb, wheel, complete bipartite graph etc. Finally we propose a conjecture for K_n. Terms that are not defined here follows from Harary [3] and Gallian [2].

2 Remainder cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, p\}$ be an injective map. For each edge uv assign the label r where r is the remainder when $f(u)$ is divided by $f(v)$ or $f(v)$ is divided by $f(u)$ according as $f(u) \geq f(v)$ or $f(v) \geq f(u)$. The function f is called a remainder cordial labeling of G if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with even integers and odd integers. A graph G with a remainder cordial labeling is called a remainder cordial graph.

Example 2.2. A simple example of a remainder cordial graph is shown in Figure 1.

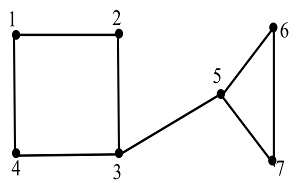

Figure 1:

First we investigate the remainder cordial labeling behavior of a path.

Theorem 2.3. *All paths are remainder cordial.*

Proof. Let P_n be the path $u_1u_2 \dots u_{n-1}u_n$.
Case (i). $n \equiv 0 \pmod 4$
Assign the labels $1, 2, 3, \dots, \frac{n}{2}$ consecutively to the vertices $u_1, u_2, \dots, u_{\frac{n}{2}}$ and then assign the labels $\frac{n}{2}+2, \frac{n}{2}+4, \dots, n$ to the vertices $u_{\frac{n}{2}+1}, u_{\frac{n}{2}+2}, \dots, u_{\frac{n}{2}+\frac{n}{4}}$ respectively. Finally assign the labels $\frac{n}{2}+1, \frac{n}{2}+3, \dots, \frac{n}{2}+\frac{n}{2}-1$ to the remaining vertices consecutively.
Case(ii). $n \equiv 1 \pmod 4$
In this case assign the labels $1, 2, 3, ..., \frac{n+1}{2}$ consecutively to the vertices u_1, u_2,..., $u_{\frac{n+1}{2}}$ respectively. Next consider the vertex $u_{\frac{n+3}{2}}$. Assign the label $\frac{n+1}{2}+2, \frac{n+1}{2}+2, \dots, \frac{3n+1}{4}$ to the vertices $u_{\frac{n+3}{2}}$, $u_{\frac{n+5}{2}}$,..., $u_{\frac{3n+1}{4}}$. Next assign the labels $\frac{n+3}{2}$, $\frac{n+7}{2}$,..., $n-1$ to the vertices $u_{\frac{3n+5}{4}}$, $u_{\frac{3n+9}{4}}$,..., u_n respectively.
Case(iii). $n \equiv 2 \pmod 4$
In this case assign the labels $1, 2, 3, \dots, \frac{n}{2}$ to the vertices $u_1, u_2, \dots, u_{\frac{n}{2}}$. Next assign the labels $\frac{n}{2}+2, \frac{n}{2}+4, \dots, n-1$ to the vertices $u_{\frac{n}{2}+1}, u_{\frac{n}{2}+2}, \dots, u_{\frac{n}{2}+\frac{n}{4}-2}$. Finally assign the labels $\frac{n}{2}+1, \frac{n}{2}+3, \dots, n$ to the vertices $u_{\frac{n}{2}+\frac{n-2}{4}+1}, u_{\frac{n}{2}+\frac{n-2}{4}2}, \dots, u_n$.
Case(iv). $n \equiv 3 \pmod 4$
Assign the integers $1, 2, \dots, \frac{n+1}{2}$ continuously to the vertices $u_1, u_2, \dots, u_{\frac{n+1}{2}}$. Next assign the labels $\frac{n+5}{2}, \frac{n+5}{2}+2, \dots, n-1$ to the vertices $u_{\frac{n+1}{2}+1}$, $u_{\frac{n+1}{2}+2}$,..., $u_{\frac{n+1}{2}+\frac{n-3}{4}}$. Finally, assign the labels $\frac{n+3}{2}$, $\frac{n+7}{2}$, ..., n to the remaining vertices $u_{\frac{3n-1}{4}+1}$, $u_{\frac{3n-1}{4}+2}$, ..., u_n. □

Next to be considered are cycle and the wheel.

