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Prisoner's Dilemma Under Global Game Framework

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ABSTRACT

This paper studies the global game and its interesting properties in the famous game; $i.e.$, prisoner's dilemma game. The game is described, its Nash equilibriums are derived and sensitivity analysis is studied. The problem is also extended. This solution is applied to study the problem of selling foreign currency before or after speculation attack. Finally, a conclusion section is given.

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1 Introduction

Many economic problems are naturally modeled as a game of incomplete information. Global games are originally developed as equilibrium selection devices. Incomplete information comes from a noisy payoff perturbation of a complete information game, such that when the noise vanishes, we recover the original game. In a global game, each player receives, with a small amount of noise, a private signal of the value of a payoff's fundamental. The noise distribution is common knowledge, so each player's signal generates beliefs not only about the fundamental but also about the other players' beliefs (over the fundamental and beliefs of their rivals and so on). The idea of this approach is to examine Nash equilibrium – of the original complete information game as a limit of the equilibrium of the payoff-perturbed game.

Global games are games of incomplete information whose type space is determined by the players each observing a noisy signal of the underlying state. With strategic complementarities, global games often have a unique, dominance solvable equilibrium, allowing analysis of a number of economic models of coordination failure. For symmetric binary action global games, equilibrium strategies in the limit (as noise becomes negligible) are simple to characterize in terms of diffuse beliefs over the actions of others.

In global game framework, as described by Carlsson and van Damme (2003), the exact payout structure is random. These games have been used to model many types of regime changes, including asset bubbles, financial crises, bank failure and revolutions. Since the publication of Morris and Shin (2007) a new research program has arisen where global games have been used to model exchange rate crises. Initial applications of game theory to currency crises typically assumed complete information. From this structure, these second generation models typically explored cases where collective action is required to bring about devaluation. This state is typically characterized by the existence of multiple equilibriums. Morris and Shin (2007) relaxed the assumption of complete information in their model. They start by assuming a continuum of agents who seek to cause a currency crisis but face transaction costs. They discuss how the existence of addition, these and other dynamic applications of global games raise many other important economical problems. While common knowledge of payoffs is relaxed in global games, there is still assumed to be common knowledge of the information structure, which is surely a no more realistic assumption. There is a large literature that studies how changes in information structure in different types of coordination games with incomplete information affects equilibrium play (see Angeletos and Pavan (2007), Colombo, Femminis, and Pavan (2012) or Iachan and Nenov (2014)). A paper by Weinstein and Yildiz (2019) shows that the exact form of the perturbation away from common knowledge of payoffs is crucial in determining the rationalizable outcome. The global game prediction is not the only possible perturbation that yields unique rationalizable outcomes. Frankel *et al*. (2023) considered games with strategic complementarities. Rather than allow for all possible payoff profiles, they restrict attention to a one-dimensional set of possible payoff functions.

The Prisoner's dilemma plays important role in studying and developing game theory. This game is studied assuming complete information hypothesis. However, the incomplete case is not studied under the global game framework. In the current paper, this game is studied using proposing the global game framework which has been worked a few before the current paper. An exception is

Parilina and Tampieri (2018) and references therein. Prisoner’s dilemma, imaginary situation employed in game theory. One version is as follows. Two prisoners are accused of a crime. If one confesses and the other does not, the one who confesses will be released immediately and the other will spend 20 years in prison. If neither confesses, each will be held only a few months. If both confess, they will each be jailed 15 years. They cannot communicate with one another. Given that neither prisoner knows whether the other has confessed, it is in the self-interest of each to confess himself. Paradoxically, when each prisoner pursues his self-interest, both end up worse off than they would have been had they acted otherwise.
The rest of paper is organized as follows. In the next section, prisoner’s dilemma game under the global game framework is defined and the Nash equilibrium is derived for both players and sensitivity analysis of these equilibriums are derived. The prisoner's dilemma game in the global framework is used to decide to sell a specific foreign currency before or after the speculation attack in section 3. A real data analysis is given in section 4. Finally, conclusions are given in section 5.
2 Equilibriums and sensitivities. The prisoner's dilemma game, in bi-matrix format, between raw player (player 1) and column player (player 2) is given by

