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Determining Triangular Fuzzy Number Bounds Using Bootstrap Confidence Intervals: An Algorithmic Framework for Fuzzy DEMATEL

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ABSTRACT

Triangular fuzzy numbers (TFNs) are extensively used in fuzzy logic and multi criteria decision making (MCDM) problems, where expert judgments are often expressed linguistically. A critical issue in applying TFNs is the determination of appropriate upper and lower bounds, particularly when the number of experts is limited. In this paper, a bootstrap based approach is proposed to estimate the upper and lower boundaries of TFNs using confidence intervals derived from expert evaluations. The proposed method enables a data driven and statistically grounded construction of fuzzy triangles that better reflect the inherent uncertainty in expert opinions. As an application, the method is integrated into the fuzzy DEMATEL technique within an MCDM framework. Simulation results indicate that the proposed approach provides a clearer discrimination of causal relationships compared to the classical fuzzy DEMATEL method. These findings confirm the effectiveness of the proposed method in improving the reliability of TFN based decision making models.

Keyword: Nonlinear Modeling, Artificial Neural Networks, Ground Vibrations, Hybrid Optimization, Open-Pit Mining.

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1 Introduction

Fuzzy set theory has been widely recognized as an effective mathematical framework for modeling uncertainty, vagueness, and imprecision inherent in real world systems. Among various types of fuzzy representations, triangular fuzzy boundaries (or triangular fuzzy numbers) have attracted considerable attention due to their conceptual simplicity, computational efficiency, and intuitive geometric interpretation. A triangular fuzzy boundary is typically characterized by three parameters representing the lower bound, the most plausible value, and the upper bound, which together define a piecewise linear membership function. This structure makes triangular fuzzy boundaries particularly suitable for applications where expert knowledge or limited data are available[1, 2]. Triangular fuzzy boundaries have been successfully employed in a wide range of disciplines, including decision making systems, risk assessment, control engineering, supply chain management, and economic modeling. Their popularity stems from the balance they offer between expressive power and analytical tractability, particularly in problems involving expert judgments or limited data. Despite their advantages, a fundamental challenge in the use of triangular fuzzy boundaries lies in the accurate determination and optimization of their parameters. In practice, the boundaries are often constructed based on subjective judgments, heuristic rules, or limited empirical observations. Such approaches may lead to biased or suboptimal fuzzy representations that fail to reflect the true uncertainty of the underlying data-generating process. Consequently, the reliability and robustness of decision-making models built upon these fuzzy boundaries can be significantly affected. To address this challenge, numerous methods have been proposed for optimizing and estimating triangular fuzzy boundaries. These include optimization based techniques, statistical estimation methods, learning based approaches, evolutionary algorithms, and hybrid fuzzy probabilistic models[3]. While these methods have contributed to improving parameter estimation, many of them rely on strong distributional assumptions, require large sample sizes, or involve high computational costs. Moreover, uncertainty quantification for the estimated fuzzy boundaries is often overlooked or treated in an ad hoc manner. In recent years, resampling techniques particularly bootstrap methods have emerged as powerful tools for statistical inference under minimal assumptions. Bootstrap procedures enable the empirical approximation of sampling distributions by repeatedly resampling from observed data, thereby facilitating the construction of confidence intervals without relying on asymptotic normality or known parametric forms. In the context of fuzzy modeling, bootstrap confidence intervals provide a principled way to quantify uncertainty associated with fuzzy boundary parameters and to enhance the robustness of fuzzy representations derived from data[4, 5]. Nevertheless, a closer examination of existing approaches reveals that many of the proposed methods ultimately rely on triangular fuzzy numerical scales that are predominantly symmetric. Such constructions often yield moderate or average-type results, with fuzzy evaluations tending to cluster around central or crisp values. As a consequence, the expressive capability of fuzziness may be weakened, and the resulting fuzzy representations may fail to fully capture the underlying uncertainty inherent in expert based assessments. In particular, the determination of appropriate lower and upper bounds for triangular fuzzy numbers remains a persistent and nontrivial issue in practice, especially when expert judgments or linguistic evaluations are employed. This observation highlights the need for a statistically grounded mechanism to define fuzzy boundaries in a way that reflects data variability rather than heuristic symmetry assumptions. Motivated by these considerations, this paper proposes a new approach to constructing and optimizing triangular fuzzy boundaries based on bootstrap confidence intervals, and we demonstrate its applicability within the fuzzy DEMATEL framework. The core idea is to integrate bootstrap-based uncertainty estimation into the construction and refinement of triangular fuzzy boundaries, allowing the lower, modal, and upper parameters to be optimized in a statistically grounded manner. By leveraging bootstrap confidence intervals, the proposed approach aims to produce fuzzy boundaries that more accurately capture data variability while maintaining the simplicity and interpretability of triangular fuzzy models. The remainder of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 presents the necessary preliminaries. Section 4 details the proposed bootstrap based method, and Section 5 illustrates its application in fuzzy DEMATEL. Sections 6 and 7 report the simulation study and experimental case study, respectively. Finally, Section 9 concludes the paper.

