Journal of Algorithms and Computation

journal homepage: http://jac.ut.ac.ir

Group $\{1, -1, i, -i\}$ Cordial Labeling of sum of C_n and K_m for some m

M.K.Karthik Chidambaram*[1], S.Athisayanathan†[2] and R.Ponraj‡[3]

[1]Department of Mathematics, St.Xavier's College ,Palayamkottai 627 002, Tamil Nadu, India.
[2]Department of Mathematics, St.Xavier's College ,Palayamkottai 627 002, Tamil Nadu, India.
[3]Department of Mathematics, Sri Paramakalyani College, Alwarkurichi–627 412, India.

ABSTRACT

Let G be a (p,q) graph and A be a group. We denote the order of an element $a \in A$ by $o(a)$. Let $f : V(G) \to A$ be a function. For each edge uv assign the label 1 if $(o(f(u)), o(f(v))) = 1$or 0 otherwise. f is called a group A Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labelled with an element x and number of edges labelled with $n(n = 0, 1)$. A graph which admits a group A Cordial labeling is called a group A Cordial graph. In this paper we define group $\{1, -1, i, -i\}$ Cordial graphs and characterize the graphs $C_n + K_m (2 \leq m \leq 5)$ that are group $\{1, -1, i, -i\}$ Cordial.

ARTICLE INFO

Article history:
Received 08, December 2016
Received in revised form 12, September 2017
Accepted 03, November 2017
Available online 01, December 2017

Keyword: Cordial labeling, group A Cordial labeling, group $\{1, -1, i, -i\}$ Cordial labeling.

AMS subject Classification: 05C78.

*Corresponding author: M.K.Karthik Chidambaram. Email: karthikmat5 @gmail.com
†athisxc @gmail.com
‡ponrajmath@gmail.com

1 Introduction

Graphs considered here are finite, undirected and simple. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$. Cahit [3] introduced the concept of Cordial labeling. Motivated by this, we defined group A cordial labeling and investigated some of its properties. We also defined group $\{1, -1, i, -i\}$ cordial labeling and discussed that labeling for some standard graphs [1] .In this paper we characterize $C_n + K_2, C_n + K_3, C_n + K_4$ and $C_n + K_5$ that are group $\{1, -1, i, -i\}$ Cordial. Terms not defined here are used in the sense of Harary[5] and Gallian [4].

The greatest common divisor of two integers m and n is denoted by (m, n) and m and n are said to be *relatively prime* if $(m, n) = 1$. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x.

Given two graphs G and H, $G + H$ is the graph with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{uv/u \in V(G), v \in V(H)\}$. We need the following theorem.

Theorem 1.1 [1]
The Complete graph K_n is group $\{1, -1, i, -i\}$ Cordial iff $n \in \{1, 2, 3, 4, 7, 14, 21\}$.

Theorem 1.2 [2]
The Wheel W_n is group $\{1, -1, i, -i\}$ Cordial iff $3 \leq n \leq 6$.

2 Group $\{1, -1, i, -i\}$ Cordial labeling of sum of C_n and K_m

Definition 1. Let G be a (p,q)graph and consider the group $A = \{1, -1, i, -i\}$ with multiplication. Let $f : V(G) \to A$ be a funtion. For each edge uv assign the label 1 if $(o(f(u)), o(f(v))) = 1$or 0 otherwise. f is called a group $\{1, -1, i, -i\}$ Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labelled with an element x and number of edges labelled with $n(n = 0, 1)$. A graph which admits a group $\{1, -1, i, -i\}$ Cordial labeling is called a group $\{1, -1, i, -i\}$ Cordial graph.

Example 2. A simple example of a group $\{1, -1, i, -i\}$ Cordial graph is given in Fig. 2.1.

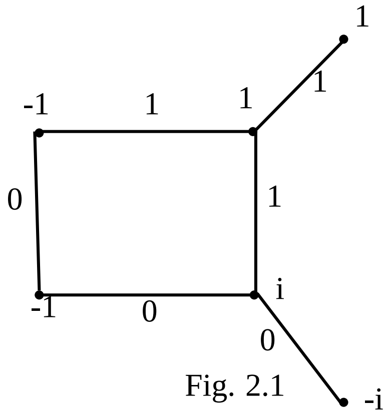

Fig. 2.1

We now investigate the group $\{1,-1,i,-i\}$ Cordial labeling of $C_n + K_m$ for $1 \leq m \leq 5$. C_n+K_1 is the Wheel and theorem 1.2 characterizes the Wheels that are group $\{1,-1,i,-i\}$ cordial.

