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Correlation Coefficients for Hesitant Fuzzy Linguistic Term Sets

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ABSTRACT

Here are many situations in real applications of decision making where we deal with uncertain conditions. Due to the different sources of uncertainty, since its original definition of fuzzy sets in 1965 [45], different generalizations and extensions of fuzzy sets have been introduced: Type-2 fuzzy sets [11, 39], Intuitionistic fuzzy sets [1], fuzzy multi-sets [44] and etc. However, in such cases, it is suitable for experts to provide their preferences or assessments by using linguistic information rather than quantitative values.

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1 Introduction

There are many situations in real applications of decision making where we deal with uncertain conditions. Due to the different sources of uncertainty, since its original definition of fuzzy sets in 1965 [45], different generalizations and extensions of fuzzy sets have been introduced: Type-2 fuzzy sets [11, 39], Intuitionistic fuzzy sets [1], fuzzy multi-sets

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[44] and etc. However, in such cases, it is suitable for experts to provide their preferences or assessments by using linguistic information rather than quantitative values. The complexity of real world decision problems is often induced by the uncertainties of the alternatives. For managing these uncertainties, the use of linguistic terms has provided successful results in many decision making problems. However, the linguistic models usually use one linguistic term to describe such uncertainties and so experts may not reflect exactly what they mean. These situations happen, for instance, when experts hesitate among several values to assess a linguistic variable.

The topic of operations on Hesitant Fuzzy Linguistic Term Sets has been studied by some authors. Below is a brief review of some studies relevant to the present work. Rodríguez et al. [36] introduced the concept of Hesitant Fuzzy Linguistic Term Sets (briefly, $HFLTS$s) to increase the flexibility and richness of linguistic elicitation based on the fuzzy linguistic approach and the use of context-free grammars to support the elicitation of linguistic information obtained by experts in hesitant situations under qualitative settings. They also presented some computational functions for ranking $HFLTS$. Then, they presented a multicriteria linguistic decision-making model in which experts provide their assessments by using linguistic expressions based on comparative terms and applied it to a decision-making problem to show the usefulness of the $HFLTS$ in decision making. In addition, they [37] presented a group decision making model that is capable of dealing with comparative linguistic expressions based on $HFLTS$s. Lee et al. [23] also extended a similar type of Rodríguez et al.'s preference degree and a similarity measure for $HFLTS$s. Beg et al. [2] modified the TOPSIS method for solving decision-making problems under the opinion of finite experts and multiple criteria represented by $HFLTS$s. Based on a proposed distance measure between $HFLTS$s, they compared alternatives using a proposed closeness coefficient. Liao et al. [24] suggested some family of distance and similarity measures for $HFLTS$s and applied them to multi-criteria decision making problems. Liu et al. [26] suggested a representation of the $HFLTS$s by means of a fuzzy envelope for fuzzy multicriteria decision making applications. Liu et al. [27] proposed the linguistic fuzzy preference relations based on the comparative linguistic expressions for $HFLTS$s. They transformed the linguistic fuzzy preference relations into linguistic 2-tuple fuzzy preference relations, and introduced an iterative method to measure and improve the additive consistency of the linguistic 2-tuple fuzzy preference relations. Wei [40] investigated the hesitant fuzzy multiple attribute decision making problem in which the attributes are in different priority levels. He developed some prioritized aggregation operators for aggregating hesitant fuzzy information including hesitant fuzzy prioritized weighted average operator and hesitant fuzzy prioritized weighted geometric operator. Liao et al. [25] extended the classical VIKOR method to accommodate hesitant fuzzy circumstances. They developed the hesitant normalized Manhattan L^p-metric, the hesitant fuzzy group utility measure, the hesitant fuzzy individual regret measure and the hesitant fuzzy compromise measure. Zhu [48] developed a concept of hesitant fuzzy linguistic preference relations as a tool to collect and present the decision makers preferences. He extended some consistency measures for hesitant fuzzy linguistic preference relations to ensure that the decision makers are neither random nor illogical. Wei et al. [41] defined some operations on $HFLTS$s and

