Journal of Algorithms and Computation

journal homepage: http://jac.ut.ac.ir

Vector Basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-Cordial Labeling of $L_n \odot mK_1$ and $T(P_n) \odot mK_1$

R. Ponraj*[1] and R. Jeya†[1]

[1]Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India.

ABSTRACT

Let G be a (p, q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors ω_1 and ω_2 by $< \omega_1, \omega_2 >$. Let $\chi : V(G) \to S$ be a function. For edge uv assign the label $< \chi(u), \chi(v) >$. Then χ is called a vector basis S-cordial labeling of G if $|\chi_{\omega_1} - \chi_{\omega_2}| \leq 1$ and $|\delta_i - \delta_j| \leq 1$ where χ_{ω_i} denotes the number of vertices labeled with the vector ω_i and δ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph . In this paper, we prove that the graphs $L_n \odot mK_1$ and $T(P_n) \odot mK_1$ are the vector basis $\{(1, 1, 1, 1), (1, 1, 1, 0), (1, 1, 0, 0), (1, 0, 0, 0)\}$-cordial.

ARTICLE INFO

Article history:
Research paper
Received 10 March 2025
Accepted 14 May 2025
Available online 14 May 2025

Keyword: path, ladder, complete graph and total graph.

AMS subject Classification: 05C78.

*Corresponding author: R. Ponraj. Email:ponrajmaths@gmail.com
†Email:jeya67205@gmail.com

1 Introduction

In this paper, we consider finite, simple, undirected and connected graph. Let $G = (V(G), E(G))$ be a graph with p vertices and q edges where $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of the graph G. Note that $p = |V(G)|$ and $q = |E(G)|$ are the size and order of the graph G respectively. Labeling of graph was introduced by Rosa [15] in 1967. Ajitha et al. [1] have introduced the square sum graphs and square sum labeling of some classes of graphs were examined in [4, 5, 16]. A dynamic survey on graph labeling is regularly updated by Gallian [6]. The notion of cordial labeling was introduced by Cahit [3]. Cordial labeling for new class of graphs was discussed in [9]. Ponraj, Subbulakshmi and Somasundaram [14] have introduced PD-prime cordial graphs. Varatharajan et al. [17] have introduced divisor cordial graphs and investigated the divisor cordial labeling behavior of some special graphs in [18]. Moreover Barasara and Thakkar [2] have shown that ladder, circular ladder and Mobius ladder, total graph of path and total graph of cycle are divisor cordial graphs. Ponraj and Prabhu [13] have introduced the pair mean cordial graphs. Additionally pair difference cordial labeling of m-copies of path, cycle, star and ladder graphs were discussed in [10]. We use some basic definitions which are needed for the upcoming section.

Definition 1.1. *[2] The graph $L_n = P_n \times P_2$ is called the ladder.*

Definition 1.2. *[5] The corona graph $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and n copies of G_2 and joining i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2, where G_1 is graph of order n.*

Definition 1.3. *[4] For a graph $G(V, E)$, the total graph $T(G)$ has the vertex set $V(G) \cup E(G)$ and two vertices are adjacent in $T(G)$ whenever their corresponding elements are either incident or adjacent in G.*

Terms not defined here are used in the sense of Harary [7] and Herstein [8]. Ponraj and Jeya have introduced the new graph labeling technique called vector basis S-cordial labeling in [11] and they have been investigated the vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of certain thorn graphs in [12]. In this present paper, we investigate the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling behavior of $L_n \odot mK_1$ and $T(P_n) \odot mK_1$.

2 Vector Basis S-Cordial Labeling

Definition 2.1. *Let G be a (p, q) graph. Let V be an inner product space with basis S. We denote the inner product of the vectors ω_1 and ω_2 by $< \omega_1, \omega_2 >$. Let $\chi : V(G) \to S$ be a function. For edge uv assign the label $< \chi(u), \chi(v) >$. Then χ is called a vector basis S-cordial labeling of G if $|\chi_{\omega_1} - \chi_{\omega_2}| \leq 1$ and $|\delta_i - \delta_j| \leq 1$ where χ_{ω_i} denotes the number of vertices labeled with the vector ω_i and δ_i denotes the number of edges labeled with the scalar i. A graph which admits a vector basis S-cordial labeling is called a vector basis S-cordial graph.*

Figure 1 illustrates a simple example of vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial graph.