2 Related Work

Since the introduction of fuzzy set theory by Zadeh (1965), fuzzy numbers have been widely recognized as effective tools for transforming mental, linguistic, and subjective judgments into mathematical representations [6, 7, 8]. In many real-world decision-making problems, particularly in multi-attribute decision making (MADM), multi-criteria decision making (MCDM), and multi-attribute group decision making (MAGDM), fuzzy scales are commonly employed to model uncertainty and vagueness in expert evaluations [9, 10]. Among various fuzzy representations, triangular fuzzy numbers (TFNs) have attracted significant attention due to their simplicity, computational efficiency, and intuitive structure. TFNs have been extensively applied in decision analysis, engineering systems, and economic modeling [11, 12]. Recent studies have further demonstrated the effectiveness of TFNs in fuzzy decision environments and practical case studies. Despite their popularity, the theoretical validity of certain triangular fuzzy constructions has been questioned in the literature. As discussed in Seresht et al. (2019), Cheng (1994) showed that the triangular weight approximation of fuzzy numbers proposed by Zeng and Li may not satisfy the essential properties of fuzzy numbers, thereby challenging their mathematical soundness [11]. To overcome this limitation, improved trapezoidal and triangular approximations were proposed to preserve the fundamental characteristics of fuzzy sets. In addition to structural issues, ranking and comparison methods for TFNs have also been widely investigated. Feng Wang (2021) argued that many existing ranking approaches rely solely on three characteristic points the minimum value, the maximum value, and the most probable value while neglecting other possible values within the fuzzy number [12]. Such simplifications may lead to information loss and inaccurate decision outcomes. To address this issue, probabilistic and possibilistic measures were combined to define preference degrees that account for the entire support of TFNs. To enhance the accuracy and robustness of MADM and MAGDM methods, various extensions and modifications of TFNs have been proposed, including improved aggregation operators and hybrid fuzzy decision making frameworks [13]. These studies have significantly contributed to the advancement of fuzzy decision models and their practical applications. However, a common limitation of many existing approaches is the reliance on predefined triangular fuzzy numerical scales, which are predominantly symmetric. Such symmetric constructions often yield moderate or average type results, with fuzzy evaluations clustering around crisp values. Consequently, the expressive capability of fuzziness may be weakened, and the resulting fuzzy representations may fail to fully capture the inherent uncertainty in expert based assessments [9, 12]. In particular, the determination of appropriate upper and lower bounds for triangular fuzzy numbers remains a persistent and unresolved issue. In most existing studies, these bounds are selected based on heuristic rules or subjective assumptions rather than statistically grounded uncertainty quantification. This gap highlights the need for a data driven and statistically principled approach for defining fuzzy boundaries, especially in contexts where expert opinions or limited samples are involved.

3 Preliminaries

3.1 Fuzzy Set Theory and Triangular Fuzzy Numbers

Fuzzy set theory provides a mathematical framework for modeling vague, imprecise, and subjective information that cannot be adequately represented using classical crisp sets. In contrast to binary logic, where membership is restricted to the values 0 or 1, fuzzy set theory allows partial membership degrees in the interval $[0, 1]$. This feature makes fuzzy sets particularly suitable for handling qualitative judgments and linguistic assessments in decision making problems. A triangular fuzzy number (TFN) $\tilde{N}$ is commonly represented by an ordered triplet (l, m, u), where l, m, and u denote the lower bound, the most plausible value, and the upper bound, respectively, with $l \leq m \leq u$. The membership function $\mu_{\tilde{N}} : \mathbb{R} \to [0, 1]$ is defined as

$$\mu_{\tilde{N}}(x) = \begin{cases} 0, & x < l, \\ \dfrac{x-l}{m-l}, & l \leq x \leq m, \\ \dfrac{u-x}{u-m}, & m \leq x \leq u, \\ 0, & x > u. \end{cases} \tag{1}$$

Due to their simple structure and computational efficiency, TFNs are widely employed in fuzzy decision-making and multi criteria analysis[14, 15].

3.2 Linguistic Variables and Fuzzy Scales

Linguistic variables are variables whose values are expressed in natural language rather than numerical form. Such variables are particularly effective for representing human judgments, which are often imprecise and subjective. In practical applications, linguistic terms are typically modeled using fuzzy numbers, with TFNs being one of the most frequently adopted representations. In this study, linguistic assessments describing the degree of influence between criteria are modeled using triangular fuzzy numbers. The linguistic scale adopted in this work is presented in Table 1.

Table 1: Linguistic terms and corresponding triangular fuzzy numbers

Linguistic term	TFN (l, m, u)
No influence	$(0, 0, 0.25)$
Very low influence	$(0, 0.25, 0.50)$
Low influence	$(0.25, 0.50, 0.75)$
High influence	$(0.50, 0.75, 1)$
Very high influence	$(0.75, 1, 1)$

While this conventional symmetric scale is widely adopted, its fixed bounds may not adequately reflect the variability inherent in expert judgments across different contexts. This motivates the proposed bootstrap-based determination of TFN bounds in the subsequent sections[16, 17].