Theorem 3. *$C_n + K_2$ is group $\{1,-1,i,-i\}$ cordial iff $n \neq 3, 9$.*

Proof. Let the vertices of C_n be labelled as $u_1, u_2, ..., u_n$ and let the vertices of K_2 be labelled as v_1, v_2 . Number of vertices of $C_n + K_2$ is $n+2$ and number of edges is $3n+1$. If n=3 , $C_3 + K_2 \approx K_5$ and by Theorem 1.1, K_5 is not group $\{1,-1,i,-i\}$ Cordial. If n=9 , $C_9 + K_2$ has 11 vertices and 28 edges. There is no choice of 2 or 3 vertices so that 14 edges get label 1. So, $C_9 + K_2$ is not group $\{1,-1,i,-i\}$ Cordial. Thus, $n \neq 3, 9$. Conversely, suppose that $n \neq 3, 9$.

Case(1): $n + 2 \equiv 0(mod\ 4)$.

Let $n + 2 = 4k(k \in \mathbb{Z}, k \geq 2)$. Now each vertex label should appear k times. As number of edges is $12k - 5$, one edge label appears $6k - 3$ times and another $6k - 2$ times. Label the vertices v_1,u_1,u_2,....,u_{k-1} by 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and k of them get label $-i$.

Case(2): $n + 2 \equiv 1(mod\ 4)$.

Let $n + 2 = 4k + 1(k \in \mathbb{Z}, k \geq 2)$. Now one vertex label should appear $k + 1$ times and each of the other three labels should appear k times. Number of edges $= 3n + 1 = 3(4k - 1) + 1 = 12k - 2$ and so each edge label appears $6k - 1$ times . Label the vertices v_1,u_1,u_2,....,u_{k-1} by 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and $k + 1$ of them get label $-i$.

Case(3): $n + 2 \equiv 2(mod\ 4)$.

Let $n + 2 = 4k + 2(k \in \mathbb{Z}, k \geq 1)$. When k=1, a group $\{1,-1,i,-i\}$ Cordial labeling of $C_4 + K_2$ is given in Table 1. Suppose $k \geq 2$. Now 2 vertex labels appear $k + 1$ times and 2 vertex labels appear k times. Number of edges $= 12k + 1$. So one edge label appears $6k$ times and another $6k+1$ times ($k \geq 2$). Label the vertices $v_1, u_1, u_2, ..., u_{k-1}$ by 1. Label the remaining vertices arbitrarily so that k vertices get label 1, $k + 1$ vertices get label i and $k + 1$ vertices get label $-i$.

Case (4): $n + 2 \equiv 3(mod\ 4)$.

Let $n + 2 = 4k + 3$. If $k = 1$, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_5 + K_2$ is given in Table 1. $k = 2$, is impossible by assumption. Suppose $k \geq 3$. In this case, 3

n	v_1	v_2	u_1	u_2	u_3	u_4	u_5
4	i	$-i$	1	1	-1	-1	
5	i	$-i$	1	-1	1	-1	i

Table 1

vertex labels appear $k + 1$ times and 1 vertex label appears k times. Number of edges $= 3(4k + 1) + 1 = 12k + 4$ and so each edge label should appear $6k + 2$ times. Label the vertices v_1,u_1,u_3,u_4,.....,u_k($k \geq 3$) with label 1. Label the other vertices arbitrarily so that $k+1$ vertices get label -1, $k+1$ vertices get label i and $k+1$ vertices get label $-i$. That $C_n + K_2$ is group $\{1, -1, i, -i\}$ Cordial for$n \neq 3, 9$ follows from Table 2. □

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(1)$	$e_f(0)$
$n + 2 \equiv 0(mod\ 4)$	k	k	k	k	$6k - 2$	$6k - 3$
$n + 2 \equiv 1(mod\ 4)$	k	k	k	$k + 1$	$6k - 1$	$6k - 1$
$n + 2 \equiv 2(mod\ 4)$	k	k	$k + 1$	$k + 1$	$6k$	$6k + 1$
$n + 2 \equiv 3(mod\ 4)$	k	$k + 1$	$k + 1$	$k + 1$	$6k + 2$	$6k + 2$

Table 2

.

