Journal of Algorithms and Computation

journal homepage: http://jac.ut.ac.ir

FP-Cordial labeling of certain graph structures

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ABSTRACT

Let $G = (V, E)$ be a (p, q) graph.
Let

$$M = \begin{cases} 1, 2, \cdots, \frac{p}{2}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{p+2}, & \text{if } p \text{ is even} \\ 1, 2, \cdots, \frac{p-1}{2}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{p+3}, & \text{if } p \text{ is odd} \end{cases}$$

Let $\chi : V(G) \to M$ be a bijection.For each edge xy assign the label $\lceil \chi(x)\chi(y) \rceil$. χ is called a fractional product cordial labeling (simply called FP-cordial labeling) if $|\Pi_\chi(0) - \Pi_\chi(1)| \leq 1$, where $\Pi_\chi(1)$ and $\Pi_\chi(0)$ respectively denotes the number of edges labelled with 1 and not labelled with 1. A graph with a fractional product cordial labeling is called a fractional product cordial graph (Simply FP-cordial graph).In this paper we investigate the fractional product cordial labeling behaviour of lotus graph, necklace graph, F-tree, Y-tree, hurdle graph, key graph, coconut tree, prism, mobious ladder, vanessa graph and udukkai graph.

Keyword: lotus graph, necklace graph, vanessa graph, udukkai graph.

ARTICLE INFO

Article history:
Research paper
Received 10, November 2025
Accepted 27, November 2025
Available online 03, December 2025

AMS subject Classification: 05C78.

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1 Introduction

We consider finite, simple and undirected graphs only. In 1980, Cahit [1] was introduced the concept of cordial labeling of graphs. Ponraj, Gayathri and Somasundaram [10] have been studied about the 4-remainder cordial of some tree related graphs like banana tree, coconut tree, double coconut tree, fire cracker, bamboo tree and caterpillar graph. Seoud and Helmi [17] have been investigate the product cordial labling behaviour of complete graph, cycle, gear graph, web graph, $T_n \odot K_1$, $C_n \odot \bar{K_w}$ and C_4-snake graph . Ponraj et al. [9] defined the k-remainder cordial labeling of a graph and have been investigate the 4- remainder cordial labeling behaviour of path, cycle, star, complete graph, wheel, comb, etc. In [2], the fibonacci prime labeling of udukkai and octopus graphs have been examined by Chandrakala and Sekar. Ponraj and prabhu [11] have been investigate the pair mean cordial labeling behaviour of hurdle graph, key, lotus, necklace, subdivided shell, uniform bow, and F-tree graph. The product cordial labeling behaviour of the path union of k-copies of helm, the path union of k-copies of closed helm, the path union of k-copies of gear graphs have been examined by Ghodasara and Vaghasiya [5]. Prajapati and Patel [14] have been investigate the edge product cordial labeling of $W_n^{(t)}$, PS_n and DPS_n. Edward Samuvel and Kalaivani [3] have proved that the prime labeling for some udukkai related graphs.In [18] the power dominator coloring for fan graph, double fan graph, planter graph, lilly graph, octopus graph, drum graph, vanessa graph, flower pot graph and umbrella graph have been examined by Uma Maheswari and Bala Samuvel. Jesintha and Hilda [7] examined all uniform bow graphs to ensure they were graceful. Petrano and Rulete [8] shows that cartesian product of $P_m \times C_n$ and $C_m \times C_n$ and generalized peterson graph are total product cordial. Rokad [15] have been studied about product cordial labeling double wheel, path union of finite copies of double wheel, joining two copies of double wheel by a path of arbitrary length, $DW_n \oplus K_{1,n}$ and $DF_n \oplus K_{1,n}$. Ponraj and Sutharson [12] have been introduced the new graph labeling called fractional product cordial labeling of graph. In [13], they have been investigate the fractional product cordial labeling behaviour of certain graphs, like web graph, dumbbell graph, jelly fish, jewel graph and tadpole graph. In this paper we investigate the FP-cordial labeling behaviour of certain graphs, like lotus graph, necklace graph, F-tree, Y-tree, hurdle graph, key graph, coconut tree, prism, mobious ladder, vanessa graph and udukkai graph. Notation and definitions not defined here are followed from Gallian [4] and Harary [6]. $\lceil x \rceil$ denotes the smallest integer $\geq x$. The number of vertices of a graph G is called the order of G and number of edges is called the size of G. $\Omega_{fpc}(G)$ denotes the set of all fractional product cordial graphs.