3.3 Bootstrap Method

The bootstrap method is a nonparametric resampling technique designed to approximate the sampling distribution of an estimator when the underlying population distribution is unknown. Let $\mathbf{x} = (x_1, \ldots, x_n)$ denote an observed sample drawn from an unknown distribution. Bootstrap resampling generates pseudo-samples $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$ by sampling with replacement from $\mathbf{x}$, such that

$$P(X^* = x_i) = \frac{1}{n}, \quad i = 1, \ldots, n.$$

Let θ be a parameter of interest and $\hat{\theta}$ its estimator. For each bootstrap replication $b = 1, \ldots, B$, an estimator $\hat{\theta}^{(b)}$ is computed from a bootstrap sample. The empirical distribution of $\{\hat{\theta}^{(1)}, \ldots, \hat{\theta}^{(B)}\}$ provides an approximation to the sampling distribution of $\hat{\theta}$.

3.4 Basic Bootstrap Confidence Interval

The basic bootstrap confidence interval is one of the classical bootstrap techniques for constructing confidence limits based on resampling. This method relies on centering the bootstrap distribution around the observed statistic and estimating confidence limits using empirical quantiles [18, 19]. Let $\hat{\theta}$ denote the estimator of θ computed from the observed data, and let $\hat{\theta}^*$ represent the corresponding bootstrap replicates. The $100(1-\alpha)\%$ basic bootstrap confidence interval for θ is given by

$$\left(2\hat{\theta} - \hat{\theta}_{(1-\alpha/2)},\ 2\hat{\theta} - \hat{\theta}_{(\alpha/2)}\right), \tag{2}$$

where $\hat{\theta}_{(\alpha)}$ denotes the empirical α-quantile of the bootstrap distribution. To illustrate the rationale behind this interval, let us assume a simplified parametric setting. Let T be an estimator of θ, and let a_α be the α-quantile of the distribution of $T - \theta$. Then

$$P(T - \theta > a_\alpha) = 1 - \alpha \quad \Rightarrow \quad P(T - a_\alpha > \theta) = 1 - \alpha.$$

Accordingly, a $100(1-2\alpha)\%$ confidence interval with equal tail probabilities α is given by

$$(t - a_{1-\alpha},\ t - a_\alpha),$$

where t is the observed value of T. In practice, the distribution of T is unknown and is approximated via bootstrap resampling. Let $\hat{\theta}_\alpha$ denote the empirical α-quantile of $\hat{\theta}^*$. Defining $\hat{b}_\alpha = \hat{\theta}_\alpha - \hat{\theta}$ yields an approximation of the quantiles of $\hat{\theta}^* - \hat{\theta}$. Consequently, the lower and upper confidence limits in (2) are obtained.

4 Proposed Method

The main objective of this study is to determine the lower and upper bounds of triangular fuzzy numbers in a statistically grounded manner when expert opinions are employed. Given the typically limited number of experts in decision making problems, predefined symmetric fuzzy scales may inadequately reflect the true uncertainty of expert judgments. Let θ denote the aggregated value of expert opinions for a given linguistic assessment, and let $\hat{\theta}$ be its estimator. Based on the basic bootstrap confidence interval, the proposed bootstrap-based triangular fuzzy number is defined as

$$\widetilde{\mathrm{TFN}}_B = \left(2\hat{\theta} - \hat{\theta}_{(1-\alpha/2)},\ \hat{\theta},\ 2\hat{\theta} - \hat{\theta}_{(\alpha/2)}\right). \tag{3}$$

It is important to emphasize that the estimator $\hat{\theta}$ in this formulation specifically denotes the aggregated central value (e.g., the mean) of expert opinions for a given pair of criteria (i, j) in the fuzzy DEMATEL context. Consequently, the proposed bootstrap procedure is applied independently to each individual cell of the direct influence matrix, rather than to the entire matrix or to a linguistic term as a whole. Furthermore, to ensure the mathematical validity of the constructed triangular fuzzy number, the condition $l \leq m \leq u$ must hold. In the proposed bootstrap framework, this condition is satisfied as long as

$$\hat{\theta}_{(\alpha/2)} \leq \hat{\theta} \leq \hat{\theta}_{(1-\alpha/2)},$$

which is generally guaranteed for the bootstrap replicates. In the rare case where this inequality is violated due to extreme skewness or a very small number of experts, the lower and upper bounds should be clamped to the feasible range of the linguistic scale (e.g., $[0, 1]$ or $[0, 4]$) to preserve the triangular fuzzy number structure. Unlike conventional symmetric scales, this construction does not impose a fixed offset for the lower and upper bounds. Instead, the boundaries are directly determined by the empirical variability of expert responses, thereby preserving the inherent asymmetry that may exist in real world judgments.

5 Application of the Proposed Method in Fuzzy DEMATEL

The DEMATEL (Decision Making Trial and Evaluation Laboratory) technique is a well established multi criteria decision-making (MCDM) method for analyzing and visualizing complex cause effect relationships among system components. Originally introduced by Fontela and Gabus, this method transforms interdependent relationships into a structured causal diagram using matrix based computations. In many real world decision making problems, expert judgments are inherently vague and imprecise, making exact numerical evaluations inadequate. To address this limitation, fuzzy logic has been widely integrated into the DEMATEL framework. In this study, the proposed *bootstrap based triangular fuzzy numbers* are incorporated into the fuzzy DEMATEL procedure in order to construct data-driven lower and upper bounds for expert evaluations. The implementation process consists of the following steps.