$$\begin{array}{ccc} & s_1 & s_2 \\ s_1 & \beta,\beta & \theta,\alpha \\ s_2 & \alpha,\theta & \gamma,\gamma \end{array}$$

where

$$\theta < \gamma < \beta < \alpha.$$

Here, s_1 stands for "contribution" of players and s_2 means the "don't contribution" state. There is a dominant equilibrium for both players at (s_1, s_1), see Osborne and Rubinstein (2018). Let

$$\alpha = \beta + l_2, \theta = \beta - l_1 \text{ and } \gamma = \beta - l_3$$

and consider the following re-parameterized game

$$\begin{array}{ccc} & s_1 & s_2 \\ s_1 & \beta,\beta & \beta - l_1, \beta + l_2 \\ s_2 & \beta + l_2, \beta - l_1 & \beta - l_3, \beta - l_3 \end{array}$$

where $l_i > 0, i = 1,2,3$. In the first prisoners dilemma, it is assumed (as above stated) that

$$l_1 > l_3.$$

However, here, this assumption is substituted with

$$l_1 < l_3.$$

Then, there are two pure Nash equilibriums at (s_1, s_1) and (s_2, s_2). Thus, there is instability in the problem. Here, according to the Carlsson and Van Damme (2003), the global game theory is applied to overcome this difficulty. Suppose that each player has his/her intuition (signal) about the state (economic fundamental) β. Thus,

$$x_i = \beta + \varepsilon_i, i = 1,2$$

and ε_i's are iid from $N(0, \sigma^2)$ distribution. The parameter σ is precision of public signals. Thus, as soon as player 1 receives signal x_1, then β is distributed with $N(x_1, \sigma^2)$ and hence x_2 has distribution

$$N(x_1, 2\sigma^2).$$

Consider the switching strategy for each player such that he contribute with another player when his signal is less than threshold K, that is

$$s_x = \begin{cases} corporate & x < K, \\ Do\ not & x \geq K, \end{cases} = \begin{cases} s_1 & x < K, \\ s_2 & x \geq K, \end{cases}$$

Player 1 assigns the probability

$$q = \Phi(\frac{K - x_1}{\sqrt{2}\sigma})$$

to the player 2 to contribute. One can see that

$$q = \frac{l_3 - l_1}{l_3 - l_1 + l_2}.$$

Thus,

$$\Phi\left(\frac{K - b(K)}{\sqrt{2}\sigma}\right) - \frac{l_3 - l_1}{l_3 - l_1 + l_2} = 0.$$

This is also true for the probability p which player 2 assigns to the player 1 to contribute. Hence,

$$p = q = \Phi\left(\frac{K - x_1}{\sqrt{2}\sigma}\right).$$

Although, in this paper, it is assumed that

$$l_1 < l_3,$$

however, in the regular prisoner's dilemma which assumes that

$$l_1 > l_3,$$

then

$$p = q = \frac{l_1 - l_3}{l_1 - l_3 - l_2}.$$

Here, it is necessary to assume that

$$0 < l_2 < l_1 - l_3.$$

The following special cases are interesting.

(*a*) Let $l_2 < \infty$ and $l_3 - l_1 \to \infty$, then $p, q \to 1$. Also, let $l_3 - l_1 < \infty$ and $l_2 \to \infty$, then $p, q \to 0$.

(*b*) Let $0 < \varepsilon < l_2 < \infty$, for some $0 < \varepsilon$ and $l_3 - l_1 \to 0$, then $p, q \to 0$. Also, let $0 < \varepsilon < l_3 - l_1 < \infty$, and $l_2 \to 0$, then $p, q \to 1$.

Remark 1. For special case, let $l_3 = l_1$. Then, $K = \infty$. Then player 1 suppose that player 2 never contribute. As well as, let $l_3 - l_1 = l_2$. Then

$$K = x_1 \text{ and } p = 0.5.$$

Following Carlsson and Van Damme (2003), it can be shown that

$$s_x = \begin{cases} s_1 & x < 0.5, \\ s_2 & x \geq 0.5, \end{cases}$$

is a Nash equilibrium for both players.