Theorem 2.4. *All cycles C_n are remainder cordial.*

Proof. Let C_n be the cycle $u_1u_2u_3 \dots u_nu_1$.
Case(i). n is odd
Clearly the vertex labeling of the path P_n given in Theorem 2.3, is also a remainder cordial labeling of C_n.
Case(ii). n is even
Assign the labels $1, 2, \dots, \frac{n}{2}, \frac{n}{2}+1$ to the vertices $u_1, u_2, \dots, u_{\frac{n}{2}}, u_{\frac{n}{2}+1}$.
Subcase(i). $n \equiv 2 \pmod 4$
In this case assign the labels $\frac{n}{2}+3, \frac{n}{2}+5, \dots, n$ to the vertices $u_{\frac{n}{2}+2}$, $u_{\frac{n}{2}+3}$, ..., $u_{\frac{3n-2}{4}+1}$. Next assign the labels $\frac{n}{2}+2$, $\frac{n}{2}+4$, ..., $n-1$ to the vertices $u_{\frac{3n-2}{4}+2}$, $u_{\frac{3n-2}{4}+3}$, ..., u_n.
Subcase(ii). $n \equiv 0 \pmod 4$
In this case assign the labels $\frac{n}{2}+3, \frac{n}{2}+5, \dots, n-1$ to the vertices $u_{\frac{n}{2}+2}$, $u_{\frac{n}{2}+3}$,..., $u_{\frac{3n-4}{4}+1}$. Finally assign the labels $\frac{n}{2}+2$, $\frac{n}{2}+4$, ..., n to the remaining vertices consecutively. □

Theorem 2.5. *The wheel W_n is remainder cordial for all even n.*

Proof. Let $W_n = C_n + K_1$, where C_n is the cycle $u_1u_2u_3 \dots u_nu_1$ and $V(K_1) = u$. We now assign the labels to the vertices of the wheel W_n. Assign the label 1 to the central vertex. Next assign the labels $2, 3, \dots, n, n+1$ consecutively to the rim vertices $u_1, u_2, u_3, \dots, u_n$. This vertex labeling f is clearly a remainder cordial labeling, since $e_f(0) = e_f(1) = n$. □

Illustration 1. An illustration of wheel W_8 for remainder cordial graph is shown in Figure 2.

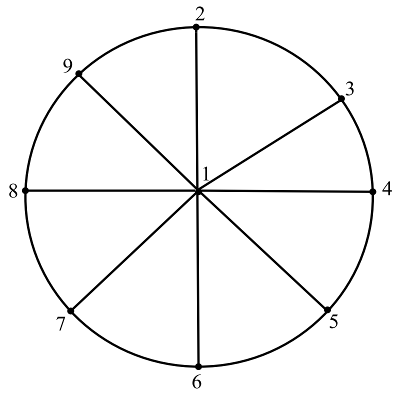

Figure 2:

We now investigate the star.

Theorem 2.6. *The star $K_{1,n}$ is remainder cordial for all values of n.*

Proof. Assign the label 2 to central vertex and assign the labels $1, 3, 4, \ldots, n, n+1$ to the pendant vertices. Clearly this vertex labeling is a remainder cordial labeling. □

Next our focus of attention is the bistar.

Theorem 2.7. *The bistar $B_{m,n}$ is remainder cordial.*

Proof. Let $V(B_{m,n}) = \{u, v, u_i, v_j : 1 \leq i \leq m,\ 1 \leq j \leq n\}$ and $E(B_{m,n}) = \{uv, uu_i, vv_j : 1 \leq i \leq m,\ 1 \leq j \leq n\}$. Clearly $B_{m,n}$ has $m+n$ vertices and $m+n+1$ edges. Assign the labels $2, 4$ respectively to the vertices u, v. Next we move to the vertices u_i. Assign the labels $1, 3, 5, 6, \ldots, m+2$ respectively to the vertices $u_1, u_2, \ldots, u_{m+2}$. Finally assign the labels $m+3, m+4, \ldots, m+n$ to the remaining vertices. Table 1, establishes that this labeling f is a remainder cordial labeling.

Nature of m and n	$e_f(0)$	$e_f(1)$
m and n are of same parity	$\frac{m+n}{2}+1$	$\frac{m+n}{2}$
m and n are of different parity	$\frac{m+n+1}{2}$	$\frac{m+n+1}{2}$

Table 1:

□

Next we investigate the crowns and the comb.