Theorem 4. *$C_n + K_3$ is group $\{1, -1, i, -i\}$ Cordial iff $n \neq 3$.*

Proof. Let the vertices of C_n be labelled as $u_1, u_2, ..., u_n$ and let the vertices of K_3 be labelled as v_1, v_2, v_3 . Number of vertices of $C_n + K_3$ is $n + 3$ and number of edges is $4n + 3$.

If n = 3, $C_3 + K_3 \approx K_6$ which is not group $\{1, -1, i, -i\}$ Cordial by Theorem 1.1. Conversely , assume $n \neq 3$. We need to prove that $C_n + K_3$ is group $\{1, -1, i, -i\}$ Cordial.

Case(1): $n + 3 \equiv 0(mod\ 4)$.

Let $n + 3 = 4k(k \in \mathbb{Z}, k \geq 2)$. Now each vertex label should appear k times. Number of edges $= 4(4k - 3) + 3 = 16k - 9$ and so one edge label appears $8k - 4$ times and another $8k - 5$ times. Label the vertices v_1,u_1,u_3,u_5,....,u_{2k-3} by 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = n + 2 + (k - 1)4 = 8k - 5$.

Case(2): $n + 3 \equiv 1(mod\ 4)$.

Let $n + 3 = 4k + 1(k \in \mathbb{Z}, k \geq 2)$. If $k = 2$, a group $\{1, -1, i, -i\}$ Cordial labeling of C_6+K_3 is given in Table 3. Suppose $k \geq 3$. Now one vertex label should appear $k+1$ times and each of the other three labels should appear k times. Number of edges is $16k - 5$. Label the vertices v_1,u_1,u_3,....,u_{2k-5}, u_{2k-4}, u_{2k-3}($k \geq 3$) by 1. Label the remaining vertices arbitrarily so that k of them get label -1, k of them get label i and k of them

get label $-i$. Number of edges with label $1 = n + 2 + (k-2)4 + 2 \times 3 = 8k - 2$.
Case(3): $n + 3 \equiv 2(mod\ 4)$.
Let $n+3 = 4k+2 (k \in \mathbb{Z}, k \geq 2)$. Now 2 vertex labels appear $k+1$ times and 2 vertex labels appear k times. Number of edges $= 16k - 1$. So one edge label appears $8k$ times and another $8k-1$ times . Label the vertices v_1, u_1,u_3,....,u_{2k-5}, u_{2k-3}, u_{2k-2} by 1. Label the remaining vertices arbitrarily so that $k+1$ vertices get label -1, k vertices get label i and k vertices get label $-i$. Number of edges with label $1 = n + 2 + (2k-2)2 + 3 = 8k$.
Case (4): $n + 3 \equiv 3(mod\ 4)$.
Let $n+3 = 4k+3 (k \geq 1)$. If $k = 1$, $n = 4$. A group $\{1, -1, i, -i\}$ Cordial labeling of $C_4 + K_3$ is given in Table 3. Suppose $k \geq 2$. In this case, 3 vertex labels appear $k+1$ times and 1 vertex label appears k times. Label the vertices v_1,u_1,u_3,u_5,.....,u_{2k-1} with label 1. Label the other vertices arbitrarily so that $k+1$ vertices get label -1, $k+1$ vertices get label i and k vertices get label $-i$. Number of edges with label $1 = 8k + 2$.
That $C_n + K_2$ is group $\{1, -1, i, -i\}$ Cordial for $n \neq 3$ follows from Table 4.

n	v_1	v_2	v_3	u_1	u_2	u_3	u_4	u_5	u_6
4	-1	-1	i	1	i	1	$-i$		
6	-1	-1	i	1	i	1	1	$-i$	$-i$

Table 3

Nature of n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(1)$	$e_f(0)$
$n+3 \equiv 0(mod\ 4)$	k	k	k	k	$8k-4$	$8k-5$
$n+3 \equiv 1(mod\ 4)$	$k+1$	k	k	k	$8k-3$	$8k-2$
$n+3 \equiv 2(mod\ 4)$	$k+1$	$k+1$	k	k	$8k-1$	$8k$
$n+3 \equiv 3(mod\ 4)$	$k+1$	$k+1$	$k+1$	k	$8k+1$	$8k+2$

Table 4

□

Theorem 5. *$C_n + K_4$ is group $\{1, -1, i, -i\}$ Cordial iff* $n \in \{3, 4, 5, 6, 7, 9, 10, 11, 13, 17\}$.

