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Skolem Odd Difference Mean Graphs

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ABSTRACT

In this paper we define a new labeling called skolem odd difference mean labeling and investigate skolem odd difference meanness of some standard graphs. Let $G = (V, E)$ be a graph with p vertices and q edges. G is said be skolem odd difference mean if there exists a function $f : V(G) \to \{0, 1, 2, 3, \ldots, p + 3q - 3\}$ satisfying f is $1-1$and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q - 1\}$ denoted by $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits skolem odd difference mean labeling is called odd difference mean graph. We call skolem odd difference mean labeling as skolem even vertex odd difference mean labeling if all the vertex labels are even.

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1 Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary [1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling. An excellent survey of graph labeling is available in [2]. The notion of mean labeling was due to S. Somasundaram and R. Ponraj [7]. A graph $G = (V, E)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0, 1, 2, \ldots, q\}$ such that each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then the resulting edge labels are distinct. The concept of odd mean labeling was introduced by K. Manickam and M. Marudai in [3]. Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said to be odd mean if there exists a function $f : V(G) \to \{0, 1, 2, 3, \ldots, 2q - 1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

is a bijection.

K. Murugan and A. Subramanian [4] introduced the concept of skolem difference mean labeling and some standard results on skolem difference mean labeling were proved in [5] and [6]. A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $\{1, 2, 3, \ldots, p + q\}$ in such a way that for each edge $e = uv$, $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, \ldots, q$. A graph that admits skolem difference mean labeling is called skolem difference mean graph. Motivated by the concepts of skolem difference mean labeling [4] and odd mean labeling [3], we introduce a new labeling called skolem odd difference mean labeling. A graph G is said to be skolem odd difference mean if there exists a function $f : V(G) \to \{1, 2, 3, \ldots, p + 3q - 3\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q - 1\}$ denoted by $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits skolem odd difference mean labeling is called skolem odd difference mean graph.
We use the following definitions in the subsequent section.

Definition 1.1. *The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is obtained by taking one copy of G_1(with p vertices) and p copies of G_2 and then join the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.*

Definition 1.2. *The bistar $B_{m,n}$ is a graph obtained from K_2 by joining m pendent edges to one end of K_2 and n pendent edges to the other end of K_2.*

Definition 1.3. *A caterpillar is a tree with a path $P_m : v_1, v_2, v_3, \ldots, v_m$ called spine with leaves(pendent vertices) known as feet attached to the vertices of the spine by edges known as legs. If every spine v_i is attached with n_i number of leaves, then the caterpillar is denoted by $S(n_1, n_2, \ldots, n_m)$.*

Definition 1.4. *The graph $P_m@P_n$ is obtained from P_m and m copies of P_n by identifying one pendent vertex of the i^{th} copy of P_n with i^{th} vertex of P_m where P_m is a path of length of $m-1$.*

2 Skolem odd difference mean labeling

Theorem 2.1. *Any path $P_n (n \geq 1)$ is a skolem odd difference mean graph.*

Proof: Let $V(P_n) = \{u_i : 1 \leq i \leq n\}$. Then $E(P_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\}$
Define $f : V(P_n) \to \{0, 1, 2, 3, \ldots, p + 3q - 3 = 4n - 6\}$ as follows:
$f(u_{2i-1}) = 4(i-1)$ for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$,
$f(u_{2i}) = 4(n-i) - 2$ for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$.
For each vertex label f the induced edge label f^* is defined as follows:
$f^*(u_i u_{i+1}) = 2(n-i) - 1$ for $1 \leq i \leq n-1$.
It can be verified that f is a skolem odd difference mean labeling of P_n. Hence P_n is a skolem odd difference mean graph. ■
For example, the skolem odd difference mean labeling of P_6 is shown in Figure 1.

Figure 1

Theorem 2.2. *Any cycle $C_n (n \geq 4)$ is a skolem odd difference mean graph.*

Proof: Let $V(C_n) = \{u_i : 1 \leq i \leq n\}$.
Then $E(C_n) = \{u_i u_{i+1}, u_n u_1 : 1 \leq i \leq n-1\}$
Define $f : V(C_n) \to \{0, 1, 2, 3, \ldots, p + 3q - 3\}$ as follows:
Case(i): $n \equiv 0 (mod\ 4)$
Let $n = 4k$.
$f(v_1) = 0$,
$f(v_{2i+1}) = 4i - 1$ for $1 \leq i \leq k$,
$f(v_{2i}) = 4(n-i) + 1$ for $1 \leq i \leq k$,
$f(v_{2k+2}) = n + 1$,
$f(v_{2k+3}) = n - 4$,
$f(v_{2k+4}) = n + 6$,

