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PD-prime cordial labeling of graphs

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ABSTRACT

Let G be a graph and $f : V(G) \rightarrow \{1, 2, 3, ... |V(G)|\}$ be a bijection. Let $p_{uv} = f(u)f(v)$ and

$$d_{uv} = \begin{cases} \left[\frac{f(u)}{f(v)}\right] & if \;\; f(u) \geq f(v) \\ \\ \left[\frac{f(v)}{f(u)}\right] & if \;\; f(v) \geq f(u) \end{cases}$$

for all edge $uv \in E(G)$. For each edge uv assign the label 1 if $gcd(p_{uv}, d_{uv}) = 1$ or 0 otherwise. f is called PD-prime cordial labeling if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with 0 and 1. A graph with admit a PD-prime cordial labeling is called PD-prime cordial graph.

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1 Introduction

Graphs in this paper are finite, simple and undirected. M.Sundaram, R.Ponraj and S.Somasundaram was introduced the concept of Prime Cordial Labeling of graphs, Gee-Choon Lau, Hong- Heng Chu, Nurulzulaiha Suhadak, Fong-Yeng Foo, Ho-kuen Ng[4] was introduced the SD-prime cordial graph and studied certain graphs for this labeling. Motivated by this, we introduced PD-prime cordial labeling of graphs. In the paper we investigate the PD-prime cordial labeling behaviour of path, bistar, subdivision of star, wheel, subdivision of bistar, fan and double fan.

2 Introduction

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3 PD-prime cordial graph

Definition 3.1. Let G be a graph and $f : V(G) \to \{1, 2, 3, ... |V(G)|\}$ be a bijection. Let $p_{uv} = f(u)f(v)$ and

$$d_{uv} = \begin{cases} \left[\frac{f(u)}{f(v)}\right] & if \quad f(u) \geq f(v) \\ \\ \left[\frac{f(v)}{f(u)}\right] & if \quad f(v) \geq f(u) \end{cases}$$

for all edge $uv \in E(G)$. For each edge uv assign the label 1 if $gcd(p_{uv}, d_{uv}) = 1$ or 0 otherwise. f is called PD-prime cordial labeling if $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ respectively denote the number of edges labelled with 0 and 1. A graph with admit PD-prime cordial labeling is called PD-prime cordial graph.

Remark 3.2. K_6 is SD-prime cordial to refer[4], but it is not PD-prime cordial.

Remark 3.3. K_8 is PD-prime cordial, but it is not SD-prime cordial.

4 Preliminaries

Definition 4.1. The *union* of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Definition 4.2. Let G_1 and G_2 be two graphs with vertex sets V_1 and V_2 and edge sets E_1 and E_2 respectively. Then their *join* $G_1 + G_2$ is the graph whose vertex set is $V_1 \cup V_2$ and edge set is $E_1 \cup E_2 \cup \{uv : u \in V_1 \text{ and } v \in V_2\}$.

Definition 4.3. If $e = uv$ is an edge of G and w is a vertex not in G then e is said to be *subdivided* when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and is denoted by $S(G)$.

Definition 4.4. $K_{1,n}$ is called a *Star*.

Definition 4.5. The *Bistar* $B_{n,n}$ is the graph obtained by joining the two central vertices of $K_{1,n}$ and $K_{1,n}$.

Definition 4.6. The graph $W_n = C_n + K_1$ is called a *wheel*. In a Wheel, a vertex of degree 3 on the cycle is called a *rim vertex*. A vertex which is adjacent to all the rim vertices is called the *central vertex*. The edges with one end incident with a rim vertex and the other incident with the central vertex are called *spokes*.

Definition 4.7. The graph $F_n = P_n + K_1$ is called a *Fan graph* where $P_n : u_1u_2 \dots u_n$ is a Path and $V(K_1) = u$.

Definition 4.8. The *double Fan* DF_n is defined as $P_n + 2K_1$.

Theorem 4.9. There are infinitely many primes[1].

Notation:
$[x]$ denote the greatest integer $\leq x$.

5 Main results

Theorem 5.1. Every graph is a subgraph of a PD-prime cordial graph.