2 Preliminaries

Definition 2.1. *[2]The Corona graph $G_1 \odot G_2$ is the graph obtained from G_1 and G_2 by taking one copy of G_1 and n copies of G_2 and joining the i^{th} vertex of G_1 with an edge to* *the every vertex in the i^{th} copy of G_2 where G_1 is the graph of order n.*

Definition 2.2. *[3]A shell graph is a cycle C_n with $(n-3)$ chords that share a common end point called the apex. A shell graph is denoted as $C(n, n-3)$.*

Definition 2.3. *[3]A graph is obtained from a shell graph by adding a vertex between each pair of adjacent vertices on the cycle and adding an edge in the apex and two or more chords are known as a Lotus graph LS_n.*

Definition 2.4. *[3]The necklace graph denoted by Ne_n is a cubic halin graph obtained by joining a cycle with all vertices of degree 1 of a caterpillar having n vertices of degree 3 and $n+2$ vertices of degree 1.*

Definition 2.5. *[3]A F-tree $F(P_n)$ is a graph obtained from path on $n \geq 3$ vertices by appending two pendant edges one to an end vertex and the other to a vertex adjacent to an end vertex.*

Definition 2.6. *[3]A Y-tree Y_{n+1}, $n \geq 2$ is a graph obtained from the path P_n by appending an edge to a vertex of the path P_n adjacent to an end vertex.*

Definition 2.7. *[3]A graph obtained from a path P_n by attaching pendent edges to every internal vertices of the path. It is called Hurdle graph with $n-2$ hurdles and is denoted by Hd_n.*

Definition 2.8. *[3]A key graph is a graph obtained from K_2 by appending one vertex of C_5 to one end point and Hoffman tree $P_n \odot K_1$ to the other end point of K_2.*

Definition 2.9. *[3]The coconut tree $CT(m, n)$ is a graph obtained from the path P_n by appending m new pendent edges at an end vertex of P_n.*

Definition 2.10. *[3]The double coconut tree $DCT(m, n, r)$ is a tree obtained by attaching $m > 1$ pendent vertices to one end of the path P_n and $r > 1$ pendent vertices to the other end of P_n.*

Definition 2.11. *[3]The mobius ladder M_n id obtained from the ladder L_n by joining the vertices a_1 with b_n and a_n with b_1.*

Definition 2.12. *[3]An Udukkai graph U_n, $n \geq 2$ is a graph constructed by joining two fan graphs F_n, $n \geq 2$ with two paths P_n, $n \geq 2$ by sharing a common vertex at the centre.*

Definition 2.13. *[3]The vanessa graph V_n, $n \geq 3$ can be constructed by two fan graphs $2F_n$, $n \geq 2$ of same order, sharing same common vertex w, with n number of pendent vertices K_n.*

Theorem 2.14. *The cycles C_3, C_4, C_5 are not fractional product cordial.*

Theorem 2.15. *The complete graph K_p is FP-cordial if and only if $p \leq 2$.*

3 Fractional Product Cordial Labeling

Definition 3.1. *Let $G = (V, E)$ be a (p, q) graph.*
Let

$$M = \begin{cases} 1, 2, \cdots, \frac{p}{2}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{p+2}, & \text{if } p \text{ is even} \\ 1, 2, \cdots, \frac{p-1}{2}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{p+3}, & \text{if } p \text{ is odd} \end{cases}$$

Let $\chi : V(G) \to M$ be a bijection.For each edge xy assign the label $\lceil \chi(x)\chi(y) \rceil$.$\chi$ is called a fractional product cordial labeling (simply called FP-cordial labeling) if $|\Pi_\chi(0) - \Pi_\chi(1)| \leq 1$, where $\Pi_\chi(1)$ and $\Pi_\chi(0)$ respectively denotes the number of edges labelled with 1 and not labelled with 1. A graph with a fractional product cordial labeling is called a fractional product cordial graph (Simply FP-cordial graph).

4 Main Results

In this paper we investigate the FP-cordial labeling behaviour of certain graphs, like lotus graph, necklace graph, F-tree, Y-tree, hurdle graph, key graph, coconut tree, prism, mobious ladder, vanessa graph and udukkai graph.