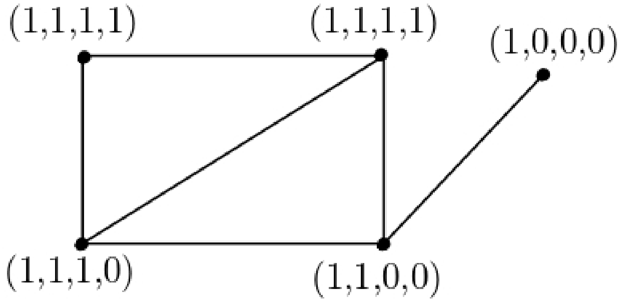

Figure 1: An Example of Vector Basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-Cordial Graph

3 Main Results

In this section, we prove that the graphs $L_n \odot mK_1$, $n \geq 4$ & $m \geq 1$ and $T(P_n) \odot mK_1$, $n \geq 4$ and $m \geq 1$ are the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial.

Theorem 3.1. *The graph $L_n \odot mK_1$ is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial for all $n \geq 4$ and $m \geq 1$.*

Proof. Let $V(L_n \odot mK_1) = \{u_i, v_i, u_{i,j}, v_{i,j} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq m\}$ and $E(L_n \odot mK_1) = \{u_iu_{i+1}, v_iv_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_iv_i, u_iu_{i,j}, v_iv_{i,j} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq m\}$ respectively be the vertex and edge sets of $L_n \odot mK_1$. Then the number of vertices and edges of $L_n \odot mK_1$ are given by $p = |V(L_n \odot mK_1)| = 2n(m+1)$ and $q = |E(L_n \odot mK_1)| = n(2m+3) - 2$ respectively. Assign the vector to the vertices in the following order $u_1, v_1, u_2, v_2, \ldots, u_n, v_n$, $u_{1,1}, u_{1,2}, \ldots, u_{1,m}, v_{1,1}, v_{1,2}, \ldots, v_{1,m}$, $u_{2,1}, u_{2,2}, \ldots, u_{2,m}, v_{2,1}, v_{2,2}, \ldots, v_{2,m}, \ldots,$ $u_{n,1}, u_{n,2}, \ldots, u_{n,m}, v_{n,1}, v_{n,2}, \ldots, v_{n,m}$. We have consider four cases

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4t_1$, $t_1 > 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Also assign the vector (1,1,1,1) to the next $2t_1$ vertices $u_2, u_3, \ldots, u_{t_1+1}$ and $v_2, v_3, \ldots, v_{t_1+1}$. Assign the vector (1,1,1,0) to the next $2t_1$ vertices $u_{t_1+2}, u_{t_1+3}, \ldots, u_{2t_1+1}$ and $v_{t_1+2}, v_{t_1+3}, \ldots, v_{2t_1+1}$. Further assign the vector (1,1,0,0) to the next $2t_1$ vertices $u_{2t_1+2}, u_{2t_1+3}, \ldots, u_{3t_1+1}$ and $v_{2t_1+2}, v_{2t_1+3}, \ldots, v_{3t_1+1}$. Finally assign the vector (1,0,0,0) to the next $2(t_1-1)$ vertices $u_{3t_1+2}, u_{3t_1+3}, \ldots, u_{4t_1}$ and $v_{3t_1+2}, v_{3t_1+3}, \ldots, v_{4t_1}$.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. Then $p = 4(8t_1t_2 + 2t_1)$ and $q = 4(8t_1t_2 + 3t_1) - 2$. Thereafter assign the vector (1,1,1,1) to the first $8t_1t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to