proposed some possibility degree formulas for comparing $HFLTS$s. They also proposed two aggregation operators for $HFLTS$s.
An important issue in real life applications is consists in evaluating the relationship between two variables. The measures of association refer to a wide variety of coefficients that measure the statistical strength of the relationship on the variables of interest. These measures of strength can be described in several ways, depending on the analysis. The correlation coefficient is one of the most broadly applied indices in many fields and also an important measure in data analysis and classification, pattern recognition, decision making and so on. It is typically a number between 0 and 1. This measure tells us how closely one object is related to the other, i.e. if there is no relationship between the objects the correlation coefficient is 0 or very low and it increases toward 1 when the strength of the relationship between the the objects increases. As it mentioned above, in real life phenomena, we usually come across many characteristics and attributes which are reported by imprecise information instead of crisp ones. During the last decades, the concept of correlation coefficients has been extended for imprecise data by some authors (see [6, 12, 13, 16, 17, 18, 19, 20, 28, 39, 43] for fuzzy data, [5, 13, 15, 17, 19, 21, 30, 35, 38, 42] for intuitionistic fuzzy sets, and [7, 29] for Hesitant Fuzzy Sets). On the other hand, as it mentioned above, the analysis of relationships between linguistic terms is a vital importance in many applications of decision making. Therefore, for investigating the relationship between this type of imprecise data, it needs to aggregate soft statistical methods and theory of $HFLTS$s.
In this paper, some formulas for correlation coefficients of $HFLTS$s are suggested and discussed. These derived correlation coefficients are also applied to do clustering analysis for $HFLTS$s. An applied example is also presented to illustrate the effectiveness and reasonability of the proposed methods.
The rest of the paper is organized as follows. In Section 2 some preliminary concepts and operations of $HFLTS$s is briefly reviewed. Then some definitions and relevant properties with respect to correlation coefficients for $HFLTS$ will be introduced and discussed. Section 3 applies a clustering algorithm based on $HFLTS$s and a real case study is developed to demonstrate an application of the proposed method for $HFLTS$s. Concluding remarks are finally made in Section 4.

2 Correlation coefficients for $HFLTS$s

In this section, some correlation coefficients for $HFLTS$s are introduced and the main properties of the proposed correlation coefficients are put into investigation. First, we recall briefly some preliminaries of $HFLTS$s that we need in this work.
Throughout this paper, the following definitions and notations are used for extending some utility concepts concerned with $HFLTS$s based on [36].
Definition 1 Assume L consists of finite linguistic terms called a linguistic term set denoted by $L = \{l_1, \ldots, l_n\}$. A $HFLTS$ is an ordered finite subset H of consecutive linguistic terms of L. The set of all such subsets of L is denoted by $\mathbb{H}(L)$.

Example 1 [2] Consider an investment company which wants to invest money in the best option based on some possible alternatives to invest the money and some criteria based on some risk factors. In such cases, to make a optimum decision, the possible evaluations of alternatives is usually reported by decision makers in some linguistic term sets such as:

$$L = \Big\{l_1 : \text{“exteremly poor”}, l_2 : \text{“very poor”}, l_3 : \text{“poor”}, l_4 : \text{“medium”}, l_5 : \text{“goog”}, \\ l_6 : \text{“very good”}, l_7 : \text{“extremely good”}\Big\}.$$

Due to different levels of skills, experience and etc, different evaluations of the alternatives are often made by the experts. These differences of the opinions may be reported by the following $HFLTS$s, for instance:

$$H_1 = \ \big\{l_1\big\}, \qquad H_2 = \ \big\{l_2,\ l_3\big\}, \qquad H_3 = \ \big\{l_2, l_3,\ l_4\big\}.$$

Definition 2 Let $L = \big\{l_1, \ldots, l_n\big\}$ be a linguistic term set. Then,

1) the empty $HFLTS$ is defined as the $HFLTS$, $H = \emptyset$,

2) the intersection between two $HFLTS$s, H_1 and H_2 is defined as the $HFLTS$:

$$H_1 \cap H_2 = \big\{l \in L : l \in H_1 \ and \ l \in H_2\big\}.$$

3) two $HFLTS$s, H_1 and H_2 are said to be equal, i.e. $H_1 = H_2$, if $H_1 \subseteq H_2$ and $H_2 \subseteq H_1$.