Theorem 4.1. *The lotus graph, LS_n is FP-cordial for all $n \geq 3$.*

Proof. Let $V(LS_n) = \{x_0, y_0, x_i, y_j : 1 \leq i \leq n, 1 \leq j \leq n+1\}$ and $E(LS_n) = \{y_0y_i, x_0y_0 : 1 \leq i \leq n+1\} \cup \{y_ix_i, x_iy_{i+1} : 1 \leq i \leq n\}$. Then it has $2n + 3$ vertices and $3n + 2$ edges. This proof divided into three cases.
Case 1: n is even and $n \geq 4$
Assign the labels $\frac{1}{3}$ and 1 to the vertices x_0 and y_0. Now assign the labels $2, 4, \cdots, n+1$ to the vertices $y_1, y_2, \ldots, y_{\frac{n}{2}+1}$ respectively and assign the labels $\frac{1}{n+2}, \frac{1}{n}, \cdots, \frac{1}{4}$ to the vertices $y_{\frac{n}{2}+2}, y_{\frac{n}{2}+3}, \ldots, y_{n+1}$ respectively. We now assign the labels $3, 5, \cdots, n-1, \frac{1}{2}$ to the vertices $x_1, x_2, \cdots, x_{\frac{n}{2}}$ and assign the labels $\frac{1}{n+3}, \frac{1}{n+1}, \cdots, \frac{1}{5}$ to the vertices $x_{\frac{n}{2}+1}, x_{\frac{n}{2}+2}, \cdots, x_n$. We have $\Pi_\chi(0) = \frac{3n+2}{2}$ and $\Pi_\chi(1) = \frac{3n+2}{2}$.
Case 2: n is odd
Assign the labels $\frac{1}{n+3}$ and 1 to the vertices x_0 and y_0. Now assign the labels $2, 4, \cdots, n-1$ to the vertices $y_1, y_2, \ldots, y_{\frac{n-1}{2}}$ respectively and assign the labels $\frac{1}{2}, \frac{1}{3}, \frac{1}{5} \cdots, \frac{1}{n+2}$ to the vertices $y_{\frac{n-1}{2}+1}, y_{\frac{n-1}{2}+2}, \ldots, y_{n+1}$ respectively. We now assign the labels $3, 5, \cdots, n+1$ to the vertices $x_1, x_2, \cdots, x_{\frac{n+1}{2}}$ and assign the labels $\frac{1}{4}, \frac{1}{6}, \cdots, \frac{1}{n+1}$ to the vertices $x_{\frac{n+1}{2}+1}, x_{\frac{n+1}{2}+2}, \cdots, x_n$. Hence $\Pi_\chi(0) = \frac{3n+1}{2}$ and $\Pi_\chi(1) = \frac{3n+3}{2}$.
Case 3: $n = 2$
suppose $LS_2 \in \Omega_{fpc}$. In this case the vertex labels are $1, 2, 3, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$. When $1, 2, , 3, \frac{1}{2}$ are the vertex labels of the adjacent vertices we have $\Pi_\chi(0) \leq 5$ a contradiction to the size of LS_2 is 13. When $1, 2, , 3, \frac{1}{2}$ are the vertex labels of the non adjacent vertices we have $\Pi_\chi(0) \leq 8$ again a contradiction to the size of LS_2 is 13. It follows that $LS_2 \notin \Omega_{fpc}$.
□

Theorem 4.2. *The necklace graph, Ne_n is FP-cordial for all $n \geq 2$.*

Proof. Let $V(LS_n) = \{x_0, y_0, x_i, y_i : 1 \leq i \leq n\}$ and $E(Ne_n) = \{x_0x_1, x_0y_1, x_iy_i, y_0y_n, y_0x_n : 1 \leq i \leq n\} \cup \{x_ix_{i+1}, y_iy_{i+1} : 1 \leq i \leq n-1\}$. Then necklace graph has $2n+2$ vertices and $3n+3$ edges. This proof break up into two cases as follows.
Case 1: n is even and $n \geq 2$
Assign the labels 1 and $\frac{2}{n+4}$ to the vertices x_0 and y_0. Now assign the labels $2, 4, \cdots, n$ to the vertices $x_1, x_2, \ldots, x_{\frac{n}{2}}$ respectively and assign the labels $\frac{1}{n+2}, \frac{1}{n+1}, \cdots, \frac{2}{n+6}$ to the vertices $x_{\frac{n}{2}+1}, x_{\frac{n}{2}+2}, \ldots, x_n$ respectively. We now assign the labels $3, 5, \cdots, n+1$ to the vertices $y_1, y_2, \cdots, y_{\frac{n}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+2}$ to the vertices $y_{\frac{n}{2}+1}, y_{\frac{n}{2}+2}, \cdots, y_n$. It is easy to verify that $\Pi_\chi(0) = \frac{3n+2}{2}$ and $\Pi_\chi(1) = \frac{3n+4}{2}$.
Case 2: n is odd and $n \geq 3$
Assign the labels 1 and $\frac{2}{n+5}$ to the vertices x_0 and y_0. Now assign the labels $2, 4, \cdots, n+1$ to the vertices $x_1, x_2, \ldots, x_{\frac{n+1}{2}}$ respectively and assign the labels $\frac{1}{n+2}, \frac{1}{n+1}, \cdots, \frac{2}{n+7}$ to the vertices $x_{\frac{n+1}{2}+1}, x_{\frac{n+1}{2}+2}, \ldots, x_n$ respectively. We now assign the labels $3, 5, \cdots, n$ to the vertices $y_1, y_2, \cdots, y_{\frac{n-1}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+3}$ to the vertices $y_{\frac{n-1}{2}+1}, y_{\frac{n-1}{2}+2}, \cdots, y_n$. Obviously $\Pi_\chi(0) = \frac{3n+3}{2}$ and $\Pi_\chi(1) = \frac{3n+3}{2}$.