Step 1. Formation of the Expert Group

A group of l experts is formed to provide knowledge and judgments regarding the causal relationships among the criteria of the system.

Step 2. Determination of Criteria and Linguistic Scale

Let $F = \{F_1, F_2, \ldots, F_n\}$ denote the set of criteria. A linguistic scale is designed to evaluate the degree of influence among criteria using the terms *No*, *Very Low*, *Low*, *High*, and *Very High*. At this stage, only the central values of triangular fuzzy numbers are employed for eliciting expert opinions, as presented in Table 1.

Step 3. Collection and Aggregation of Expert Judgments

Each expert k ($k = 1, 2, \ldots, l$) provides pairwise comparisons among criteria. Accordingly, l direct-influence matrices are obtained as

$$\tilde{Z}^k = [\tilde{z}_{ij}^k]_{n\times n}, \quad k = 1, 2, \ldots, l,$$

where

$$\tilde{z}_{ij}^k = m_{ij}^{(k)}, \quad i, j = 1, 2, \ldots, n,$$

and $\tilde{z}_{ii}^k = 0$.
The aggregated estimator of expert opinions is computed as

$$\hat{\theta}_{ij} = \frac{1}{l}\sum_{k=1}^{l} \tilde{z}_{ij}^k.$$

Step 4. Construction of the Bootstrap-Based Fuzzy Direct-Influence Matrix

To capture the uncertainty inherent in expert evaluations, the basic bootstrap confidence interval is applied to the aggregated estimator $\hat{\theta}_{ij}$. The resulting *bootstrap based triangular fuzzy number (B-TFN)* is defined as:

$$\tilde{Z}_{B,ij} = \left(2\hat{\theta}_{ij} - \hat{\theta}_{ij}^{(1-\alpha/2)},\ \hat{\theta}_{ij},\ 2\hat{\theta}_{ij} - \hat{\theta}_{ij}^{(\alpha/2)}\right),$$

where $\hat{\theta}_{ij}^{(\alpha)}$ denotes the empirical α-quantile of the bootstrap replicates.

$$\tilde{Z}_B = [\tilde{Z}_{B,ij}]_{n\times n}.$$

The matrix composed of these B-TFNs is called the bootstrap based fuzzy direct influence matrix $\tilde{Z}_B$.

Step 5. Normalization of the Direct-Influence Matrix

The normalized fuzzy direct-influence matrix $\tilde{X}$ is obtained as

$$\tilde{X} = \frac{\tilde{Z}_B}{r},$$

where

$$r = \max_{1\le i\le n}\left(\sum_{j=1}^{n} z_{ij}^{(3)}\right),$$

and $z_{ij}^{(3)}$ represents the upper bound of the triangular fuzzy number $\tilde{Z}_{B,ij}$.

Step 6. Computation of the Total-Influence Matrix

The fuzzy total influence matrix is computed as

$$\tilde{T} = \lim_{h\to\infty} \left(\tilde{X}^1 + \tilde{X}^2 + \cdots + \tilde{X}^h\right) = \tilde{X}(I - \tilde{X})^{-1},$$

provided that $\lim_{h\to\infty} \tilde{X}^h = O$.
Each triangular fuzzy element of $\tilde{T}$ is decomposed as

$$\tilde{T} = (T_1, T_2, T_3),$$

where

$$T_k = X_k(I - X_k)^{-1}, \quad k = 1, 2, 3.$$

Step 7. Derivation of Cause–Effect Relationships

For each criterion F_i, the sum of rows and columns of the total influence matrix are computed as

$$\tilde{R}_i = \sum_{j=1}^{n} \tilde{t}_{ij}, \quad \tilde{C}_i = \sum_{j=1}^{n} \tilde{t}_{ji}.$$

The indices $\tilde{R}_i + \tilde{C}_i$ and $\tilde{R}_i - \tilde{C}_i$ represent the prominence and causal role of each criterion, respectively.

Step 8. Defuzzification and Impact Relation Map

The fuzzy values are transformed into crisp values using the defuzzification formula

$$y = \frac{l + 2m + u}{4}.$$

Finally, the impact relation map (IRM) is constructed by plotting the ordered pairs

$$\left((R_i + C_i)^{\text{def}},\ (R_i - C_i)^{\text{def}}\right).$$

6 Simulation Study

6.1 Simulation Design and Rationale

In the DEMATEL technique, conventional error based accuracy measures such as the Mean Absolute Percentage Error (MAPE) and the Mean Absolute Deviation (MAD) are not directly applicable, as the method is not designed for forecasting but for uncovering structural relationships. Consequently, prior studies have often relied on simulation and experimental examples to demonstrate the efficacy of new DEMATEL variants [20]. In line with this approach, the present study first employs a controlled simulation to validate the proposed Bootstrap based Fuzzy DEMATEL (BFD) method against the conventional Fuzzy DEMATEL (FD) approach under a known causal structure. Subsequently, a real-world case study is used to illustrate its practical utility. The simulation involves five criteria $\{F_1, F_2, F_3, F_4, F_5\}$, corresponding respectively to the linguistic influence levels: *No influence*, *Very low influence*, *Low influence*, *High influence*, and *Very high influence* (Table 2).