Theorem 2.8. *All crowns $C_n \odot K_1$ are remainder cordial.*

Proof. Let C_n be the cycle $u_1u_2u_3 \ldots u_nu_1$ and v_i $(1 \leq i \leq n)$ be the pendant vertex adjacent to u_i. Assign the labels $1, 3, 5, \ldots, 2n-1$ respectively to the vertices $u_1, u_3, u_5, \ldots, u_{2n-1}$. Next move to the pendant vertices. Assign the even labels $2, 4, \ldots, 2n$ to the vertices $v_1, v_2, \ldots, v_{2n}$ respectively. Clearly this vertex labeling f is a remainder cordial labeling, since $e_f(0) = e_f(1) = n$. □

Corollary 2.9. *The Comb $P_n \odot K_1$ is remainder cordial.*

Proof. The labeling given in Theorem 2.8 obviously a remainder cordial labeling of $P_n \odot K_1$. □

Now we investigate the complete bipartite graph $K_{2,n}$.

Theorem 2.10. *The complete bipartite graph $K_{2,n}$ is remainder cordial for all even values of n.*

Proof. Let $V(K_{2,n}) = \{u, v, u_i : 1 \leq i \leq n\}$ and $E(K_{2,n}) = \{uu_i, vv_i : 1 \leq i \leq n\}$. Assign the labels $2, 4$ respectively to the vertices u, v and assign the labels $1, 3, 5, 6, \ldots, n$ to the remaining non-labelled vertices in any order. Clearly this vertex labeling f is a remainder cordial labeling, since $e_f(0) = e_f(1) = n$. □

Our next investigation is about the complete graph K_n. According to the visual basic program 2.11 and its output 2.12 it is seen that, K_n $(4 \leq n \leq 510)$ is not remainder cordial.

Program 2.11.

```
SORT OUT REMAINDER ODD OR EVEN CODING
Dim con As New ADODB.Connection
Dim rs As New ADODB.Recordset

Private Sub Command1_Click()
If rs.BOF And rs.EOF Then rs.AddNew
rs.MoveLast


Dim no,c1, f, c2, gno, gno2, n, odd, eve, div, i, j, k, s  As Double

c1 = 0
c2 = 0
f = 0
odd = 0
eve = 0
gno = Val(Text1.Text)
gno2 = Val(Text2.Text)
n = gno2
For no = gno To gno2 Step 1
'hide these 6 statement if u dont want listbox
List1.AddItem "-----------------------"
List1.AddItem "---For Number = " & n
```

```
List1.AddItem "-----------------------"
List2.AddItem "-----------------------"
List2.AddItem "---For Number = " & n
List2.AddItem "-----------------------"

For i = n To 2 Step -1
k = i - 1
For j = k To 1 Step -1
s = i Mod j
If s Mod 2 = 0 Then
eve = eve + 1
'hide these 3 statement if u dont want listbox
List1.AddItem i & " % " & j & " = " & s
List1.Visible = True
lbleven.Visible = True
c1 = c1 + 1
Else
odd = odd + 1
'hide these 3 statement if u dont want listbox
List2.AddItem i & " % " & j & " = " & s
List2.Visible = True
lblodd.Visible = True
c2 = c2 + 1
End If
Next j
Next i


c3 = c1 - c2
rs.MoveFirst
Do While Not rs.EOF
If n = rs(0) Then
f = 1
Exit Do
End If
rs.MoveNext
Loop
If found = 0 Then
rs.MoveLast
rs.AddNew
rs(0) = n
rs(1) = c1
rs(2) = c2
```

```
rs(3) = c3
rs.Update
End If
n = n - 1
c1 = 0
c2 = 0
c3 = 0
f = 0
Next no
MsgBox "recorded"
End Sub

Private Sub Command2_Click()
List1.Clear
List2.Clear
Text1.Text = ""
List1.Visible = False
List2.Visible = False
lblodd.Visible = False
lbleven.Visible = False

Text1.SetFocus

End Sub

Private Sub Command3_Click()
End
End Sub

Private Sub Form_Load()

con.Open "Provider=Microsoft.Jet.OLEDB.4.0;

Data Source=D:\genoddeven\result.mdb;Persist Security Info=False;"

rs.Open "select * from res", con, adOpenDynamic, adLockOptimistic
End Sub
```

Output 2.12.