□

Theorem 4.3. *The F-tree graph, $F(P_n)$ is FP-cordial for all $n \geq 4$.*

Proof. Let $V(F(P_n)) = \{u, v, u_i : 1 \leq i \leq n\}$ and $E(F(P_n)) = \{u_iu_{i+1}, uu_{n-1}, vu_n : 1 \leq i \leq n-1\}$. Then it has $n+2$ vertices and $n+1$ edges. This proof divided into three cases as follows.
Case 1: n is even and $n \geq 4$
Assign the labels $1, 2, \cdots, \frac{n+2}{2}$ to the vertices $u_1, u_2, \ldots, u_{\frac{n+2}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n}$ to the vertices $u_{\frac{n+2}{2}+1}, u_{\frac{n+2}{2}+2}, \cdots, u_n$. Now assign the labels $\frac{2}{n+2}$ and $\frac{2}{n+4}$ to the vertices u and v. Therefore $\Pi_\chi(0) = \frac{n+2}{2}$ and $\Pi_\chi(1) = \frac{n}{2}$.
Case 2: n is odd and $n \geq 5$
Assign the labels $1, 2, \cdots, \frac{n+1}{2}$ to the vertices $u_1, u_2, \ldots, u_{\frac{n+1}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+1}$ to the vertices $u_{\frac{n+1}{2}+1}, u_{\frac{n+1}{2}+2}, \cdots, u_n$. Now assign the labels $\frac{2}{n+3}$ and $\frac{2}{n+5}$ to the vertices u and v. Hence $\Pi_\chi(0) = \frac{n+1}{2}$ and $\Pi_\chi(1) = \frac{n+1}{2}$.
Case 3: $n = 3$
Suppose $F(P_3) \in \Omega_{fpc}$. In this case the vertex labels are $1, 2, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. When 1 and 2 are labels of adjacent vertices we have $\Pi_\chi(0) = 1$ and $\Pi_\chi(1) = 3$, a contradiction. When 1 and 2 are labels of non adjacent vertices we have $\Pi_\chi(0) = 0$ and $\Pi_\chi(1) = 4$, not possible.

□

Corollary 4.4. *The Y-tree graph, Y_{n+1} is FP-cordial for all $n \geq 2$, except $n = 3$.*

Proof. Let $V(Y_{n+1}) = \{u, v, u_i : 1 \leq i \leq n\}$ and $E(Y_{n+1}) = \{u_iu_{i+1}, uu_n, vu_n : 1 \leq i \leq n-1\}$. Then it has $n+2$ vertices and $n+1$ edges. This proof follows from theorem 4.3.

□

Theorem 4.5. *The hurdle graph, Hd_n is FP-cordial for all $n \geq 3$.*

Proof. Let $V(Hd_n) = \{v_i, w_j : 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and $E(Hd_n) = \{v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{v_{i+1}w_i : 1 \leq i \leq n-2\}$. Then it has $n+2$ vertices and $n+1$ edges.
When $n \geq 4$, assign the labels $1, 2, \cdots, n-1$ to the vertices $u_1, u_2, \cdots, u_{n-1}$ and assign the label $\frac{1}{2}$ to the vertex u_n. Now assign the labels $\frac{1}{3}, \frac{1}{4}, \cdots, \frac{1}{n}$ to the vertices $v_1, v_2, \cdots, v_{n-2}$. Therefore $\Pi_\chi(0) = n-1$ and $\Pi_\chi(1) = n-2$.
When $n = 3$, Hd_3 is FP-cordial follows from Figure 1.