For more operations on $HFLTS$s see [36].
In the sequel, we will propose some formulas to evaluate the correlation between $HFLTS$s.
Definition 3 Let L be a given linguistic term set. A function $\rho : \mathbb{H}(L) \times \mathbb{H}(L) \to [0, \infty)$ is called a correlation coefficient if for all H_1, H_2 in $\mathbb{H}(L)$, it satisfies the following conditions:
1) $\rho(H_1, H_2) \in [0, 1]$,
2) $\rho(H_1, H_2) = 1$ if and only if $H_1 = H_2$,
3) $\rho(H_1, H_2) = \rho(H_2, H_1)$,
4) $\rho(H_1, H_2) = 0$ if and only if $H_1 \cap H_2 = \emptyset$.
Theorem 1 Given a linguistic term set $L = \{l_1, \ldots,\ l_n\}$, the function defined for all nonempty $H_1, H_2 \in \mathbb{H}(L)$ by

$$\rho_1(H_1, H_2) = \sum_{i \in \{1,2,\ldots,n\}} \sqrt{a_{H_1}(i) a_{H_2}(i)}, \tag{2.1}$$

is a correlation coefficient, where for all $H \in \mathbb{H}(L)$

$$a_H(i) = \frac{I_H(i)}{|H|},\ I_H(i) = \begin{cases} 1 & \text{if} \quad i \in Ins(H), \\ 0 & \text{if} \quad i \notin Ins(H), \end{cases}$$

$$Ins(H) = \big\{i \in \{1, 2, \ldots, n\} : l_i \in H\big\},$$

and $|H|$ denotes the cardinal number of H.
Proof. To proof the theorem, we need to verify the conditions 1)-4) in Definition 2. let $a_i = a_{H_1}(i)$ and $b_i = b_{H_2}(i)$ for $i = 1, 2, \ldots, n$. Here, we recall that all nonnegative sequences of real numbers $\{a_i\}_{i=1}^n$ and $\{b_i\}_{i=1}^n$ where $\sum_{i=1}^n a_i = \sum_{i=1}^n b_i = 1$ satisfy $0 \leq \sum_{i=1}^n \sqrt{a_i b_i} \leq 1$. The equality holds if and only if $a_i = b_i$ for any $i = 1, 2, \ldots, n$ [4]. So, the conditions 1), 3) and 4) are easily verified. Now, let $H_1, H_2 \in \mathbb{H}(L)$ where $H_1 = H_2$, therefore it is easily seen that $\rho(H_1, H_2) = 1$. In reverse, assume $\rho(H_1, H_2) = 1$, i.e. $a_{H_1}(i) = a_{H_2}(i)$ for all $i \in Ins(L)$, then it follows that $H_1 = H_2$. If not, i.e. $H_1 \neq H_2$, there exist at least one element $i \in Ins(L)$ such that $i \in Ins(H_1)$ but $i \notin Ins(H_2)$. Therefore, it concludes that $\frac{1}{|H_1|} = a_{H_l}(i) \neq a_{H_2}(i) = 0$ which is a contradiction. So the condition 2) is verified and thus the proof is completed.
Theorem 2 Let $L = \{l_1, \ldots, l_n\}$ be a linguistic term set. Then the function defined by

$$\rho_1(H_1, H_2) = \frac{\sum_{i \in \{1,2,\ldots,n\}} k_{H_1}(i) k_{H_2}(i)}{\sqrt{\left(\sum_{i \in \{1,2,\ldots,n\}} (k_{H_1}(i))^2\right)\left(\sum_{i \in \{1,2,\ldots,n\}} (k_{H_2}(i))^2\right)}}, \tag{2.2}$$