Table 2: Mapping of simulation criteria to linguistic influence levels

Linguistic influence level	Assigned criterion
No influence	F_1
Very low influence	F_2
Low influence	F_3
High influence	F_4
Very high influence	F_5

A panel of eight experts ($l = 8$) is simulated. For each expert k ($k = 1, \ldots, 8$), a direct influence matrix $\tilde{Z}^k = [\tilde{z}_{ij}^k]_{5\times 5}$ is generated, where $\tilde{z}_{ii}^k = 0$. The off diagonal elements $\tilde{z}_{ij}^k$ represent the central value (the most plausible value m) of a triangular fuzzy assessment, following the approach of [13]. These central values are generated from discrete probability distributions designed to reflect the underlying influence pattern, as outlined in Table 3. Weaker influences are assigned higher probabilities to smaller numerical codes, while stronger influences skew toward larger codes. This design ensures a known causal benchmark: factors F_5 and F_1 are expected to have influence values of similar magnitude but opposite signs in the $R - C$ vector; factors F_2 and F_4 should behave similarly; and factor F_3 should be approximately neutral ($R - C \approx 0$).
The simulation is repeated 1000 times. In each iteration, the aggregated central value $\hat{\theta}_{ij}$ is computed as the mean of the eight expert assessments. Bootstrap resampling with $B = 1000$ replicates and a confidence level of $\alpha = 0.05$ is then applied to construct the bootstrap-based triangular fuzzy number $\tilde{Z}_{B,ij}$ for each pair (i, j), forming the matrix $\tilde{Z}_B$. The conventional FD method is applied in parallel using the predefined symmetric fuzzy scale from Table 1. Finally, the vectors R, C, $R + C$, and $R - C$ are computed for both methods, and their averages over the 1000 runs are reported.

6.2 Simulation Results and Comparative Analysis

The averaged results from the conventional FD method and the proposed BFD method are presented in Tables 4 and 5, respectively.
The results highlight significant differences between the two methods. As designed, the benchmark expected $R - C$ values for F_5 and F_1 to be of similar magnitude but opposite signs, for F_2 and F_4 to exhibit similar behavior, and for F_3 to converge near zero. The FD method results (Table 4) deviate from this benchmark: the $R - C$ values of F_4 and F_5 are overly close (0.5112 vs. 0.6711), and F_3 does not sufficiently converge to zero (0.1023).

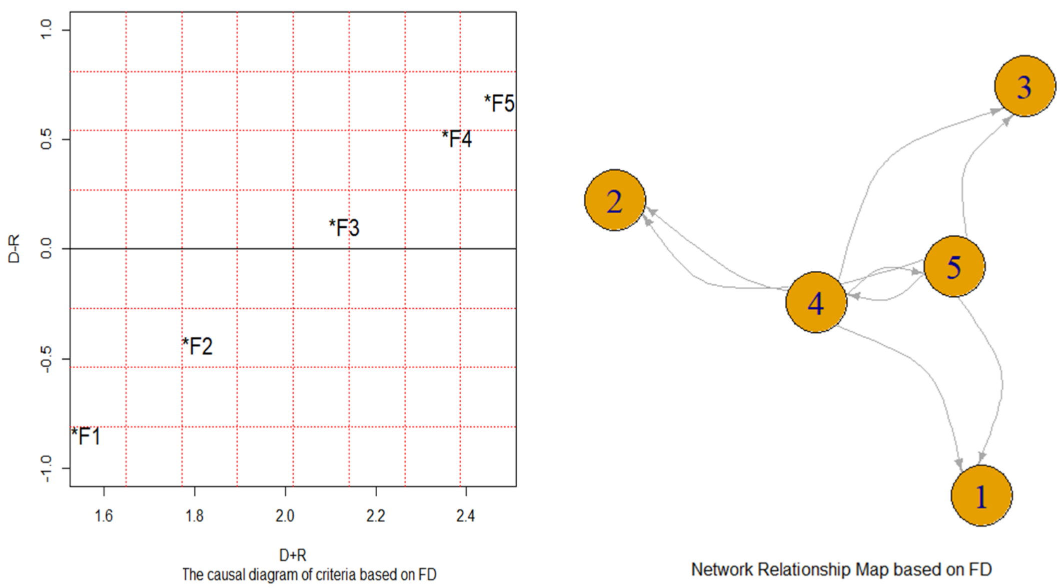

Figure 1: Visualization of the Fuzzy DEMATEL (FD) results from the simulation study, showing (a) the causal diagram and (b) the network relationship map.