□

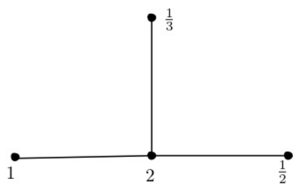

Figure 1: FP-Cordial Labeling of Hd_3

Theorem 4.6. *The key graph, Ky_n is FP-cordial for all $n \geq 1$.*

Proof. Let $V(Ky_n) = V(C_5) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(Ky_n) = E(C_5) \cup \{u_1v_1, v_iw_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-1\}$. Then it has $2n+5$ vertices and $2n+5$ edges. This proof divided into two cases.
Case 1: n is even
When $n \geq 4$, assign the labels $n+2, n+1, n, n-1, n-2$ to the vertices u_5, u_4, u_3, u_2, u_1. Now assign the labels $n-3, n-5, \cdots, 3, 1$ to the vertices $v_1, v_2, \cdots, v_{\frac{n-2}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+4}$ to the vertices $v_{\frac{n-2}{2}+1}, v_{\frac{n-2}{2}+2}, \cdots, v_n$. We now assign the labels $n-4, n-6, \cdots, 4, 2$ to the vertices $w_1, w_2, \cdots, w_{\frac{n-4}{2}}$ and assign the labels $\frac{2}{n+6}, \frac{2}{n+8}, \cdots, \frac{1}{n+4}$ to the vertices $w_{\frac{n-4}{2}+1}, w_{\frac{n-4}{2}+2}, \cdots, w_n$. Therefore $\Pi_\chi(0) = n+2$ and $\Pi_\chi(1) = n+3$.
When $n = 2$, A FP-cordial labeling of Ky_2 is given in Figure 2.

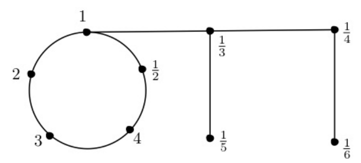

Figure 2: FP-Cordial Labeling of Ky_2

Case 2: n is odd
When $n \geq 3$, assign the labels $n+2, n+1, n, n-1, n-2$ to the vertices u_5, u_4, u_3, u_2, u_1.

Now assign the labels $n-3, n-5, \cdots, 4, 2$ to the vertices $v_1, v_2, \cdots, v_{\frac{n-3}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+5}$ to the vertices $v_{\frac{n-3}{2}+1}, v_{\frac{n-3}{2}+2}, \cdots, v_n$. We now assign the labels $n-4, n-6, \cdots, 3, 1$ to the vertices $w_1, w_2, \cdots, w_{\frac{n-3}{2}}$ and assign the labels $\frac{2}{n+7}, \frac{2}{n+9}, \cdots, \frac{1}{n+4}$ to the vertices $w_{\frac{n-3}{2}+1}, w_{\frac{n-3}{2}+2}, \cdots, w_n$. Therefore $\Pi_\chi(0) = n+2$ and $\Pi_\chi(1) = n+3$.
When $n = 1$, Ky_1 is FP-cordial follows from Figure 3.

□

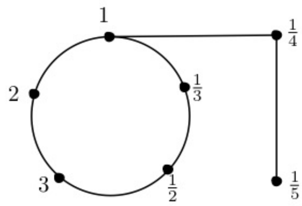

Figure 3: FP-Cordial Labeling of Ky_1

Theorem 4.7. *The coconut tree graph, $CT(n, n)$ is FP-cordial for all $n \geq 2$.*

Proof. Let P_n be the $w_1w_2\cdots w_n$ path. Let $V(CT(n,n)) = V(P_n) \cup \{u_i : 1 \leq i \leq n\}$ and $E(CT(n,n)) = E(P_n) \cup \{w_1u_i : 1 \leq i \leq n\}$. Then the coconut tree graph has $2n$ vertices and $2n-1$ edges.
Assign the labels $1, 2, \cdots, n$ to the vertices $w_1, w_2, \cdots, w_n$ and assign the labels

$$\frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n+1}$$

to the vertices $u_1, u_2, \cdots, u_n$. Hence $\Pi_\chi(0) = n-1$ and $\Pi_\chi(1) = n$.