Remark 2. Above results are derived assuming independence between $\varepsilon_i, i = 1,2$. Now, suppose that there is a correlation ρ between them. It is easily seen that

$$q_\rho = \Phi\left(\frac{K - x_1}{\sqrt{2(1+\rho)}\sigma}\right) = \frac{l_3 - l_1}{l_3 - l_1 + l_2}.$$

Notice that when $\rho \to -1$, if $K - x_1 > 0$, then $q_\rho = 1$ and if $K - x_1 < 0$, then $q_\rho = 0$. Also, notice that

$$\frac{\Phi^{-1}(q_\rho)}{\Phi^{-1}(q)} = \frac{1}{\sqrt{1+\rho}}.$$

Notice that as $\rho \to 0$, then q_ρ and q are similar. As $\rho \to 1$, then

$$\frac{\Phi^{-1}(q_\rho)}{\Phi^{-1}(q)} \to \frac{1}{\sqrt{2}} < 1,$$

thus

$$\Phi^{-1}(q_\rho) < \Phi^{-1}(q)$$

and since Φ is increasing function, hence, $q_\rho < q$. That is, if there is a strong positive correlation between $\varepsilon_i, i = 1,2$ and if player 1 mistakes and uses q instead of q_ρ, he/she overestimates the probability of contribution assigned to player 2.

Remark 3. As it is mentioned, there are two equilibrium states (s_1, s_1) with probability

$$pr_1 = pq = (\frac{l_3-l_1}{l_3-l_1+l_2})^2$$

and (s_2, s_2) with probability of

$$pr_2 = (1-p)(1-q) = (\frac{l_2}{l_3-l_1+l_2})^2.$$

Notice that

$$\frac{pr_1}{pr_2} = (\frac{l_3-l_1}{l_2})^2.$$

As $l_3 - l_1$ is large or l_2 is small, then (s_1, s_1) is more probable than (s_2, s_2). Conversely, as $l_3 - l_1$ is small or l_2 is large, then(s_2, s_2) is more probable.

In this section, Nash equilibriums for players 1 and 2 are given and their sensitivities are studied. Also, another formulation of the global game in the case of prisoner's dilemma is given.

*Nash equilibrium***.** Here, Nash equilibriums are derived. Suppose that K_2 is the threshold is considered for switching strategy of player 2 by player 1 and K_1 is defined, analogously. Then,

$$\Phi\left(\frac{K_2-K_1}{\sqrt{2}\sigma}\right) = \Phi\left(\frac{K_1-K_2}{\sqrt{2}\sigma}\right).$$

It is seen that

$$K_1 = K_2 \text{ and } p = q = 0.5.$$

The necessary and sufficient condition for $p = q = 0.5$ is

$$l_3 - l_1 = l_2.$$

Sensitivity analysis. Here, the sensitivity of Nash equilibriums with respect to changes of precision of public signals, i.e. σ is studied. To this end, let $\sigma \to 0$, when $x_1 > k_2$, $p, q \to 0$ and (s_2, s_2) is more probable than (s_1, s_1) and if $x_1 < k_2$, then (s_1, s_1) is more probable. Also, suppose that $\sigma \to \infty$, then both equilibriums (s_1, s_1) and (s_2, s_2) have assigned probability 0.25. As well as, the sensitivities of equilibriums to change of correlation ρ is studied. To this end, notice that when $\rho \to -1$, if $K - x_1 > 0$, then $q_\rho = 1$. In this case, (s_1, s_1) is more probable and conversely, if $K - x_1 < 0$, then $q_\rho = 0$, then (s_2, s_2) is more probable than (s_1, s_1).