Proof. Let the vertices of C_n be labelled as $u_1, u_2, ..., u_n$ and let the vertices of K_4 be labelled as v_1, v_2, v_3, v_4. Number of vertices of $C_n + K_4$ is $n + 4$.
Number of edges is $5n + 6$.
Case(1): $n + 4 \equiv 0(mod\ 4)$.
Let $n + 4 = 4k (k \geq 2)$. If 3 v_i's are given label 1, we get $(n+3) + (n+2) + (n+1) = 3n + 6 = 12k - 6$ edges with label 1. But we need only $10k - 7$ edges with label 1. So at most 2 v_i's are given label 1.
Subcase(i): 2 v_i's are given label 1.
If k=2, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_4 + K_4$ is given in Table 5.
Suppose $k \geq 3$. Minimum number of edges that can get label 1 using k vertices is $(8k -$

$3) + 4 + (k - 3)3 = 11k - 8$. So , the necessary condition is , $10k - 7 \geq 11k - 8$ and so $k \leq 1$ which is a contradiction.

Subcase(ii): One v_i is given label 1.

Maximum number of edges that can get label 1 now is $(4k - 1) + (k - 1)5 = 9k - 6$. To get a group $\{1, -1, i, -i\}$ Cordial labeling we need to have $9k - 6 \geq 10k - 7$ i.e. $k \leq 1$, which is impossible.

Subcase(iii): No v_i is given label 1.

We need to have $k.6 \geq 10k - 7 => 4k \leq 7$ which is a contradiction. Thus in case 1 , we get $n = 4$.

Case(2): $n + 4 \equiv 1(mod\ 4)$.

Let $n + 4 = 4k + 1(k \geq 2)$. If 3 v_i's are given label 1, we get $(n + 3) + (n + 2) + (n + 1) = 3n + 6 = 12k - 3$ edges with label 1. But we need only at most $10k - 4$ edges with label 1. So at most 2 v_i's are given label 1.

Subcase(i): 2 v_i's are given label 1.

If $k = 2$, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_5 + K_5$ is given in Table 5.

Suppose $k \geq 3$. Minimum number of edges that can get label 1 using $k + 1$ vertices is $(8k - 1) + 4 + (k - 2)3 = 11k - 3$. So the necessary condition is, $10k - 4 \geq 11k - 3$ or $10k - 5 \geq 11k - 3$ i.e. $k \leq -1$ or $k \leq -2$, both not possible. Minimum number of edges that can get label 1 using k vertices is, $(8k - 1) + 4 + (k - 3)3 = 11k - 1 + 4 - 9 = 11k - 6$. So the necessary condition is , $10k - 4 \geq 11k - 6$ or $10k - 5 \geq 11k - 6$ i.e. $k \leq 2$ or $k \leq 1$.

Subcase(ii): One v_i is given label 1.

Now , maximum number of edges that can get label 1 using k+1 vertices is, $(n+3)+k.5 = 4k + 5k = 9k$. So the necessary condition to get a group $\{1, -1, i, -i\}$ Cordial labeling is $9k \geq 10k - 4$ or $9k \geq 10k - 5$ i.e. $k \leq 5$. If $3 \leq k \leq 5$, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_k + K_5$ is given in Tables 5 and 6.

n	v_1	v_2	v_3	v_4	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
4	1	1	-1	-1	i	i	$-i$	$-i$						
5	1	1	-1	-1	i	i	i	$-i$	$-i$					
9	1	-1	-1	-1	1	i	1	1	i	i	$-i$	$-i$	$-i$	
13	1	-1	-1	-1	1	-1	1	i	1	i	1	i	i	$-i$
17	1	-1	-1	-1	1	-1	1	-1	1	i	1	i	1	i

Table 5

n	u_{11}	u_{12}	u_{13}	u_{14}	u_{15}	u_{16}	u_{17}
13	$-i$	$-i$	$-i$				
17	i	i	$-i$	$-i$	$-i$	$-i$	$-i$

Table 6

Subcase(iii): No v_i is given label 1.
We need to have $6(k+1) \geq 10k-5 => 4k \leq 11 => k \leq \frac{11}{4}$. Thus in Case 2,we have $n \in \{5, 9, 13, 17\}$.