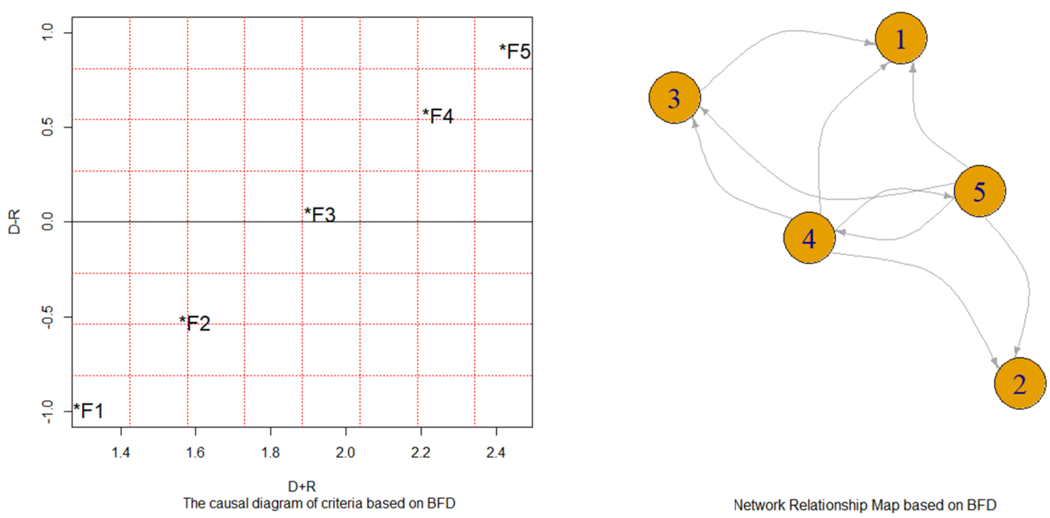

Figure 2: Visualization of the Bootstrap based Fuzzy DEMATEL (BFD) results from the simulation study, showing (a) the causal diagram and (b) the network relationship map.

Table 3: Simulation design: Probability distributions for generating expert assessments

Influencing factor	Influenced factor(s)	Distribution of $\tilde{z}_{ij}^k$ (central value)
F_1	F_2, F_3, F_4, F_5	$f(z) = \begin{cases} 0.75, & z = 0 \\ 0.25, & z = 1 \end{cases}$
F_2	F_1, F_3, F_4, F_5	$f(z) = \begin{cases} 0.25, & z = 0 \\ 0.50, & z = 1 \\ 0.25, & z = 2 \end{cases}$
F_3	F_1, F_2, F_4, F_5	$f(z) = \begin{cases} 0.25, & z = 1 \\ 0.50, & z = 2 \\ 0.25, & z = 3 \end{cases}$
F_4	F_1, F_2, F_3, F_5	$f(z) = \begin{cases} 0.25, & z = 2 \\ 0.50, & z = 3 \\ 0.25, & z = 4 \end{cases}$
F_5	F_1, F_2, F_3, F_4	$f(z) = \begin{cases} 0.75, & z = 3 \\ 0.25, & z = 4 \end{cases}$

Table 4: Averaged simulation results: Conventional Fuzzy DEMATEL (FD) method

Criteria	R	C	$R+C$	$R-C$
F_1	0.3568	1.2049	1.5617	-0.8480
F_2	0.6857	1.1223	1.8080	-0.4366
F_3	1.1173	1.0150	2.1323	0.1023
F_4	1.4458	0.9346	2.3803	0.5112
F_5	1.5723	0.9012	2.4735	0.6711

Table 5: Averaged simulation results: Bootstrap based Fuzzy DEMATEL (BFD) method

Criteria	R	C	$R+C$	$R-C$
F_1	0.1639	1.1525	1.3164	-0.9886
F_2	0.5366	1.0635	1.6001	-0.5269
F_3	0.9884	0.9439	1.9323	0.0445
F_4	1.4057	0.8409	2.2466	0.5648
F_5	1.6786	0.7724	2.4511	0.9062

Furthermore, the network relation map derived from the FD method (not shown) groups F_1, F_2, and F_3 together, which contradicts the distinct influence levels assigned in the simulation. In contrast, the BFD method results (Table 5) align much more closely with the simulated causal structure. The separation between cause and effect factors is more pronounced: the $R-C$ values of F_5 and F_1 show a clearer opposition (0.9062 vs. -0.9886), and F_3 approaches the neutral position (0.0445). The dispersion of factors in the influence relation map is also more distinct, enhancing interpretability. The superior performance of the BFD method can be attributed to its use of bootstrap confidence intervals for determining the lower and upper bounds of the triangular fuzzy numbers. By empirically capturing the variability in expert judgments, the BFD method mitigates the bias toward moderate values inherent in fixed symmetric fuzzy scales, leading to a more accurate and discriminative representation of the underlying influence structure.

7 Experimental Case Study: Analysis of Inflation Determinants

7.1 Case Description and Data Collection

To demonstrate the practical applicability of the proposed BFD method, a real world case study investigating the determinants of inflation is conducted. Based on an extensive review of economic literature, five key factors influencing inflation are identified: liquidity, government expenditures, gross domestic product (GDP), inflation of the previous period, and world oil prices. A structured DEMATEL questionnaire was designed to assess the causal relationships among these factors. Evaluations were collected from eight domain experts using a snowball sampling method. The linguistic judgments were then processed using both the conventional FD method (with the predefined symmetric scale) and the proposed BFD method (with bootstrap-derived bounds, $B = 1000$, $\alpha = 0.05$). The factors and their codes are listed in Table 6.

Table 6: Factors affecting inflation considered in the case study

Factor	Code
Liquidity	F_1
Government expenditures	F_2
Gross domestic product (GDP)	F_3
Inflation of the previous period	F_4
World oil prices	F_5

7.2 Results of FD and BFD Models

The aggregated total-influence matrices from both methods were defuzzified using the center of gravity formula $y = (l + 2m + u)/4$. The resulting R, C, $R + C$, and $R - C$ vectors for the FD and BFD methods are presented in Tables 7 and 8, respectively.