n	$e_f(0)$	$e_f(1)$	$\lvert e_f(0) - e_f(1)\rvert$	n	$e_f(0)$	$e_f(1)$	$\lvert e_f(0) - e_f(1)\rvert$
1	0	0	0	2	1	0	1

3	2	1	1	4	4	2	2
5	6	4	2	6	10	5	5
7	12	9	3	8	17	11	6
9	21	15	6	10	27	18	9
11	31	24	7	12	39	27	12
13	43	35	8	14	53	38	15
15	59	46	13	16	68	52	16
17	75	61	14	18	87	66	21
19	93	78	15	20	107	83	24
21	115	95	20	22	129	102	27
23	137	116	21	24	153	123	30
25	162	138	24	26	179	146	33
27	190	161	29	28	207	171	36
29	218	188	30	30	239	196	43
31	248	217	31	32	270	226	44
33	282	246	36	34	304	257	47
35	318	277	41	36	341	289	52
37	354	312	42	38	379	324	55
39	394	347	47	40	419	361	58
41	434	386	48	42	463	398	65
43	476	427	49	44	507	439	68
45	524	466	58	46	553	482	71
47	570	511	59	48	601	527	74
49	619	557	62	50	652	573	79
51	671	604	67	52	704	622	82
53	723	655	68	54	760	671	89
55	779	706	73	56	816	724	92
57	837	759	78	58	874	779	95
59	895	816	79	60	936	834	102
61	955	875	80	62	998	893	105
63	1021	932	89	64	1061	955	106
65	1087	993	94	66	1129	1016	113
67	1153	1058	95	68	1197	1081	116
69	1223	1123	100	70	1269	1146	123
71	1293	1192	101	72	1342	1214	128
73	1365	1263	102	74	1416	1285	131
75	1443	1332	111	76	1492	1358	134
77	1521	1405	116	78	1572	1431	141
79	1599	1482	117	80	1652	1508	144
81	1682	1558	124	82	1734	1587	147
83	1764	1639	125	84	1820	1666	154

85	1850	1720	130	86	1906	1749	157
87	1938	1803	135	88	1994	1834	160
89	2026	1890	136	90	2088	1917	171
91	2118	1977	141	92	2180	2006	174
93	2212	2066	146	94	2274	2097	177
95	2308	2157	151	96	2370	2190	180
97	2404	2252	152	98	2469	2284	185
99	2506	2345	161	100	2570	2380	190
101	2606	2444	162	102	2674	2477	197
103	2708	2545	163	104	2778	2578	200
105	2818	2642	176	106	2884	2681	203
107	2924	2747	177	108	2994	2784	210
109	3032	2854	178	110	3106	2889	217
111	3144	2961	183	112	3218	2998	220
113	3256	3072	184	114	3334	3107	227
115	3372	3183	189	116	3450	3220	230
117	3492	3294	198	118	3568	3335	233
119	3612	3409	203	120	3690	3450	240
121	3733	3527	206	122	3812	3569	243
123	3857	3646	211	124	3936	3690	246
125	3983	3767	216	126	4066	3809	257
127	4109	3892	217	128	4193	3935	258
129	4239	4017	222	130	4325	4060	265
131	4369	4146	223	132	4459	4187	272
133	4503	4275	228	134	4593	4318	275
135	4643	4402	241	136	4729	4451	278
137	4779	4537	242	138	4869	4584	285
139	4917	4674	243	140	5011	4719	292
141	5059	4811	248	142	5153	4858	295
143	5203	4950	253	144	5298	4998	300
145	5349	5091	258	146	54447	5141	303
147	5499	5232	267	148	5592	52867	306
149	5647	5379	268	150	5746	5429	317
151	5797	5528	269	152	5898	5578	320
153	5953	5675	278	154	6054	5727	327
155	6109	5826	283	156	6212	5878	334
157	6265	5981	284	158	6370	6033	337
159	6425	6136	289	160	6530	6190	340
161	6587	6293	294	162	6695	6346	349
163	6749	6454	295	164	6859	6507	352
165	6919	6611	308	166	7025	6670	355