Case(3): $n+4 \equiv 2(mod\ 4)$.
Let $n+4 = 4k+2(k \geq 2)$. If 3 v_i's are given label 1, we get $3n+6 = 12k$ edges with label 1. But we need only at most $10k-2$ edges with label 1. So at most 2 v_i's are given label 1. **Subcase(i):** 2 v_i's are given label 1.
Thus at least $2n+5 = 8k+1$ edges will have label 1. So $k \leq \frac{3}{2}$ which is impossible.
Subcase(ii): One v_i is given label 1.
Maximum number of edges that can get label 1 using $k+1$ vertices is $(n+3)+k.5 = (4k-2+3)+5k = 9k+1$. So the necessary condition to get a group $\{1, -1, i, -i\}$ Cordial labeling is $9k+1 \geq 10k-2 => k \leq 3$. If $k = 2, n = 6$ and if $k = 3$, $n = 10$. A group $\{1, -1, i, -i\}$ Cordial labeling of $C_6 + K_4$ and $C_{10} + K_4$ are given in Table 7.

n	v_1	v_2	v_3	v_4	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}
6	1	-1	-1	-1	1	1	i	i	$-i$	$-i$				
10	1	-1	-1	-1	1	-1	1	i	1	i	i	$-i$	$-i$	$-i$

Table 7

Subcase(iii): No v_i is given label 1.
We need to have $6(k+1) \geq 10k-2 => 4k \leq 8 => k \leq 2$. Thus in Case 3 , we have $n \in \{6, 10\}$.
Case(4): $n+4 \equiv 3(mod\ 4)$.
Let $n+4 = 4k+3(k \geq 1)$. Number of edges $= 5(4k-1)+6 = 20k+1$. If 3 v_i's are given label 1, we get $3n+6 = 12k+3$ edges with label 1. But we need only at most $10k+1$ edges with label 1. So at most 2 v_i's are given label 1. **Subcase(i):** 2 v_i's are given label 1.
Minimum number of edges that can get label 1 using k vertices is $(n+3)+(n+2)+4+(k-3)3 = 11k-2$ and minimum number of edges that can get label 1 using $k+1$ vertices is $11k+1$. Thus $10k+1 \geq 11k-2 => k \leq 3$ or $10k+1 \geq 11k+1 => k \leq 0$, both impossible. If $k = 1$, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_3 + K_4$ is given in Table 8.
If $k = 2$, and if v_1 and v_2 are labelled with 1 then 19 edges get label 1. Thus , there is no choice of 2 or 3 vertices so that 20 or 21 edges get label 1.
If k = 3, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_{11} + K_4$ is given in Table 8.
Subcase(ii): One v_i is given label 1.
Maximum number of edges that can get label 1 using k vertices is $(n+3)+(k-1)5 = (4k-1+3)+5k-5 = 9k-3$. Maximum number of edges that can get label 1 using $k+1$ vertices is $(n+3)+k.5 = (4k-1)+3+5k = 9k+2$. Thus $9k+2 \geq 10k+1$ or $9k+2 \geq 10k$ so that $k \leq 1$ or $k \leq 2$. Also $9k-3 \geq 10k+1 => k \leq -4$ which is a contradiction. When k = 2, a group $\{1, -1, i, -i\}$ Cordial labeling of $C_7 + K_4$ is given in table 8.

Subcase(iii): No v_i is given label 1.

n	v_1	v_2	v_3	v_4	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}
3	1	1	-1	-1	i	i	$-i$								
7	1	-1	-1	-1	1	i	1	i	i	$-i$	$-i$				
11	1	1	-1	-1	1	-1	-1	i	i	i	i	$-i$	$-i$	$-i$	$-i$

Table 8

We need to have $6(k+1) \geq 10k+1$ or $6(k+1) \geq 10k$ so that $4k \leq 5$ or $4k \leq 6$. Otherwise we have, $6k \geq 10k+1$ or $6k \geq 10k$, both impossible. Thus in this Case , $n \in \{3, 7, 11\}$. The labelings given for $n \in \{3, 4, 5, 6, 7, 9, 10, 11, 13, 17\}$ are group $\{1, -1, i, -i\}$ Cordial is clear from Table 9.

n	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(0)$	$e_f(1)$
3	2	2	2	1	10	11
4	2	2	2	2	13	13
5	2	2	3	2	16	15
6	3	3	2	2	18	18
7	3	3	3	2	21	20
9	4	3	3	3	25	26
10	4	4	3	3	28	28
11	3	4	4	4	30	31
13	5	4	4	4	35	36
17	6	5	5	5	46	45