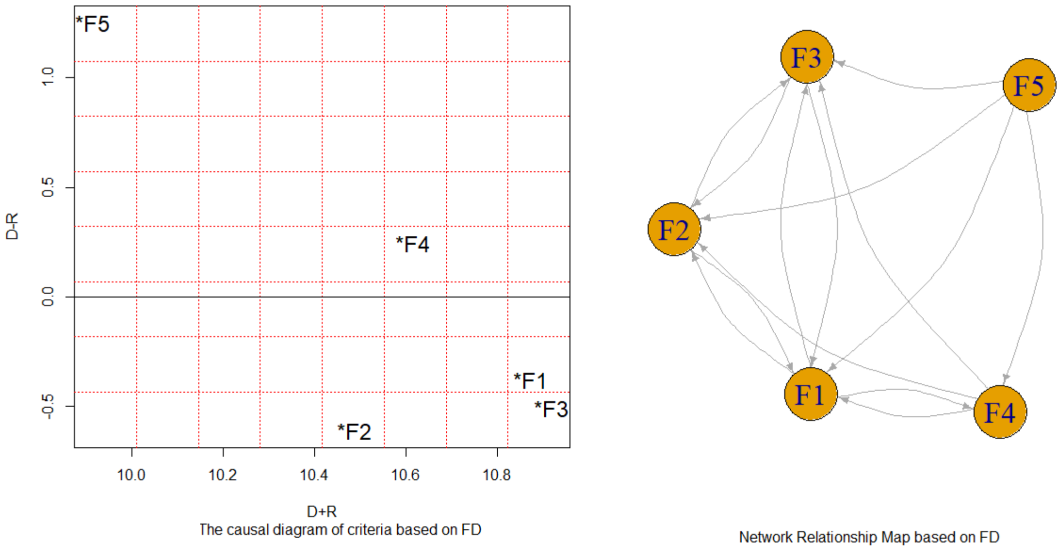

Figure 3: Visualization of the Fuzzy DEMATEL (FD) results from the inflation determinants case study, showing (a) the causal diagram and (b) the network relationship map.

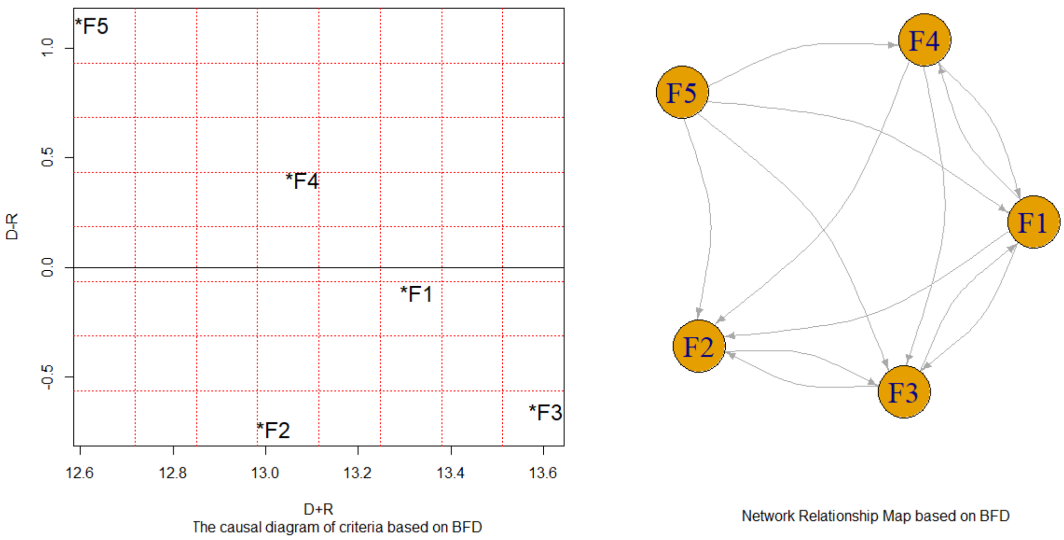

Figure 4: Visualization of the Bootstrap-based Fuzzy DEMATEL (BFD) results from the inflation determinants case study, showing (a) the causal diagram and (b) the network relationship map.

Table 7: Case study results: Conventional Fuzzy DEMATEL (FD) method

Criteria	R	C	$R+C$	$R-C$
F_1	5.2476	5.6269	10.8745	-0.3793
F_2	4.9380	5.5501	10.4881	-0.6121
F_3	5.2056	5.7134	10.9191	-0.5078
F_4	5.4309	5.1853	10.6163	0.2456
F_5	5.5841	4.3306	9.9147	1.2535

Table 8: Case study results: Bootstrap-based Fuzzy DEMATEL (BFD) method

Criteria	R	C	$R+C$	$R-C$
F_1	6.6068	6.7220	13.3288	-0.1152
F_2	6.1402	6.8783	13.0185	-0.7380
F_3	6.4753	7.1297	13.6050	-0.6543
F_4	6.7395	6.3403	13.0798	0.3993
F_5	6.8665	5.7582	12.6248	1.1083