Other formulation. Here, assume that $l_i, i = 1,2,3$ are unknown and

$$\frac{l_3 - l_1}{l_2} = \pi\beta$$

for some known π. Then,

$$p = q = \frac{\pi\beta}{\pi\beta+1}.$$

However, since β is unknown, it is replaced with x. Thus,

$$\Phi\left(\frac{K-x}{\sqrt{2}\sigma}\right) - \frac{\pi\beta}{\pi\beta+1} = 0.$$

Indeed, thresholds for both players 1 and 2 are satisfied in the following equations

$$\begin{cases} \Phi\left(\frac{K_1-K_2}{\sqrt{2}\sigma}\right)-\frac{\pi K_2}{\pi K_2+1}=0, \\ \Phi\left(\frac{K_2-K_1}{\sqrt{2}\sigma}\right)-\frac{\pi K_1}{\pi K_1+1}=0. \end{cases}$$

Following Carlsson and Van Damme (2003), it is seen that unique surviving strategies by iterated deletion of strictly interim-dominated strategies are switching strategies defined by thresholds

$$K_1=K_2=\frac{1}{\pi}.$$

3 Selling currency. Here, a game is considered between two sellers (players) of a foreign currency (say, dollar). Each seller wants to sell dollar and to give Japanese yen. However, suppose that it is expected yen is devaluating and currency attack is probable. Thus, players decide to sell or not dollar. Speculation attack, or currency attack, happens when a central bank of a country (here, Japan) pegs its currency to a fixed–exchange rate regime. In these cases, speculators sells currency short and the currency will float with a shadow exchange rate. Suppose that, at the current time, the exchange rate USD/JPY is e^*, however, when attack is done, it will float to shadow rate δ_β ($\delta_\beta > e^*$) where β is a fundamental economic state, say the amount of reserved dollar by Japan central bank. For more details about currency attack, see Morris and Shin (2007).

3.1 Game description. Here, the prisoner's dilemma framework under the global game is applied to solve the selling currency game. Considering above mentioned circumstances, two sellers will play as follows: If both players sell their foreign currencies (their dollars), they receive the exchange rate e^*. If one of them sells the dollar now and other does not sell and waits until the speculation attack happens, the first receives e^* and the second seller will be paid exchange rate $\delta_\beta - a$. There is a reason for supposing that the exchange rate is less than δ_β. Because, dollar supply of the first player decreases the exchange rate δ_β. Finally, if both sellers wait until attack event happens and after it sell their foreign currencies, they will be paid exchange rate δ_β. Indeed, if both sellers believe that speculation attack will happen soon and wait after the attack to sell their dollars (i.e., if they corporate) they will receive more amounts of Japanese yen. This is why, this game is a member of prisoner's dilemma games family. The following bi-matrix summaries the selling currency game:

	Before	After
Before	e^*, e^*	$e^*, \delta_\beta - a$
After	$\delta_\beta - a, e^*$	$\delta_\beta, \delta_\beta$.

Here, "Before" and "After" means selling dollar before and after the attack, respectively. When, it is assumed that

$$\delta_\beta - a > e^*,$$

then there is a unique dominant strategy (After, After). Assuming

$$\delta_\beta - a < e^*,$$

then there are two pure Nash equilibriums (Before, Before) and (After, After) for this game.

As follows, the mixture Nash equilibrium is studied. Let q be the probability (assigned probability to player 2 by player 1) of selling dollar before speculation attack (at the current time) and p is defined analogously. It is easy to show that the mixture Nash equilibriums occur at

$$p=q=\frac{\delta_\beta-e^*}{a}\in(0,1).$$

However, because of uncertainty in parameter β, each seller (player$_i$,i=1,2) receives a signal $\delta_i, i=1,2$ about δ_β, that is

$$\delta_i = \delta_\beta + \varepsilon_i, i = 1,2,$$

whereas previous sections, ε_i's are independent with common distribution $N(0,\sigma^2)$. As soon as player 1 observes signal δ_1, following section 1, then δ_2 has $N(\delta_1, 2\sigma^2)$. Next, consider the following switching strategies s_δ for both sellers

$$s_\delta = \begin{cases} \text{Before} & \delta \leq K, \\ \text{After} & \delta > K. \end{cases}$$

Thus,

$$q = P(\delta_2 \leq K_2) = \Phi\left(\frac{K_2-\delta_1}{\sqrt{2}\sigma}\right).$$