□

Theorem 4.8. *The double coconut tree graph, $DCT(n, n, n)$ is FP-cordial for all $n \geq 2$.*

Proof. Let P_n be the $w_1w_2\cdots w_n$ path. Let $V(DCT(n,n,n)) = V(P_n) \cup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(DCT(n,n,n)) = E(P_n) \cup \{u_iw_n, v_iw_1 : 1 \leq i \leq n\}$. Then it has $3n$ vertices and $3n-1$ edges. This proof divided into two cases as follows.
Case 1: n is even and $n \geq 2$
Assign the labels $\frac{3n}{2}, \frac{3n}{2} - 1 \cdots, \frac{n+2}{2}$ to the vertices $v_1, v_2, \cdots, v_n$ and assign the labels $\frac{n}{2}, \frac{n}{2} - 1, \cdots, 2, 1$ to the vertices $w_1, w_2, \cdots, w_{\frac{n}{2}}$. We now assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+2}$ to the vertices $w_{\frac{n}{2}+1}, w_{\frac{n}{2}+2}, \cdots, w_n$ and assign the labels $\frac{2}{n+4}, \frac{2}{n+6}, \cdots, \frac{2}{3n+2}$ to the vertices $u_1, u_2, \cdots, u_n$. Obviously $\Pi_\chi(0) = \frac{3n-2}{2}$ and $\Pi_\chi(1) = \frac{3n}{2}$.
Case 2: n is odd nd $n \geq 3$
Assign the labels $1, 2, \cdots, n$ to the vertices $v_1, v_2, \cdots, v_n$ and assign the labels $n+1, n+2, \cdots, \frac{3n-1}{2}$ to the vertices $w_1, w_2, \cdots, w_{\frac{n-1}{2}}$. We now assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+3}$ to the

vertices $w_{\frac{n-1}{2}+1}, w_{\frac{n-1}{2}+2}, \cdots, w_n$ and assign the labels $\frac{2}{n+5}, \frac{2}{n+7}, \cdots, \frac{2}{3n+3}$ to the vertices $u_1, u_2, \cdots, u_n$. We have $\Pi_\chi(0) = \frac{3n-1}{2}$ and $\Pi_\chi(1) = \frac{3n-1}{2}$.

□

Theorem 4.9. *The prism graph, $C_n \times P_2$ is FP-cordial for all $n \geq 3$.*

Proof. Let $V(C_n \times P_2) = \{x_i, y_i : 1 \leq i \leq n\}$ and $E(C_n \times P_2) = \{x_iy_i, x_1x_n, y_1y_n : 1 \leq i \leq n\} \cup \{x_ix_{i+1}, y_iy_{i+1} : 1 \leq i \leq n-1\}$. Then it has $2n$ vertices and $3n$ edges. This proof break up into two cases.
Case 1: n is even and $n \geq 4$
Assign the labels $1, 2, \cdots, n$ to the vertices $x_1, x_2, \cdots, x_n$. We now assign the labels $\frac{2}{n+4}, \frac{2}{n+6}, \cdots, \frac{1}{n+1}$ to the vertices $y_1, y_2, \cdots, y_{\frac{n}{2}}$ and assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+2}$ to the vertices $y_{\frac{n}{2}+1}, y_{\frac{n}{2}+2}, \cdots, y_n$. Therefore $\Pi_\chi(0) = \frac{3n}{2}$ and $\Pi_\chi(1) = \frac{3n}{2}$.
Case 2: n is odd and $n \geq 3$
Assign the labels $1, 2, \cdots, n$ to the vertices $x_1, x_2, \cdots, x_n$ and assign the labels $\frac{1}{n+1}, \frac{1}{n}, \cdots, \frac{1}{2}$ to the vertices $y_1, y_2, \cdots, y_n$. Hence $\Pi_\chi(0) = \frac{3n-1}{2}$ and $\Pi_\chi(1) = \frac{3n+1}{2}$.