where for $H \in \mathbb{H}(L)$,

$$k_H(i) = \begin{cases} 1 & \text{if} \quad i \in Ins(H), \\ 0 & \text{if} \quad i \notin Ins(H), \end{cases}$$

is a correlation coefficient.
Proof. Similar to the previous theorem, based on the properties of Cauchy-Schwarz inequality [4], the proof is easily verified.
Example 2 Let $L = \{l_1, \ldots, l_7\}$ be a linguistic term set and $H_1 = \{l_3, l_4, l_5\}$, $H_2 = \{l_4, l_5\}$ be two $HFLTS$s on $\mathbb{H}(L)$. Then, from Eq. (2.1), we get

$$\rho_1(H_1, H_2) = \sqrt{\frac{0}{2} \times \frac{0}{3}} + \sqrt{\frac{0}{2} \times \frac{0}{3}} + \sqrt{\frac{1}{3} \times \frac{0}{2}} + \sqrt{\frac{1}{2} \times \frac{1}{3}} + \sqrt{\frac{1}{2} \times \frac{1}{3}} + \sqrt{\frac{0}{2} \times \frac{0}{3}} = 0.8165.$$

In addition, from Eq. (2.2), we obtain

$$\rho_2(H_1, H_2) = \frac{4 \times 4 + 5 \times 5}{\sqrt{(3^2 + 4^2 + 5^2) \times (4^2 + 5^2)}} = 0.9055.$$

3 Clustering algorithm for $HFLTS$s

Cluster analysis is a major technique for classifying a set of information into manageable meaningful piles. Cluster analysis, in fact, is a data reduction tool that creates subgroups that are more manageable than individual datum by examining the full complement of inter-relationships between objects. Clustering methods for vague data, however, allow the objects to belong to several clusters simultaneously. As one of the widely-adopted tools in handling imprecise information, it has been applied to many fields of decision making such as pattern recognition [44], data mining [14], information retrieval [31, 33], and other real world problems concerning social, medical, biological, climatic, financial systems [8, 22, 34, 46, 47]. As it is mentioned by Chen et al. [7], under the group decision making situations, however, the evaluation information provided by different experts may have an obvious difference and the fuzzy clustering schemes mentioned above are unable to incorporate the differences in the opinions of different experts. This is the reason that we apply the clustering algorithm introduced by Chen et al. [7] in this paper. In the sequel, based on the correlation coefficient formulas for $HFLTS$s proposed in the previous section, we develop the algorithm proposed by Chen et al. [7] to do clustering under $HFLTS$s as follows:

(Step 1.) Let $\{H_1, H_2, \ldots, H_m\}$ be a set of $HFLTS$s in $L = \{l_1, l_2, \ldots, l_n\}$. Using Eq. (2.1) or Eq. (2.2), the correlation coefficients of the $HFLTS$s are calculated leading to a correlation matrix $C = [\rho_{ij}]_{mm}$, where $\rho_{ij} = \rho(H_i, H_j)$.

(Step 2.) Check whether $C = [\rho_{ij}]_{mm}$ is an equivalent correlation matrix, i.e. check whether it satisfies $C^2 \subseteq C$, where

$$C^2 = [\overline{\rho}_{ij}]_{mm},\ \overline{\rho}_{ij} = \max_k\{\min(\rho_{ik}, \rho_{kj})\},\ i, j = 1, 2, \ldots, m,$$

if it does not hold, the equivalent correlation matrix C^{2k} is constructed as follows:

$$C \to C^2 \to C^4 \to \ldots C^{2^k} \to \ldots,\ \text{until } C^{2^k} = C^{2^{(k+1)}}.$$