167	7085	6776	309	168	7195	6833	362
169	7254	6942	312	170	7367	6998	369
171	7428	7107	321	172	7539	7167	372
173	7600	7278	322	174	7715	7336	379
175	7778	7447	331	176	7891	7509	382
177	7956	7620	336	178	8069	7684	385
179	8134	7797	337	180	8253	7857	396
181	8314	7976	338	182	8437	8034	403
183	8498	8155	343	184	8621	8215	406
185	8684	8336	348	186	8809	8396	413
187	8872	8519	353	188	8997	8581	416
189	9066	8700	366	190	9189	8766	423
191	9256	8889	367	192	9381	8955	426
193	9448	9080	368	194	9575	9146	429
195	9648	9267	381	196	9772	9338	434
197	9844	9462	382	198	9974	9529	445
199	10042	9659	383	200	10175	9725	450
201	10244	9856	388	202	10377	9924	453
203	10448	10055	393	204	10583	10123	460
205	10654	10256	398	206	10789	10326	463
207	10864	10457	407	208	10997	10531	466
209	11074	10662	412	210	11213	10732	481
211	11284	10871	413	212	11425	10941	484
213	11498	11080	418	214	11639	11152	487
215	11714	11291	423	216	11857	11363	494
217	11932	11504	428	218	12075	11578	497
219	12152	11719	433	220	12297	11793	504
221	12374	11936	438	222	12521	12010	511
223	12596	12157	439	224	12745	12231	514
225	12827	12373	454	226	12971	12454	517
227	13053	12598	455	228	13201	12677	524
229	13281	12825	456	230	13433	12902	531
231	13517	13048	469	232	13665	13131	534
233	13749	13279	470	234	13903	13358	545
235	13985	13510	475	236	14139	13591	548
237	14223	13743	480	238	14379	13824	555
239	14461	13980	481	240	14621	14059	562
241	14701	14219	482	242	14864	14297	567
243	14947	14456	491	244	15108	14538	570
245	15195	14695	500	246	15356	14779	577
247	15443	14938	505	248	15604	15024	580

249	15693	15183	510	250	15856	15269	587
251	15943	15432	511	252	16112	15514	598
253	16197	15681	516	254	16366	15765	601
255	16457	15928	529	256	16621	16019	602
257	16713	16183	530	258	16881	16272	609
259	16973	16438	535	260	17143	16527	616
261	17237	16693	544	262	17405	16786	619
263	17499	16954	545	264	17671	17045	626
265	17765	17215	550	266	17939	17306	633
267	18033	17478	555	268	18207	17571	636
269	18301	17745	556	270	18483	17832	651
271	18571	18014	557	272	18755	18101	654
273	18849	18279	570	274	19029	18372	657
275	19127	18548	579	276	19307	18643	664
277	19403	18823	580	278	19585	18918	667
279	19685	19096	589	280	19867	19193	674
281	19965	19375	590	282	20151	19470	681
283	20247	19656	591	284	20435	19751	684
285	20537	19933	604	286	20723	20032	691
287	20825	20216	609	288	21012	20316	696
289	21114	20502	612	290	21304	20601	703
291	21406	20789	617	292	21596	20890	706
293	21698	21080	618	294	21894	21177	717
295	21994	21371	623	296	22190	21470	720
297	22296	21660	636	298	22488	21765	723
299	22596	21955	641	300	22792	22058	734
301	22898	22252	646	302	23094	22357	737
303	23202	22551	651	304	23398	22658	740
305	23508	22852	656	306	23708	22957	751
307	23814	23157	657	308	24018	23260	758
309	24124	23462	662	310	24330	23565	765
311	24434	23771	663	312	24644	23872	772
313	24746	24082	664	314	24958	24183	775
315	25070	24385	685	316	25274	24496	778
317	25386	24700	686	318	25594	24809	785
319	25706	25015	691	320	25914	25126	788
321	26028	25332	696	322	26238	25443	795
323	26352	25651	701	324	26565	25761	804
325	26680	25970	710	326	26891	26084	807
327	27008	26293	715	328	27219	26409	810
329	27338	26618	720	330	27555	26730	825