the next $8t_1t_2$ pendant vertices. Also assign the vector (1,1,0,0) to the next $8t_1t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2$ pendant vertices.
Subcase (ii): $m \equiv 1 \pmod 4$
Let $m = 4t_2 + 1$, $t_2 > 0$. Then $p = 4(8t_1t_2 + 4t_1)$ and $q = 4(8t_1t_2 + 5t_1) - 2$. Also assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 2$ pendant vertices.
Subcase (iii): $m \equiv 2 \pmod 4$
Let $m = 4t_2 + 2$, $t_2 > 0$. Then $p = 4(8t_1t_2 + 6t_1)$ and $q = 4(8t_1t_2 + 7t_1)$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1$ pendant vertices. More over assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 2$ pendant vertices.
Subcase (iv): $m \equiv 3 \pmod 4$
Let $m = 4t_2 + 3$, $t_2 > 0$. Then $p = 4(8t_1t_2 + 8t_1)$ and $q = 4(8t_1t_2 + 9t_1) - 2$. Now assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1$ pendant vertices. So assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 2$ pendant vertices.
Case (ii): $n \equiv 1 \pmod 4$
Let $n = 4t_1 + 1$, $t_1 > 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2t_1$ vertices $u_2, u_3, \ldots, u_{t_1+1}$ and $v_2, v_3, \ldots, v_{t_1+1}$. Assign the vector (1,1,1,0) to the next $2t_1$ vertices $u_{t_1+2}, u_{t_1+3}, \ldots, u_{2t_1+1}$ and $v_{t_1+2}, v_{t_1+3}, \ldots, v_{2t_1+1}$. Further assign the vector (1,1,0,0) to the next $2t_1$ vertices $u_{2t_1+2}, u_{2t_1+3}, \ldots, u_{3t_1+1}$ and $v_{2t_1+2}, v_{2t_1+3}, \ldots, v_{3t_1+1}$. Finally assign the vector (1,0,0,0) to the next $2t_1$ vertices u_{3t_1+2}, $u_{3t_1+3}, \ldots, u_{4t_1+1}$ and $v_{3t_1+2}, v_{3t_1+3}, \ldots, v_{4t_1+1}$.
Subcase (i): $m \equiv 0 \pmod 4$
Let $m = 4t_2$, $t_2 > 0$. Then $p = 4(8t_1t_2 + 2t_1 + 2t_2) + 2$. So assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_2 + 1$ pendant vertices.
Subcase (ii): $m \equiv 1 \pmod 4$
Let $m = 4t_2 + 1$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 4t_1 + 2t_2 + 1)$. Further assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 2t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 2t_2 + 1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 2t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 2t_2 + 1$ pendant vertices.
Subcase (iii): $m \equiv 2 \pmod 4$
Let $m = 4t_2 + 2$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 6t_1 + 2t_2) + 2$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 2t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + 2t_2 + 1$ pendant vertices. Moreover assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + 2t_2 + 2$ pendant vertices and assign the vector (1,0,0,0) to the next

$8t_1t_2 + 4t_1 + 2t_2 + 2$ pendant vertices.
Subcase (iv): $m \equiv 3 \pmod 4$
Let $m = 4t_2 + 3$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 8t_1 + 2t_2 + 2)$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + 2t_2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + 2t_2 + 2$ pendant vertices. Moreover assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + 2t_2 + 2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 2t_2 + 2$ pendant vertices.
Case (iii): $n \equiv 2 \pmod 4$
Let $n = 4t_1 + 2$, $t_1 \geq 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2(t_1 + 1)$ vertices $u_2, u_3, \ldots, u_{t_1+2}$ and $v_2, v_3, \ldots, v_{t_1+2}$. Thereafter assign the vector (1,1,1,0) to the next $2t_1$ vertices $u_{t_1+3}, u_{t_1+4}, \ldots, u_{2t_1+2}$ and $v_{t_1+3}, v_{t_1+4}, \ldots, v_{2t_1+2}$. Then assign the vector (1,1,0,0) to the next $2t_1$ vertices $u_{2t_1+3}, u_{2t_1+4}, \ldots, u_{3t_1+2}$ and $v_{2t_1+3}, v_{2t_1+4}, \ldots, v_{3t_1+2}$. Finally assign the vector (1,0,0,0) to the next $2t_1$ vertices $u_{3t_1+3}, u_{3t_1+4}, \ldots, u_{4t_1+2}$ and $v_{3t_1+3}, v_{3t_1+4}, \ldots, v_{4t_1+2}$.
Subcase (i): $m \equiv 0 \pmod 4$
Let $m = 4t_2$, $t_2 > 0$. Then $p = 4(8t_1t_2 + 2t_1 + 4t_2 + 1)$. So assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_2 - 3$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_2 + 1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_2 + 1$ pendant vertices.
Subcase (ii): $m \equiv 1 \pmod 4$
Let $m = 4t_2 + 1$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 4t_1 + 4t_2 + 2)$. Also assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 4t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 4t_2 + 2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 4t_2 + 2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 4t_2 + 2$ pendant vertices.
Subcase (iii): $m \equiv 2 \pmod 4$
Let $m = 4t_2 + 2$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 6t_1 + 4t_2 + 3)$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 4t_2 + 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + 4t_2 + 3$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + 4t_2 + 3$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 4t_2 + 3$ pendant vertices.
Subcase (iv): $m \equiv 3 \pmod 4$
Let $m = 4t_2 + 3$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 4t_1 + 4t_2 + 4)$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 4t_2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 4t_2 + 4$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 4t_2 + 4$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 4t_2 + 4$ pendant vertices.
Case (iv): $n \equiv 3 \pmod 4$
Let $n = 4t_1 + 3$, $t_1 \geq 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2(t_1 + 1)$ vertices $u_2, u_3, \ldots, u_{t_1+2}$ and $v_2, v_3, \ldots, v_{t_1+2}$. Thereafter assign the vector (1,1,1,0) to the next $2(t_1 + 1)$ vertices $u_{t_1+3}, u_{t_1+4}, \ldots, u_{2t_1+3}$ and $v_{t_1+3}, v_{t_1+4}, \ldots, v_{2t_1+3}$. Moreover assign the vector (1,1,0,0) to the next $2t_1$ vertices $u_{2t_1+4}, u_{2t_1+5}, \ldots, u_{3t_1+3}$ and $v_{2t_1+4}, v_{2t_1+5}, \ldots, v_{3t_1+3}$. Finally assign the vector (1,0,0,0) to