(Step 3.) For a confidence level $\alpha \in [0, 1]$, a α-cutting matrix $C_\alpha = [\rho^\alpha_{ij}]_{mm}$ is constructed where

$$\rho^\alpha_{ij} = \begin{cases} 1 & \text{if } \rho_{ij} \geq \alpha, \\ 0 & \text{if } \rho_{ij} < \alpha, \end{cases}$$

in order to classify the $HFLTS$s, H_j $(j = 1, 2, \ldots, m)$. If all elements of the ith line (column) in C_α are the same as the corresponding elements of the jth line (column) in C_α, then the $HFLTS$s H_i and H_j are of the same type. Using this principle, all $HFLTS$s, Hj $(j = 1, 2, \ldots, m)$ can be classified. Here an numerical example is employed to illustrate the possible application of the clustering algorithm based on $HFLTS$s:

Example [41] A practical application of the proposed approaches involves the evaluation of university faculty for tenure and promotion. The criteria used at some universities are teaching ($u1$), research ($u2$), and service ($u3$). Suppose there are five candidates x_i $(i = 1, 2, 3, 4, 5, 6)$ to be evaluated by three experts d_k $(k = 1, 2, 3)$ under these three attributes. Assume that the possible evaluating values of attributes is labeled as a linguistic term set $S = \{l_1 : \text{“}\textit{nothing}\text{”}, l_2 : \text{“very low”}, l_3 : \text{“low”}, l_4 : \text{“medium”}, l_5 : \text{“high”}, l_6 : \text{“very high”}, l_7 : \text{“perfect”}\}$. The experts who make such an evaluation have different backgrounds and levels of knowledge, skills, experience and personality, etc. So, this leads to a difference in the evaluation of the alternatives u_i $(i = 1, 2, 3)$. To clearly reflect the differences of the opinions of different experts, assume that the evaluation information are represented by the $HFLTS$s listed bellow:

$$\begin{array}{llll} H_1 = & \{l_1,\ l_2\}, & H_2 = & \{l_2,\ l_3,\ l_4\}, \\ H_3 = & \{l_3,\ l_4\}, & H_4 = & \{l_3,\ l_4,\ l_5\}, \\ H_5 = & \{l_5\}, & H_6 = & \{l_5,\ l_6,\ l_7\}, \end{array}$$

To better evaluate the evaluation of tenure and promotion H_i $(i = 1, 2, 3, 4, 5, 6)$ for the university faculty, we perform the following clustering algorithm according to the attributes u_1-u_3.

(Step 1.) Calculate the correlation coefficients of the $HFLTS$s H_i $(i = 1, 2, \ldots, 6)$ by using Eq. (2.2). So, the derived correlation matrix is:

$$C = \begin{pmatrix} 1.0000 & 0.4082 & 0 & 0 & 0 & 0 \\ 0.4082 & 1.0000 & 0.8165 & 0.6667 & 0 & 0 \\ 0 & 0.8165 & 1.0000 & 0.6667 & 0 & 0 \\ 0 & 0.6667 & 0.8165 & 1.0000 & 0.5774 & 0.3333 \\ 0 & 0 & 0 & 0.5774 & 1.0000 & 0.5774 \\ 0 & 0 & 0 & 0.3333 & 0.5774 & 1.0000 \end{pmatrix}$$

(Step 2.) Construct the equivalent correlation matrix and calculate:

$$C^2 = \begin{pmatrix} 1.0000 & 0.4082 & 0.4082 & 0.4082 & 0 & 0 \\ 0.4082 & 1.0000 & 0.8165 & 0.6667 & 0.5774 & 0.3333 \\ 0.4082 & 0.8165 & 1.0000 & 0.6667 & 0.5774 & 0.3333 \\ 0.4082 & 0.8165 & 0.8165 & 1.0000 & 0.5774 & 0.5774 \\ 0 & 0.5774 & 0.5774 & 0.5774 & 1.0000 & 0.5774 \\ 0 & 0.3333 & 0.3333 & 0.5774 & 0.5774 & 1.0000 \end{pmatrix}$$