331	27668	26947	721	332	27887	27059	828
333	28004	27274	730	334	28221	27390	831
335	28340	27605	735	336	28559	27721	838
337	28676	27940	736	338	28898	28055	843
339	29016	28275	741	340	29240	28390	850
341	29358	28612	746	342	29586	28725	861
343	29702	28951	751	344	29930	29066	864
345	30052	29288	764	346	30276	29409	867
347	30398	29633	765	348	30626	29752	874
349	30746	29980	766	350	30980	30095	885
351	31102	30323	779	352	31332	30444	888
353	31454	30674	780	354	31688	30793	895
355	31810	31025	785	356	32044	31146	898
357	32172	31374	798	358	32402	31501	901
359	32530	31731	799	360	32766	31854	912
361	32891	32089	802	362	33128	32213	915
363	33257	32446	811	364	33494	32572	922
365	33623	32807	816	366	33862	32933	929
367	33989	33172	817	368	34230	33298	932
369	34361	33535	826	370	34602	33663	939
371	34733	33902	831	372	34976	34030	946
373	35105	34273	832	374	35352	34399	953
375	35485	34640	845	376	35728	34772	956
377	35863	35013	850	378	36112	35141	971
379	36241	35390	851	380	36494	35516	978
381	36623	35767	856	382	36876	35895	981
383	37005	36148	857	384	37260	36276	984
385	37395	36525	870	386	37646	36659	987
387	37785	36906	879	388	38034	37044	990
389	38173	37293	880	390	38430	37425	1005
391	38565	37680	885	392	38823	37813	1010
393	38959	38069	890	394	39217	38204	1013
395	39355	38460	895	396	39617	38593	1024
397	39751	38855	896	398	40015	38988	1027
399	40155	39246	909	400	40416	39384	1032
401	40555	39645	910	402	40820	39781	1039
403	40959	40044	915	404	41224	40182	1042
405	41371	40439	932	406	41632	40583	1049
407	41779	40842	937	408	42042	40986	1056
409	42187	41249	938	410	42454	41391	1063
411	42599	41656	943	412	42866	41800	1066

413	43013	42065	948	414	43284	42207	1077
415	43429	42476	953	416	43700	42620	1080
417	43847	42889	958	418	44120	43033	1087
419	44265	43306	959	420	44546	43444	1102
421	44685	43725	960	422	44968	43863	1105
423	45111	44142	969	424	45392	44284	1108
425	45539	44561	978	426	45820	44705	1115
427	45967	44984	983	428	46248	45130	1118
429	46401	45405	996	430	46680	45555	1125
431	46831	45834	997	432	47114	45982	1132
433	47263	46265	998	434	47550	46411	1139
435	47703	46692	1011	436	47986	46844	1142
437	48141	47125	1016	438	48426	47277	1149
439	48579	47562	1017	440	48868	47712	1156
441	49026	47994	1032	442	49312	48149	1163
443	49468	48435	1033	444	49758	48588	1170
445	49914	48876	1038	446	50204	49031	1173
447	50362	49319	1043	448	50652	49476	1176
449	50810	49766	1044	450	51109	49916	1193
451	51262	50213	1049	452	51561	50365	1196
453	51716	50662	1054	454	52015	50816	1199
455	52176	51109	1067	456	52473	51267	1206
457	52632	51564	1068	458	52931	51722	1209
459	53096	52015	1081	460	53393	52177	1216
461	53556	52474	1082	462	53861	52630	1231
463	54018	52935	1083	464	54325	53091	1234
465	54488	53392	1096	466	54791	53554	1237
467	54954	53857	1097	468	55263	54015	1248
469	55424	54322	1102	470	55735	54480	1255
471	55896	54789	1107	472	56207	54949	1258
473	56370	55258	1112	474	56683	55418	1265
475	56848	55727	1121	476	57161	55889	1272
477	57328	56198	1130	478	57639	56364	1275
479	57806	56675	1131	480	58121	56839	1282
481	58288	57152	1136	482	58603	57318	1285
483	58776	57627	1149	484	59088	57798	1290
485	59262	58108	1154	486	59578	58277	1301
487	59748	58593	1155	488	60066	58762	1304
489	60238	59078	1160	490	60560	59245	1315
491	60728	59567	1161	492	61054	59732	1322
493	61222	60056	1166	494	61550	60221	1329

495	61726	60539	1187	496	62046	60714	1332
497	62224	61032	1192	498	62546	61207	1339
499	62722	61529	1193	500	63048	61702	1346
501	63224	62026	1198	502	63550	62201	1349
503	63726	62527	1199	504	64058	62698	1360
505	64232	63028	1204	506	64566	63199	1367
507	64742	63529	1213	508	65074	63704	1370
509	65250	64036	1214	510	65590	64205	1385

The output 2.12 naturally leads us to propose the following conjecture.

Conjecture 2.13. *The complete graph K_n is remainder cordial iff $n \leq 3$.*

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