7.3 Comparative Analysis and Discussion

In both models, the sign of $R-C$ determines the causal role: positive values indicate net causal factors, while negative values indicate net effect factors. The FD model (Table 7) identifies F_4 and F_5 as causes, and F_1, F_2, F_3 as effects. It suggests that government expenditures (F_2), GDP (F_3), previous period inflation (F_4), and world oil prices (F_5) are primary drivers of liquidity (F_1). The BFD model (Table 8) provides a more nuanced and economically interpretable structure. While it also identifies F_4 and F_5 as causes and the others as effects, the relative strengths and separations are different. More importantly, the causal influence on liquidity (F_1) is more clearly attributed to previous period inflation (F_4), GDP (F_3), and world oil prices (F_5), while the role of government expenditures (F_2) is relatively diminished. This distinction is economically meaningful, as liquidity is typically influenced more directly by monetary conditions, aggregate output, and external price shocks than by fiscal expenditures in the short to medium term. The BFD model's enhanced discriminative power stems from its data-driven approach to uncertainty. By using bootstrap confidence intervals to define fuzzy boundaries, it reduces the compression effect of symmetric scales and allows the inherent variability in expert judgments to shape the fuzzy numbers. This leads to a causal map that better reflects the experts' collective perception and aligns more closely with established economic theory. In summary, both the simulation and the real-world case study demonstrate that the proposed BFD method offers a robust, data-adaptive alternative to conventional fuzzy DEMATEL. It improves the accuracy of causal representation under uncertainty and yields results that are both statistically grounded and contextually interpretable.

8 Discussion

This study introduced a bootstrap-based method for constructing Triangular Fuzzy Numbers (TFNs) and demonstrated its integration into the fuzzy DEMATEL technique. The core contribution lies in shifting from predefined symmetric fuzzy scales to a data driven approach where the lower and upper bounds of TFNs are derived from bootstrap confidence intervals of expert judgments. The simulation and experimental results collectively offer strong evidence supporting the efficacy and advantages of the proposed method. The simulation study, conducted under a known causal structure, provided a controlled validation. The Bootstrap based Fuzzy DEMATEL (BFD) method demonstrated superior performance compared to the conventional Fuzzy DEMATEL (FD) method. Specifically, the BFD method yielded a causal map that aligned more accurately with the predefined benchmark: it achieved a clearer separation between cause and effect groups (evident in the more distinct $R-C$ values for F_5 and F_1), and successfully identified the neutral factor (F_3) with greater precision. This improvement can be directly attributed to the method's ability to modulate the width and asymmetry of the fuzzy numbers based on the empirical variability in the simulated expert data, thereby reducing the compression bias inherent in fixed symmetric scales. The practical utility of the method was further substantiated through the real-world case study on inflation determinants. While both FD and BFD models identified similar causal directions, the BFD model provided a more nuanced and economically interpretable hierarchy of influences. Its refinement of the causal structure particularly in clarifying the primary drivers of liquidity highlights its enhanced discriminative power. The results suggest that the BFD model can better capture the collective perception and uncertainty of domain experts, leading to conclusions that resonate more closely with established economic theory. However, several limitations of the proposed approach should be acknowledged, which also point to directions for future research. First, the method's performance is inherently dependent on the quality and consistency of the initial expert judgments. While bootstrap accounts for variability, it cannot correct for systematic bias in the expert panel. Second, the current implementation uses the basic bootstrap confidence interval. Future work could explore the impact of other interval types (e.g., percentile, BCa, or studentized) on the stability and properties of the resulting fuzzy numbers. Third, the computational load increases with the number of bootstrap resamples (B) and the size of the decision matrix. Although not prohibitive for typical DEMATEL problems, efficiency optimization for very large-scale models could be investigated. Finally, the application was focused on DEMATEL; testing the proposed B-TFN construction within other fuzzy MCDM frameworks (e.g., ANP, TOPSIS, or VIKOR) would be a valuable extension to demonstrate its generalizability. Despite these limitations, the proposed method offers a statistically grounded, flexible, and robust alternative for handling uncertainty in expert driven decision models. It provides a principled way to let the data "speak" in defining fuzzy boundaries, moving beyond heuristic symmetry assumptions.

9 Conclusion

In this paper, a bootstrap based approach was proposed to determine the lower and upper bounds of triangular fuzzy numbers when aggregating expert judgments. By employing bootstrap confidence intervals, the proposed method addresses the uncertainty arising from limited expert samples and provides a statistically grounded mechanism for constructing fuzzy triangles. This approach enables the selection of an optimal fuzzy representation that better reflects the variability and inconsistency inherent in expert evaluations. The proposed method was integrated into the fuzzy DEMATEL technique and its performance was examined through both simulation studies and a real world experimental example. The results demonstrate that the bootstrap-based fuzzy DEMATEL model yields more consistent and interpretable cause effect relationships compared to the conventional fuzzy DEMATEL approach. In particular, the proposed method shows a stronger capability in distinguishing causal and effect factors and in capturing the underlying structure of complex systems. Given its flexibility and robustness, the proposed bootstrap based fuzzy triangle construction can be effectively applied to various multi criteria decision making problems that rely on expert judgments. Therefore, this study provides a novel and reliable framework for improving fuzzy modeling and causal analysis in decision support systems.

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