the next $2t_1$ vertices $u_{3t_1+4}, u_{3t_1+5}, \ldots, u_{4t_1+3}$ and $v_{3t_1+4}, v_{3t_1+5}, \ldots, v_{4t_1+3}$.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We get $p = 4(8t_1t_2+2t_1+6t_2+1)+2$. Now assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2+6t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2+6t_2+1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_2 + 1$ pendant vertices.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 4t_1 + 6t_2 + 3)$. Further assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 6t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 6t_2 + 1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 6t_2 + 3$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 6t_2 + 3$ pendant vertices.

Subcase (iii): $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 6t_1 + 6t_2 + 4) + 2$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 6t_2 + 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + 6t_2 + 2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + 6t_2 + 4$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 6t_2 + 5$ pendant vertices.

Subcase (iv): $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 8t_1 + 6t_2 + 6)$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + 6t_2 + 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + 6t_2 + 4$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + 6t_2 + 6$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 6t_2 + 6$ pendant vertices.

Clearly the above vertex labeling gives a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling to the graph $L_n \odot mK_1$. □

Example 3.2. *Figure 2 illustrates the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling of* $L_6 \odot 3K_1$.

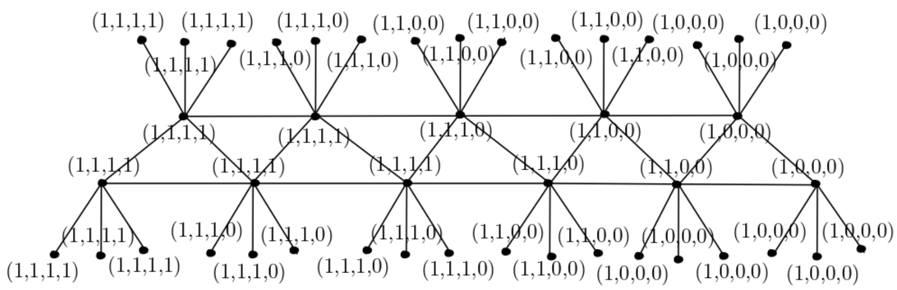

Figure 2: Vector Basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-Cordial Labeling of $L_6 \odot 3K_1$

Theorem 3.3. *The graph* $T(P_n) \odot mK_1$ *is a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial for all* $n \geq 4$ *and* $m \geq 1$.