□

Theorem 4.10. *The mobious ladder graph, M_n is FP-cordial for all $n \geq 4$.*

Proof. Let $V(M_n) = \{a_i, b_i : 1 \leq i \leq n\}$ and $E(M_n) = \{a_ib_i : 1 \leq i \leq n\} \cup \{a_1b_n, a_nb_1\} \cup \{a_ia_{i+1}, b_ib_{i+1} : 1 \leq i \leq n-1\}$. Then it has $2n$ vertices and $3n$ edges. This proof divided into two cases as follows.
Case 1: n is odd
This case break up into two subcases as follows.
Subcase 1: $n \geq 5$
Assign the labels $1, 2, \cdots, n-1$ to the vertices $a_1, a_2, \cdots, a_{n-1}$. We now assign the labels $\frac{1}{n+1}, \frac{1}{n}, \cdots, \frac{1}{3}$ to the vertices $b_1, b_2, \cdots, b_{n-1}$ and assign the labels $\frac{1}{2}$ and n to the vertices a_n and b_n. Therefore $\Pi_\chi(0) = \frac{3n+1}{2}$ and $\Pi_\chi(1) = \frac{3n-1}{2}$.
Subcase 2: $n = 3$
Suppose $M_3 \in \Omega_{fpc}$. In this case the vertex labels are $1, 2, 3, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. When $1, 2, 3, \frac{1}{2}$ are the vertex labels of the adjacent vertices, $\Pi_\chi(0) \leq 3$ a contradiction to the size M_3 is 9. When $1, 2, 3, \frac{1}{2}$ are the vertex labels of the non adjacent vertices, we have $\Pi_\chi(0) = 0$ and $\Pi_\chi(1) = 9$, again a contradiction.
Case 2: n is even
This case also break up into two subcases as follows.
Subcase 1: $n \geq 4$
This proof follows from case(1). Hence $\Pi_\chi(0) = \frac{3n}{2}$ and $\Pi_\chi(1) = \frac{3n}{2}$.
Subcase 2: $n = 2$
Suppose $M_2 \in \Omega_{fpc}$, then $M_2 \cong K_4$ the proof follows from theorem 2.15

□

Theorem 4.11. *The vanessa graph, V_n is FP-cordial for all $n \geq 2$.*

Proof. Let $V(V_n) = \{x_i, w, w_i, y_i : 1 \leq i \leq n\}$ and $E(V_n) = \{wx_i, ww_i, wy_i : 1 \leq i \leq n\} \cup \{x_i x_{i+1}, y_i y_{i+1} : 1 \leq i \leq n-1\}$. Then it has $3n+1$ vertices and $5n-2$ edges. This proof divided into two cases.
Case 1: n is even
When $n \geq 4$, Fix the central vertex w and assigned the label 3 to w. Assign the labels $1, 2, 4, \cdots, n+1$ to the vertices $x_1, x_2, \cdots, x_n$ and assign the labels $n+2, n+3, \cdots, \frac{3n}{2}$ to the vertices $w_1, w_2, \cdots, w_{\frac{n-2}{2}}$. We now assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+4}$ to the vertices $w_{\frac{n-2}{2}+1}, w_{\frac{n-2}{2}+2}, \cdots, w_n$ and assign the labels $\frac{2}{n+6}, \frac{2}{n+8}, \cdots, \frac{2}{3n+4}$ to the vertices $y_1, y_2, \cdots, y_n$. Therefore $\Pi_\chi(0) = \frac{5n-2}{2}$ and $\Pi_\chi(1) = \frac{5n-2}{2}$.
When $n = 2$, A FP-cordial labeling of V_2 is given in Figure 4.

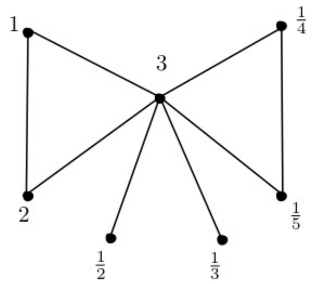

Figure 4: FP-Cordial Labeling of V_2

Case 2: n is odd
Fix the central vertex w and assigned the label 3 to w. Assign the labels $1, 2, 4, \cdots, n+1$ to the vertices $x_1, x_2, \cdots, x_n$ and assign the labels $n+2, n+3, \cdots, \frac{3n+1}{2}$ to the vertices $w_1, w_2, \cdots, w_{\frac{n-1}{2}}$. We now assign the labels $\frac{1}{2}, \frac{1}{3}, \cdots, \frac{2}{n+3}$ to the vertices

$$w_{\frac{n-1}{2}+1}, w_{\frac{n-1}{2}+2}, \cdots, w_n$$

and assign the labels $\frac{2}{n+5}, \frac{2}{n+7}, \cdots, \frac{2}{3n+3}$ to the vertices $y_1, y_2, \cdots, y_n$. It is easy to verify that $\Pi_\chi(0) = \frac{5n-1}{2}$ and $\Pi_\chi(1) = \frac{5n-3}{2}$.