Table 9

□

Theorem 6. *$C_n + K_5$ is group $\{1, -1, i, -i\}$ Cordial iff n satisfies one of the following:*
(i) $n+5 \equiv 0(mod\ 4)$ where $n \geq 7$.
(ii) $n+5 \equiv 1(mod\ 4)$ where $n \geq 8$.
(iii) $n+5 \equiv 2(mod\ 4)$ where $n \geq 17$.
(iv) $n+5 \equiv 3(mod\ 4)$ where $n \geq 22$.

Proof. Let the vertices of C_n be labelled as $u_1, u_2, ..., u_n$ and let the vertices of K_5 be labelled as v_1, v_2, v_3, v_4, v_5 . Number of vertices of $C_n + K_5$ is $n+5$ and number of edges is $6n+10$.
Case(1): $n+5 \equiv 0(mod\ 4)$.
Let $n+5 = 4k(k \in \mathbb{Z}, k \geq 2)$. Now each vertex label should appear k times. Number of edges $= 6n+10 = 6(4k-5)+10 = 24k-20$ and so each edge label appears $12k-10$ times.
If $k = 2$, there is no choice of 2 vertices so that 14 edges get label 1. Suppose $k \geq 3$. Label the k vertices v_1, v_2, u_1,u_2,u_3,....,u_{k-2} by 1. Label the remaining vertices arbitrarily

so that k of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = (n+4)+(n+3)+5+(k-3)4 = 12k-10$. That this labeling is a group $\{1,-1,i,-i\}$ Cordial labeling of $C_n + K_5 (n \geq 7)$ is evident from Table 10.

Case(2): $n+5 \equiv 1(mod\ 4)$.

Let $n+5 = 4k+1 (k \in \mathbb{Z}, k \geq 2)$. Number of edges $= 6n+10 = 6(4k-4)+10 = 24k-14$ and so each edge label appears $12k-7$ times.

If $k = 2$, there is no choice of 2 or 3 vertices so that 17 edges get label 1.For $k = 3$, a group $\{1,-1,i,-i\}$ Cordial labeling of C_8+K_5 is given in table 10. Suppose $k \geq 4$. Label the k vertices v_1, v_2, u_1,u_3,u_4,u_5,....,u_{k-1} by 1. Label the remaining vertices arbitrarily so that $k+1$ of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = (n+4)+(n+3)+2.5+(k-4)4 = 12k-7$. That this labeling is a group $\{1,-1,i,-i\}$ Cordial labeling of $C_n + K_5 (n \geq 8)$ is evident from Table 10.

$n+5$	$v_f(1)$	$v_f(-1)$	$v_f(i)$	$v_f(-i)$	$e_f(1)$	$e_f(0)$
$4k(k \geq 3)$	k	k	k	k	$12k-10$	$12k-10$
$4k+1(k \geq 3)$	k	$k+1$	k	k	$12k-7$	$12k-7$
$4k+2(k \geq 5)$	k	k	$k+1$	$k+1$	$12k-4$	$12k-4$
$4k+3(k \geq 6)$	k	$k+1$	$k+1$	$k+1$	$12k-1$	$12k-1$

Table 10

Case(3): $n+5 \equiv 2(mod\ 4)$.

Let $n+5 = 4k+2 (k \in \mathbb{Z}, k \geq 2)$. Number of edges is $24k-8$. So each edge label appears $12k-4$ times. If 3 v_i's are given label 1, we get $(n+4)+(n+3)+(n+2) = 3n+9 = 12k$ edges with label 1. But we need only $12k-4$ edges with label 1. So at most 2 v_i's are given label 1.

Subcase(i): 2 v_i's are given label 1.

Maximum number of edges that can get label 1 using k vertices is $(n+4)+(n+3)+(k-2)5 = 2(4k-3)+7+5k-10 = 13k-9$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $13k-9 \geq 12k-4 => k \geq 5$. Maximum number of edges that can get label 1 using $k+1$ vertices is $(2n+7)+(k-1)5 = 2(4k-3)+7+5k-5 = 13k-4$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $13k-4 \geq 12k-4 => k \geq 0$. For $k \leq 4$, it is easy to observe that there is no group $\{1,-1,i,-i\}$ Cordial labeling. For $k \geq 5$, label the k vertices v_1, v_2, u_1,u_3,u_5,u_6,u_7,....,u_k by 1. Label the remaining vertices arbitrarily so that k of them get label -1, $k+1$ of them get label i and $k+1$ of them get label $-i$. Number of edges with label $1 = (n+4)+(n+3)+15+(k-5)4 = 12k-4$. That this labeling is a group $\{1,-1,i,-i\}$ Cordial labeling of $C_n + K_5 (n \geq 8)$ is evident from Table 10.