Proof. Let $V(T(P_n) \odot mK_1) = \{u_i, v_i, u_{i,j}, v_{i,j} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq m\}$ and $E(T(P_n) \odot mK_1) = \{u_iu_{i+1}, v_iu_{i+1}, u_iv_i, , v_iv_{i,j} \mid 1 \leq i \leq n-1 \,\text{and}\, 1 \leq j \leq m\} \cup \{v_iv_{i+1} \mid 1 \leq i \leq n-2\} \cup \{u_iu_{i,j} \mid 1 \leq i \leq n \,\text{and}\, 1 \leq j \leq m\}$ respectively be the vertex and edge sets of $L_n \odot mK_1$. Then the number of vertices and edges of $T(P_n) \odot mK_1$ are given by $p = |V(T(P_n) \odot mK_1)| = (2n-1)(m+1)$ and $q = |E(T(P_n) \odot mK_1)| = (2n-1)(m+2)-3$ respectively. Assign the vector to the vertices in the following order $u_1, v_1, u_2, v_2, \ldots, u_n, v_n$, $u_{1,1}, u_{1,2}, \ldots, u_{1,m}, v_{1,1}, v_{1,2}, \ldots, v_{1,m}, u_{2,1}, u_{2,2}, \ldots, u_{2,m}, v_{2,1}, v_{2,2}, \ldots, v_{2,m}, \ldots, u_{n-1,1}, u_{n-1,2}$, $\ldots, u_{n-1,m}, v_{n-1,1}, v_{n-1,2}, \ldots, v_{n-1,m}$, $u_{n,1}, u_{n,2}, \ldots, u_{n,m}$. We have consider four cases

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4t_1$, $t_1 > 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2t_1$ vertices and assign the vector (1,1,1,0) to the next $2t_1 - 1$ vertices. Further assign the vector (1,1,0,0) to the next $2t_1 - 1$ vertices and assign the vector (1,0,0,0) to the next $2t_1 - 1$ vertices.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We obtain $p = 4(8t_1t_2 + 2t_1 - t_2) - 1$. Then assign the vector (1,1,1,1) to the first $8t_1t_2 - t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 - t_2 + 1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 - t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 - t_2$ pendant vertices.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 4t_1 - t_2) - 2$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 - t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 - t_2 + 1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 - t_2 - 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 - t_2$ pendant vertices.

Subcase (iii): $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 6t_1 - t_2) - 3$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 - t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 - t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 - t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 - t_2$ pendant vertices.

Subcase (iv): $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 8t_1 - t_2 - 1)$. Also assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 - t_2 - 3$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 - t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 - t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 - t_2$ pendant vertices.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4t_1 + 1$, $t_2 > 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2t_1$ vertices and assign the vector (1,1,1,0) to the next $2t_1$ vertices. Further assign the vector (1,1,0,0) to the next $2t_1$ vertices and assign the vector (1,0,0,0) to the next $2t_1 - 1$ vertices.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We have $p = 4(8t_1t_2 + 2t_1 + t_2) + 1$. Further assign the vector

(1,1,1,1) to the first $8t_1t_2 + t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + t_2 + 1$ pendant vertices.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 4t_1 + t_2) + 2$. Further assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + t_2 + 2$ pendant vertices.

Subcase (iii): $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. We get $p = 4(8t_1t_2 + 6t_1 + t_2) + 3$. So assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + t_2 + 1$ pendant vertices. More over assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + t_2 + 2$ pendant vertices.

Subcase (iv): $m \equiv 3 \pmod 4$

Let $m = 4t_2 + 3$, $t_2 \geq 0$. Then $p = 4(8t_1t_2 + 8t_1 + t_2 + 1)$. Now assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + t_2 + 1$ pendant vertices. More over assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + t_2 + 2$ pendant vertices.

Case (iii): $n \equiv 2 \pmod 4$

Let $n = 4t_1 + 2$, $t_1 \geq 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2t_1 + 1$ vertices and assign the vector (1,1,1,0) to the next $2t_1$ vertices. Further assign the vector (1,1,0,0) to the next $2t_1$ vertices and assign the vector (1,0,0,0) to the next $2t_1$ vertices.

Subcase (i): $m \equiv 0 \pmod 4$

Let $m = 4t_2$, $t_2 > 0$. We have $p = 4(8t_1t_2 + 2t_1 + 3t_2) + 3$. So assign the vector (1,1,1,1) to the first $8t_1t_2 + 3t_2 - 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 3t_2 + 1$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 3t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 3t_2$ pendant vertices.

Subcase (ii): $m \equiv 1 \pmod 4$

Let $m = 4t_2 + 1$, $t_2 \geq 0$. We have $p = 4(8t_1t_2 + 4t_1 + 3t_2 + 1) + 2$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 3t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 3t_2 + 1$ pendant vertices. Further assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 3t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 3t_2 + 2$ pendant vertices.