Also,

$$q = \frac{\delta_\beta - e^*}{a}.$$

Since δ_β is unknown, it is substituted by its expectation, i.e., δ_1. Thus,

$$\Phi\left(\frac{K_2-\delta_1}{\sqrt{2}\sigma}\right) - \frac{\delta_1 - e^*}{a} = 0.$$

It is easy to see that, thresholds for both players 1 and 2 are satisfied in the following equations

$$\begin{cases} \Phi\left(\frac{K_2-K_1}{\sqrt{2}\sigma}\right) - \frac{K_1 - e^*}{a} = 0, \\ \Phi\left(\frac{K_1-K_2}{\sqrt{2}\sigma}\right) - \frac{K_2 - e^*}{a} = 0. \end{cases}$$

One can see that

$$K_1 = K_2 = e^* + 0.5a.$$

Thus, following Carlsson and Van Damme (2003), it is seen that unique Nash strategy strategies are given by

$$s_{\delta_i} = \begin{cases} \text{Before} & \delta_i \leq e^* + 0.5a, \\ \text{After} & \delta_i > e^* + 0.5a. \end{cases}$$

An interesting feature of this answer is that it is independent of σ. Here,

$$q = \Phi\left(\frac{e^*+0.5a-\delta_1}{\sqrt{2}\sigma}\right).$$

As $e^*, a \to \infty$, then $q \to 1$, as $\delta_1 \to \infty$, then $q \to 0$. Also, as $\sigma \to 0$, then $q \to 1$.

3.2 Simulation. Here, Nash equilibriums are revisited using simulation example. For a hypothetical currency, let $e^* = 0.15, \delta_1 = 0.2$ and $\delta_2 = 0.25$. Thus,

$$s_{\delta_1} = \begin{cases} \text{Before} & a \geq 0.1, \\ \text{After} & a < 0.1, \end{cases}$$

and

$$s_{\delta_2} = \begin{cases} \text{Before} & a \geq 0.2, \\ \text{After} & a < 0.2. \end{cases}$$

Suppose that for economical uncertainty, parameter a is random and comes from uniform $U(0,1)$ distribution. The following table gives the Monte Carlo estimate (with 1000 repetitions) of probability of each cell in bi-matrix payoff function of selling currency example

	Before	After
Before	0.792	0.105
After	0	0103.

That is, both sellers probably will trade before attack. The following figure gives the rolling estimates of probabilities p_{aa}, p_{ab}, p_{ba} and p_{bb} over a window with length 10. For example, p_{aa} is the probability of state (after, after).

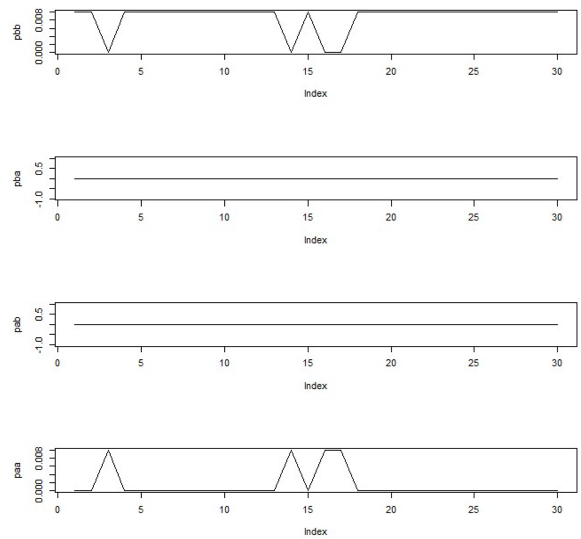

Figure 1: Time series of probabilities p_{aa}, p_{ab}, p_{ba} and p_{bb}

4 Conclusions. In a prisoner's dilemma setting, the switching strategy of a global game is identified by farness (closeness) of payoffs of different cells of payoff of strictly dominated strategy of prisoner's dilemma game. As an application, in the sell or hold then sell game framework, in the presence of high possibility of currency crisis, it is seen that the thresholds of switching strategies of both players are linear function of current (before crisis) of exchange rate.

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