□

Theorem 4.12. *The udukkai graph, U_n is FP-cordial for all $n \geq 2$.*

Proof. Let $V(U_n) = \{w, w_i : 1 \leq i \leq n\}$ and $E(U_n) = \{ww_i : 1 \leq i \leq n\} \cup \{ww_{2n-1}, ww_{2n}, ww_i : 3n-1 \leq i \leq 4n-2\}$. Then it has $4n-1$ vertices and $6n-4$ edges.
Assign the label 3 to the vertex w. Assign the labels $1, 2, 4, \cdots, n+1$ to the vertices $w_1, w_2, \cdots, w_n$ and assign the labels $\frac{1}{2}, n+2, \cdots, 2n-1$ to the vertices

$$w_{n+1}, w_{n+2}, \cdots, w_{2n-1}.$$

Now assign the labels $\frac{1}{3}, \frac{1}{4}, \cdots, \frac{1}{n+1}$ to the vertices $w_{2n}, w_{2n+1}, \cdots, w_{3n-2}$ and assign the labels $\frac{1}{n+2}, \frac{1}{n+3}, \cdots, \frac{1}{2n+1}$ to the vertices $w_{3n-1}, w_{3n}, \cdots, w_{4n-2}$. Hence $\Pi_\chi(0) = 3n-2$ and $\Pi_\chi(1) = 3n-2$.

□

5 Conclusion

In this paper we investigate the FP-cordial labeling behaviour of certain graphs, like lotus graph, necklace graph, F-tree, Y-tree, hurdle graph, key graph, coconut tree, prism, mobious ladder, vanessa graph and udukkai graph. To examine the FP-cordial labeling for different types of graphs and some other special graphs are the open area of research work.

References

[1] Cahit, I., Cordial graphs: a weaker version of graceful and harmonious graphs, Ars comb., 23, 201-207, 1987.

[2] Chandrakala S., Sekar C., Fibonacci prime labeling of udukkai and octopus graphs, International Journal of Scientific Research and Reviews, 7(2), 589-598, 2018.

[3] Edward samuvel, A., Kalaivani, S., Prime labeling to udukkai graphs, International Journal of Mathematical Archive, 9(3), 129-133, 2018.

[4] Gallian, J. A., A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 27, 1-712, 2024.

[5] Ghodasara G. V., and Vaghasiya S. M., Product cordial labeling of graphs related to helm, closed helm and gear graph, Internat. J. Pure and Appl. Math., 91 No. 4, 495-504, 2014.

[6] Harary F., Graph theory, Addison Wesely, New Delhi, 1972.

[7] Jesintha J. J., & Hilda K. E., All uniform bow graphs are graceful, Mathematics in Computer Science, Vol. 9, No. 2, pp. 185-191, 2015.

[8] Petrano A., & Rulete R., On total product cordial labeling of some graphs, Internat. J. Math. Appl., Vol. 5, No. 2B, 273-284, 2017.

[9] Ponraj R., Annathurai K., & Kala R., k-remainder cordial graphs, Journal of Algorithms and Computation, Vol. 49 (2), 41-52, 2017.

[10] Ponraj R., Gayathri A., & Somasundaram S., 4-remainder cordial of some tree related graphs, Intenational J. Math. Combin. Vol. 2, 56-71, 2022.

[11] Ponraj R., et al., Pair mean cordial labeling of hurdle, key, lotus and necklace graphs, J. Math. and App., 18(4), 2795-2804, 2024.

[12] Ponraj R., & Sutharson T., Fractional product cordial labeling of graph, J. Appl. and Pure Math., Vol. 7, No. 1-2, pp. 49-58, 2025.

[13] Ponraj R., & Sutharson T., Fractional product cordial labeling of web graph, dumbbell graph, jelly fish graph, jewel graph and tadpole graph, Journal of Indian Acad. Math., Vol. 46, No. 2, pp. 303-312, 2024.

[14] Prajapati U. M., & Patel N. B., Edge product cordial labeling of some graphs, J. Appl. Math. Comput. Mech., 18(1), 69-76, 2019.

[15] Rokad A. H., Product cordial labeling of double wheel and double fan related graphs, Kragujevac J. Math., 43(1) 7-13, 2019.

[16] Seoud M. A., & Maqsoud A. A., On cordial and balanced labeling ofgraphs, J. Egyptian Math. Soc, Vol. 7, No. 1, pp. 127-135, 1999.

[17] Seoud M. A. & Helmi E. F., On product cordial graphs, Ars Combin., 101, 519-529, 2011.

[18] Uma Maheswari A., Bala Samuvel J., A study on graphs with power dominator chromatic number 3, International journal of Mechanical Engineering, Vol. 7 No. 4, 1077-1084, 2022.