Subcase (iii): $m \equiv 2 \pmod 4$

Let $m = 4t_2 + 2$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 6t_1 + 3t_2 + 2) + 1$. Now assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 3t_2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + 3t_2 + 2$ pendant vertices. Further assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + 3t_2 + 2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 3t_2 + 2$ pendant vertices.

Subcase (iv): $m \equiv 3 \pmod 4$
Let $m = 4t_2 + 3$, $t_2 \geq 0$. We have $p = 4(8t_1t_2 + 8t_1 + 3t_2 + 3)$. Also assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + 3t_2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + 3t_2 + 3$ pendant vertices. More over assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + 3t_2 + 3$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 3t_2 + 3$ pendant vertices.
Case (iv): $n \equiv 3 \pmod 4$
Let $n = 4t_1 + 3$, $t_1 \geq 0$. Then assign the vector (1,1,1,1) to the vertices u_1 and v_1. Assign the vector (1,1,1,1) to the next $2t_1 + 1$ vertices and assign the vector (1,1,1,0) to the next $2t_1 + 1$ vertices. So assign the vector (1,1,0,0) to the next $2t_1 + 1$ vertices and assign the vector (1,0,0,0) to the next $2t_1$ vertices.
Subcase (i): $m \equiv 0 \pmod 4$
Let $m = 4t_2$, $t_2 > 0$. We have $p = 4(8t_1t_2 + 2t_1 + 5t_2 + 1) + 1$. Also assign the vector (1,1,1,1) to the first $8t_1t_2 + 5t_2 - 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 5t_2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 5t_2$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2+5t_2+1$ pendant vertices.
Subcase (ii): $m \equiv 1 \pmod 4$
Let $m = 4t_2 + 1$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 4t_1 + 5t_2 + 2) + 2$. Assign the vector (1,1,1,1) to the first $8t_1t_2 + 2t_1 + 5t_2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 2t_1 + 5t_2 + 1$ pendant vertices. Further assign the vector (1,1,0,0) to the next $8t_1t_2 + 2t_1 + 5t_2 + 1$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 2t_1 + 5t_2 + 3$ pendant vertices.
Subcase (iii): $m \equiv 2 \pmod 4$
Let $m = 4t_2 + 2$, $t_2 \geq 0$. We obtain $p = 4(8t_1t_2 + 6t_1 + 5t_2 + 3) + 3$. So assign the vector (1,1,1,1) to the first $8t_1t_2 + 4t_1 + 5t_2 + 1$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 4t_1 + 5t_2 + 2$ pendant vertices. Assign the vector (1,1,0,0) to the next $8t_1t_2 + 4t_1 + 5t_2 + 3$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 4t_1 + 5t_2 + 4$ pendant vertices.
Subcase (iv): $m \equiv 3 \pmod 4$
Let $m = 4t_2 + 3$, $t_2 \geq 0$. We have $p = 4(8t_1t_2 + 8t_1 + 5t_2 + 5)$. Also assign the vector (1,1,1,1) to the first $8t_1t_2 + 6t_1 + 5t_2 + 2$ pendant vertices and assign the vector (1,1,1,0) to the next $8t_1t_2 + 6t_1 + 5t_2 + 4$ pendant vertices. More over assign the vector (1,1,0,0) to the next $8t_1t_2 + 6t_1 + 5t_2 + 4$ pendant vertices and assign the vector (1,0,0,0) to the next $8t_1t_2 + 6t_1 + 5t_2 + 4$ pendant vertices.
Thus the above vertex labeling method gives a vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling to the graph $T(P_n) \odot mK_1$. □

Example 3.4. *Figure* 3 *illustrates the vector basis* {*(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)*}*-cordial labeling of* $T(P_5) \odot 2K_1$.