It can be seen that $C^2 \subseteq C$ does not hold. therefore, the correlation matrix C is not an equivalent correlation matrix. So, we further calculate:

$$C^4 = \begin{pmatrix} 1.0000 & 0.4082 & 0.4082 & 0.4082 & 0.4082 & 0.4082 \\ 0.4082 & 1.0000 & 0.8165 & 0.6667 & 0.5774 & 0.5774 \\ 0.4082 & 0.8165 & 1.0000 & 0.6667 & 0.5774 & 0.5774 \\ 0.4082 & 0.8165 & 0.8165 & 1.0000 & 0.5774 & 0.5774 \\ 0.4082 & 0.5774 & 0.5774 & 0.5774 & 1.0000 & 0.5774 \\ 0.4082 & 0.5774 & 0.5774 & 0.5774 & 0.5774 & 1.0000 \end{pmatrix}$$

It is observed that $C^4 \subseteq C^2$ is not an equivalent correlation matrix. Therefore, we further calculate:

$$C^8 = \begin{pmatrix} 1.0000 & 0.4082 & 0.4082 & 0.4082 & 0.4082 & 0.4082 \\ 0.4082 & 1.0000 & 0.8165 & 0.6667 & 0.5774 & 0.5774 \\ 0.4082 & 0.8165 & 1.0000 & 0.6667 & 0.5774 & 0.5774 \\ 0.4082 & 0.8165 & 0.8165 & 1.0000 & 0.5774 & 0.5774 \\ 0.4082 & 0.5774 & 0.5774 & 0.5774 & 1.0000 & 0.5774 \\ 0.4082 & 0.5774 & 0.5774 & 0.5774 & 0.5774 & 1.0000 \end{pmatrix} = C^4.$$

Hence, it is observed that C^8 is an equivalent correlation matrix. (Step 3.) For a confidence level α, to do clustering for $HFLTS$s, by constructing the α-cutting matrix

$C_\alpha = (\rho^{\alpha}_{1,ij})_{mm}$, we get all possible classifications of H_i $(j = 1, 2, \ldots, 6)$ as follows:
(1) If $0 \leq \alpha \leq 0.4082$, then H_i $(j = 1, 2, \ldots, 6)$ are of the same type:

$$\{H_1, H_2, H_3, H_4, H_5, H_6\}.$$

(2) If $0.4082 < \alpha \leq 0.5774$, then H_i $(j = 1, 2, \ldots, 6)$ are classified into two types:

$$\{H_1\},\ \{H_2, H_3, H_4, H_5, H_6\}.$$

(3) If $0.5774 < \alpha \leq 0.8165$, then H_i $(j = 1, 2, \ldots, 6)$ are classified into four types:

$$\{H_1\},\ \{H_2, H_3, H_4\},\ \{H_5\},\ \{H_6\}.$$

(4) If $0.8165 < \alpha \leq 1$, then H_i $(j = 1, 2, \ldots, 6)$ are classified into five types:

$$\{H_1\},\ \{H_2\},\ \{H_3\},\ \{H_4\},\ \{H_5\},\ \{H_6\}.$$

4 Conclusion

The theory of $HFLTS$s is a convenient and flexible tool to reflect the decision maker's preferences in decision making in case where there are situations in which there is hesitancy in providing linguistic assessments. As a tool for further applications of $HFLTS$s in decision making, in this paper, some formulas of correlation coefficients for $HFLTS$s are introduced in which an $HFLTS$ consists of finite linguistic terms. The properties of these correlation coefficient was also investigated and discussed. An approach to clustering analysis under $HFLTS$s is also developed and the assessment of a multicriteria decision-making problem is selected to illustrate the actual application of clustering algorithm under $HFLTS$s. The application clearly indicates the need of evaluations of correlation coefficients based on $HFLTS$s, since such a clustering algorithm can automatically account for the differences of the evaluation linguistic term sets given by different experts. The correlation coefficients under $HFLTS$s is therefore of considerable practicality in many fields of decision making and consequently it constitutes a potentially useful tool to handle those decision issues involving $HFLTS$s.

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