Proof. Let G be a (p, q) graph, consider the complete graph K_p with $V(K_p) = \{u_i : 1 \leq i \leq p\}$. Assign the labels $p_1, p_2, ..., p_p$ to the vertices $u_1, u_2, ..., u_p$ where p_i, $1 \leq i \leq p$ are primes and $gcd(p_{uv}, d_{uv}) = 1$ for any two edges uv. Such primes are exist by Theorem 3.1. Let $m = \binom{p}{2}$. Consider star $K_{1,m}$ with $V(K_{1,m}) = \{v, v_i : 1 \leq i \leq m\}$ and $E(K_{1,m}) = \{vv_i : 1 \leq i \leq m\}$. Assign the label 1 to the central vertex, next assign the label 2 to the vertex v_1. Consider the smallest integer which is not Used as a label of the vertices of K_p. Say r_1. Clearly $r_1 = 4$. Assign the label r_1 to u_2. we now Consider the smallest integer which is not used as a label say r_2.
Obviously $r_2 = 6$. Assign the label r_2 to u_3. Similarly $r_3 = 8$ $r_4 = 9$ $r_5 = 10$ $r_6 = 12$ and so on. Proceeding like this assign the label $r_3, r_4, r_5, ...$ to the vertex $u_4, u_5, u_6, ...$
Let $s = p_p - 2m - 1$. we now consider the s pendent vertices $w_1, w_2, ...w_s$ and assign the labels to $w_i (i \leq s)$ which are not used as a label of K_p and $K_{1,m}$.
Clearly $e_f(0) = e_f(1) = m$. Note that G is a subgraph of a PD-prime cordial graph $K_p \cup K_{1,m} \cup sK_1$. □

Theorem 5.2. Any path is PD-prime cordial.

Proof. Let P_n be the path $u_1, u_2, u_3, ..., u_n$. Let S_1 be the set with starting from 2 and next element is the twice the previous element and not exceeding n. That is $S_1 = \{x = 2, 4, 8, 16, ...$ and $x \leq n\}$. We now construct the set S_2 as follows. Let the first element of S_1 be the smallest integer which is not in S_1. Next element is the twice the previous one and not exceeding n. That is $S_2 = \{x = 3, 6, 12, 24, ...$ and $x \leq n\}$, similarly $S_3 = \{x = 5, 10, 20, 40, ...$and $x \leq n\}$ and so on.We now assign the labels to the vertices of the path. Put the label 1 to u_1. Next assign the label to the vertices $u_2, u_3, ...$ from the set S_1 in ascending order until the number of edges with label 0 is $\lceil \frac{n}{2} \rceil - 1$. If all the element in S_1 is exhausted and $e_f(0) < \lceil \frac{n}{2} \rceil - 1$, then choose the element from S_2 in ascending order and assign the label to the next consecutive vertices until $e_f(0) = \lceil \frac{n}{2} \rceil - 1$. Each time count the edge with label 0. If $e_f(0) < \lceil \frac{n}{2} \rceil - 1$, then stop the process. If $e_f(0) < \lceil \frac{n}{2} \rceil - 1$,then we consider the set S_3 and proceed as before. At the finite statge we get $e_f(0) = \lceil \frac{n}{2} \rceil - 1$. Let $T = \{m_i / 1 \leq i \leq n$, m_i is not used as labels, $m_i \leq m_{i+1}\}$.We now assign the label to the non-labelled vertices from the set s in the ascending order. Clearly this labeling pattern is a PD-prime cordial labeling of the path P_n. □

Theorem 5.3. All bistars $B_{n,n}$ are PD-prime cordial.

Proof. Let u, v be the centre vertices of the bistar $B_{n,n}$. Let u_i $(1 \leq i \leq n)$ be the pendent vertices adjacent to u and v_i $(1 \leq i \leq n)$ be the pendent vertices adjacent to v.

$E(B_{n,n}) = \{uv\} \cup \{uu_i, vv_i : 1 \le i \le n\}$. Assign the label $2n+2$, 1 to the vertices u, v. we now move to the pendent vertices $v_1, v_2, ..., v_n$. Assign the label $2, 3, 4, ..., n+1$ to the vertices $v_1, v_2, ..., v_n$. Now we consider the other side pendent vertices $u_1, u_2, ..., u_n$. Assign the label $n+2, n+3, ..., 2n+1$ to the vertices $u_1, u_2, ..., u_n$.
Clearly $e_f(0) = n+1$ and $e_f(1) = n$.
Hence $B_{n,n}$ is PD-prime cordial. □

Theorem 5.4. The graph $S(K_{1,n})$ is PD-prime cordial.

Proof. Let $u, u_1, u_2, ..., u_n$ be the vertices of $K_{1,n}$. Let $V(S(K_{1,n})) = \{u, u_i, v_i : 1 \le i \le n\}$ and $E(S(K_{1,n})) = \{uv_i, v_iu_i : 1 \le i \le n\}$. Assign the label 1 to the vertex u. Next consider the pendent vertices $u_1, u_2, ..., u_n$. Assign the label $2, 4, 6, ..., 2n$ to the vertices $u_1, u_2, ..., u_n$. Next consider the subdivision of vertices $v_1, v_2, ..., v_n$ and assign the label $3, 5, 7, ..., 2n+1$ to the vertices $v_1, v_2, ..., v_n$. Obviously $e_f(0) = e_f(1) = n$.
Hence $S(K_{1,n})$ is PD-prime cordial. □

Theorem 5.5. The wheel W_n is PD-prime cordial if and only if $n > 3$.