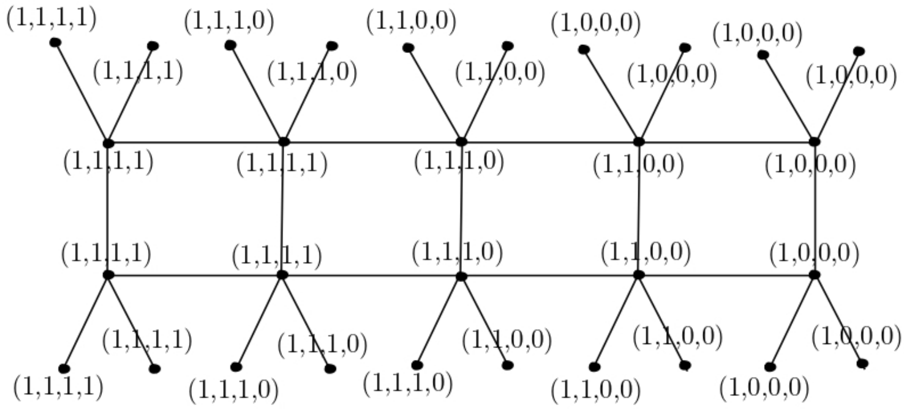

Figure 3: Vector Basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-Cordial Labeling of $T(P_5) \odot 2K_1$

4 Conclusion

In this paper, we have investigate the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling behavior of $L_n \odot mK_1$ and $T(P_n) \odot mK_1$. To examine the vector basis {(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)}-cordial labeling for different types of graphs and some standard graphs with corona operations, middle graph of a graph, union of two graphs and square of a graph are the open area of research work.

References

[1] Ajitha, V., Arumugam, S., & Germina, K. A., On square sum graphs. AKCE International Journal of Graphs and Combinatorics, 6(1), 1-10, 2009.

[2] Barasara, C. M., & Thakkar, Y. B., Divisor cordial labeling for ladders and total graph of some graphs. Advances and Applications in Mathematical Sciences, 21(7), 3577-3594, 2022.

[3] Cahit, I., Cordial graphs: a weaker version of graceful and harmonious graphs. Ars Combin., 23, 201-207, 1987.

[4] Elumalai, A., Middle and total graphs of square sum labeling. Malaya Journal of Mathematik, S(2), 4031-4032, 2020.

[5] Ganeshan, M., & Paulraj M. S., Square sum labeling of sierpinski gasket graphs and corona graph. International Journal of Research and Analytical Reviews, 6(1), 840-844, 2019.

[6] Gallian, J. A., A dynamic survey of graph labeling. The Electronic Journal of Combinatorics, 27, 1-712, 2024.

[7] Harary, F., Graph theory. Addison Wesely, New Delhi, 1972.

[8] Herstein, I. N., Topics in Algebra. John Wiley and Sons, New York, 1991.

[9] Jeba Jesintha, J., K. Subashini, K., & Cathrine Silvya Jabarani, P., Cordial labeling for new class of graphs. South East Asian J. of Mathematics and Mathematical Sciences, 17(3), 373-380, 2021.

[10] Ponraj, R., Gayathri, A., & Sivakumar, M., Pair difference cordial labeling of m-copies of path, cycle, star and ladder graphs. Journal of Algorithms and Computation, 54(2), 37-47, 2022.

[11] Ponraj, R., & Jeya, R., Vector Basis S-cordial lebeling of graphs. J. Math. Comput. Sci., 15:5, 1 - 13, 2025.

[12] Ponraj, R., & Jeya, R., Certain VB $\{(1,1,1,1),(1,1,1,0),(1,1,0,0),(1,0,0,0)\}$-cordial thorn graphs. Global Journal of Pure and Applied Mathematics, 21(1), 1-14, 2025.

[13] Ponraj, R., & Prabhu, S., Pair mean cordial labeling of graphs. Journal of Algorithms and Computation, 54(1), 1-10, 2022.

[14] Ponraj, R., Subbulakshmi, S., & Somasundaram, S., PD-prime cordial labeling of graphs. Journal of Algorithms and Computation, 51(2), 1-7, 2019.

[15] Rosa, A., On certain valuations of the vertices of a graph. Theory of Graphs (Internat. Symposium, Rome, July 1966) Gordon and Breach, N. Y. and Dunod Paris, 349-355, 1967.

[16] Sebastian, R., & Germina, K. A., Square sum labeling of class of planar graphs. Proyecciones Journal of Mathematics, 34(1), 55-68, 2015.

[17] Varatharajan, R., Navaneethakrishnan, S., & Nagarajan, K., Divisor cordial graphs. International Journal of Mathematics and Combinatorics, 4, 15-25, 2011.

[18] Varatharajan, R., Navaneethakrishnan, S., & Nagarajan, K., Special classes of divisor cordial graphs. International Mathematical Forum, 7(35), 1737-1749, 2012.