Subcase(ii): One v_i is given label 1.

Maximum number of edges that can get label 1 using k vertices is $(n+3)+(k-1)6 = 10k-6$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $10k-6 \geq 12k-4 => k \leq -1$, which is a contradiction. Maximum number of edges that can get label 1 using $k+1$ vertices is $(n+3)+6k = 10k$. So a necessary condition to get a group

$\{1,-1,i,-i\}$ Cordial labeling is $10k \geq 12k-4 => k \leq 2$. If $k=2, n=5$, and a group $\{1,-1,i,-i\}$ Cordial labeling of C_5+K_5 is given in table 11.

Case(4): $n+5 \equiv 3(mod\ 4)$.

Let $n+5 = 4k+3(k \in \mathbb{Z}, k \geq 2)$. Number of edges $= 6n+10 = 6(4k-2)+10 = 24k-2$. So each edge label appears $12k-1$ times. If 3 v_i's are given label 1, we get $(n+4)+(n+3)+(n+2) = 3n+9 = 12k+3$ edges with label 1. But we need only $12k-1$ edges with label 1. So at most 2 v_i's are given label 1.

Subcase(i): 2 v_i's are given label 1.

Maximum number of edges that can get label 1 using k vertices is $(n+4)+(n+3)+(k-2)5 = 2(4k-2)+7+5k-10 = 13k-7$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $13k-7 \geq 12k-1 => k \geq 6$. Maximum number of edges that can get label 1 using $k+1$ vertices is $(2n+7)+(k-1)5 = 2(4k-2)+7+5k-5 = 13k-2$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $13k-2 \geq 12k-1 => k \geq 1$. But for $1 \leq k \leq 5$, we observe that there is no group $\{1,-1,i,-i\}$ Cordial labeling . For $k \geq 6$, label the k vertices v_1, v_2, u_1,u_3,u_5,u_7,u_8,u_9,....,u_{k+1} by 1. Label the remaining vertices arbitrarily so that $k+1$ of them get label -1, $k+1$ of them get label i and $k+1$ of them get label $-i$. Number of edges with label $1 = 2n+7+20+(k-6)4 = 12k-1$. That this labeling is a group $\{1,-1,i,-i\}$ Cordial labeling of $C_n+K_5(n \geq 8)$ is evident from Table 10.

Subcase(ii): One v_i is given label 1.

Maximum number of edges that can get label 1 using k vertices is $(n+3)+(k-1)6 = 10k-5$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $10k-5 \geq 12k-1 => k \leq -2$, which is a contradiction. Maximum number of edges that can get label 1 using $k+1$ vertices is $(n+3)+6k = 10k+1$. So a necessary condition to get a group $\{1,-1,i,-i\}$ Cordial labeling is $10k+1 \geq 12k-1 => k \leq 1$, which is a contradiction.

n	v_1	v_2	v_3	v_4	v_5	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
5	-1	-1	-1	i	i	1	$-i$	1	1	$-i$			
8	1	-1	-1	-1	i	1	i	1	1	i	$-i$	$-i$	$-i$

Table 11

□

References

[1] Athisayanathan, S., Ponraj, R. and Karthik Chidambaram, M., K., Group A Cordial labeling of Graphs, Preprint submitted to the Journal of Discrete Mathematical Sciences and Cryptography.

[2] Athisayanathan, S., Ponraj, R. and Karthik Chidambaram, M., K., Group $\{1,-1,i,-i\}$ Cordial Labeling of Some Graphs,Preprint submitted to the Journal of Prime Research in Mathematics.

[3] Cahit, I., Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin. 23(1987) 201-207.

[4] Gallian, J. A, A Dynamic survey of Graph Labeling, The Electronic Journal of Combinatorics Dec 7(2015), No.D56.

[5] Harary, F., Graph Theory, Addison Wesley, Reading Mass, 1972..