Proof. Let C_n be the cycle $u_1, u_2, ..., u_n, u_1$.
$V(W_n) = V(C_n) \cup \{u\}$ and $E(W_n) = E(C_n) \cup \{uu_i : 1 \le i \le n\}$.

Case 1. $n = 3$.
Suppose f is a PD-prime cordial labeling. Then $e_f(0) = 4$ and $e_f(1) = 2$, a contradiction.
Hence W_3 is not PD-prime cordial.

Case 2. $n = 4$.

vertex	u	u_1	u_2	u_3	u_4
Label	3	1	2	4	5

Table 1:

Clearly $e_f(0) = e_f(1) = 4$
Hence W_4 is PD-prime cordial.

Case 3. $n = 5$.
Clearly $e_f(0) = e_f(1) = 5$
Hence W_5 is PD-prime cordial.

Case 4. $n \ge 6$

vertex	u	u_1	u_2	u_3	u_4	u_5
Label	6	1	2	3	4	5

Table 2:

Let p be the largest prime number $\leq n$ and $\left[\frac{p}{2}\right]$ is an odd number. Assign the label 1, p and n to the vertices u, u_n and u_{p-1}. Assign the label $2, 3, 4, ..., p-1, p+1, ..., n-1$ to the vertices $u_1, u_2, ...u_{p-2}, u_p, ..., u_{n-1}$. Clearly $e_f(0) = e_f(1) = n$.
Hence W_n is PD-prime cordial.

□

Theorem 5.6. The graph $S(B_{n,n})$ is PD-prime cordial.

Proof. Take the vertex set and edge set of $B_{n,n}$ as in Theorem 4.3. Let x_i and y_i be the newly vertices which subdivided the edges uu_i and vv_i respectively. Also x be the newly vertex subdivided the edge uv.Assign the label 1,2 and 3 to the vertices u,v and x. we now move to the pendent vertices $v_1, v_2, ..., v_n$. Assign the label $5, 9, 13, ..., 4n + 1$ to the vertices $v_1, v_2, ..., v_n$. Now we consider the other side pendent vertices $u_1, u_2, ..., u_n$. Assign the label $7, 11, 15........., 4n + 3$ to the vertices $u_1, u_2, ..., u_n$. Assign the label $6, 10, 14, ...4n + 2$ to the vertices $x_1, x_2, ..., x_n$. Now we consider the other side assign the label $4, 8, 12, ...4n$ to the vertices $y_1, y_2, ..., y_n$. Clearly $e_f(1) = e_f(0) = 2n + 1$.
Hence $S(B_{n,n})$ is SD-prime cordial. □

Theorem 5.7. The fan graph F_n is PD-prime cordial.

Proof. Let $u_1, u_2, ..., u_n$ be the vertices of path P_n and $V(F_n) = \{u\} \cup \{u_i : 1 \leq i \leq n\}$ and $E(F_n) = \{uu_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$. Assign the label 1 to the vertex u. Now we consider the vetices of path $u_1, u_2, ..., u_n$. Assign the label $2, 3, 4, ..., n + 1$ to the vertices $u_1, u_2, ..., u_n$. Clearly $e_f(0) = n$ and $e_f(1) = n - 1$.
Hence F_n is PD-prime cordial.

□

Theorem 5.8. The double fan graph DF_n is PD-prime cordial.

Proof. Let $u_1, u_2, ..., u_n$ be the vertices of path P_n and $V(DF_n) = \{u, v, u_i : 1 \leq i \leq n\}$ and $E(DF_n) = \{uv_i, vv_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-1\}$.

Case 1. n is odd.
Assign the label 1 and $n+1$ to the vertices u and v. Now consider the vertices of the path $u_1, u_2, ..., u_{n-1}$. Assign the label $2, 3, 4, ..., n$ to the vertices $u_1, u_2, ..., u_{n-1}$ and assign the lebel $n + 2$ to the remaining vertex u_n. Clearly $e_f(0) = e_f(1) = n + 2$.

Hence DF_n is PD-prime cordial.

Case 2. n is even.
Assign the label 1 and $n+2$ to the vertices u and v. Now consider the vertices of the path $u_1, u_2, \ldots\ldots\ldots\ldots, u_{n-1}$. Assign the label $2, 3, 4, \ldots\ldots, n$ to the vertices $u_1, u_2, \ldots\ldots\ldots\ldots, u_{n-1}$ and assign the lebel $n+1$ to the remaining vertex u_n. Clearly $e_f(0) = n+3$ and $e_f(1) = n+2$.
Hence DF_n is PD-prime cordial.

□

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