$f(v_{2k+3+2i}) = n - 4(i+1)$ for $1 \leq i \leq k-2$,
$f(v_{2k+4+2i}) = n + 4(i+1) + 2$ for $1 \leq i \leq k-2$.
For each vertex label, the induced labeling f^* is defined as follows:
Let $e_j = v_j v_{j+1} (1 \leq j \leq n-1)$ and $e_n = v_n v_1$.
$f^*(e_j) = 2n + 1 - 2j$ for $1 \leq j \leq 2k+1$,
$f^*(e_j) = 2(i - 2k) - 3$ for $2k+2 \leq j \leq 4k+2$.
It can be verified that f is a skolem odd difference mean labeling of C_n.
Case(ii): $n \equiv 1 (mod\ 4)$
Let $n = 4k+1$.
$f(v_1) = 0$,
$f(v_{2i+1}) = 4i - 1$ for $1 \leq i \leq k$,
$f(v_{2i}) = 4(n-i)$ for $1 \leq i \leq k$,
$f(v_{2k+2}) = n$,
$f(v_{2k+3}) = n + 5$,
$f(v_{2k+4}) = n - 5$,
$f(v_{2k+3+2i}) = n + 5 + 4i$ for $1 \leq i \leq k-1$,
$f(v_{2k+4+2i}) = n - 5 - 4i$ for $1 \leq i \leq k-2$.
For each vertex label, the induced labeling f^* is defined as follows:
Let $e_j = v_j v_{j+1} (1 \leq j \leq n-1)$ and $e_n = v_n v_1$.
$f^*(e_j) = 2n + 1 - 2j$ for $1 \leq j \leq 2k$,
$f^*(e_j) = 2(i - 2k) - 1$ for $2k+1 \leq j \leq 4k+1$.
It can be verified that f is a skolem odd difference mean labeling of C_n.
Case(iii): $n \equiv 2 (mod\ 4)$
Let $n = 4k+2$.
$f(v_1) = 0$,
$f(v_{2i+1}) = 4i - 1$ for $1 \leq i \leq k$,
$f(v_{2i}) = 4(n-i) + 1$ for $1 \leq i \leq k+1$,
$f(v_{2k+3}) = 3n - 3$,
$f(v_{2k+4}) = 3n - 8$,
$f(v_{2k+5}) = 3n - 18$,
$f(v_{2k+4+2i}) = 3n - 8 - 4i$ for $1 \leq i \leq k-1$,
$f(v_{2k+5+2i}) = 3n - 18 + 4i$ for $1 \leq i \leq k-2$.
For each vertex label, the induced labeling f^* is defined as follows:
Let $e_j = v_j v_{j+1} (1 \leq j \leq n-1)$ and $e_n = v_n v_1$.
$f^*(e_j) = 2n + 1 - 2j$ for $1 \leq j \leq 2k+1$,
$f^*(e_j) = 2(i - 2k) - 3$ for $2k+2 \leq j \leq 4k = 2$.
It can be verified that f is a skolem odd difference mean labeling of C_n.
Case(iv): $n \equiv 3 (mod\ 4)$
Let $n = 4k+3$.

$f(v_1) = 0$,
$f(v_{2i+1}) = 4i - 1$ for $1 \leq i \leq k+1$,
$f(v_{2i}) = 4(n-i) + 1$ for $1 \leq i \leq k+1$,
$f(v_{2k+4}) = n - 2$,
$f(v_{2k+5}) = n + 3$,
$f(v_{2k+6}) = n - 7$,
$f(v_{2k+4+2i}) = n - 7 - 4i$ for $1 \leq i \leq k-2$.
$f(v_{2k+5+2i}) = n + 3 + 4i$ for $1 \leq i \leq k-1$.
For each vertex label, the induced labeling f^* is defined as follows:
Let $e_j = v_j v_{j+1} (1 \leq j \leq n-1)$ and $e_n = v_n v_1$.
$f^*(e_j) = 2n + 1 - 2j$ for $1 \leq j \leq 2k$,
$f^*(e_j) = 2(j - 2k - 2) - 1$ for $2k+3 \leq j \leq 4k+3$.
It can be verified that f is a skolem odd difference mean labeling of C_n. ■
The skolem odd difference mean labeling of C_7 is shown in Figure 2.

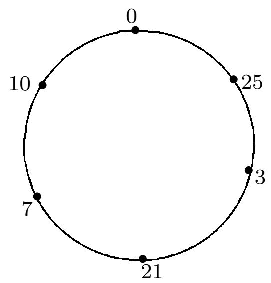

Figure 2

Theorem 2.3. *The star graph $K_{1,n} (n \geq 1)$ admits skolem odd difference mean labeling.*

Proof: Let v_0 be the central vertex and $v_i (1 \leq i \leq n)$ be the pendent vertices of the star $K_{1,n}$.
Then $E(K_{1,n}) = \{v_0 v_i : 1 \leq i \leq n\}$
Define $f : V(K_{1,n}) \to \{0, 1, 2, 3, 4, \ldots, p + 3q - 3 = 4n - 2\}$ as follows:
$f(v_0) = 0$,
$f(v_i) = 4i - 2$ for $1 \leq i \leq n$.
For each vertex label f the induced edge label f^* is defined as follows:
$f^*(v_0 v_i) = 2i - 1$ for $1 \leq i \leq n$.
It can be verified that f is a skolem odd difference mean labeling of $K_{1,n}$. Hence $K_{1,n}$ admits skolem odd difference mean labeling. ■
The skolem odd difference mean labeling of $K_{1,6}$ is shown in Figure 3.

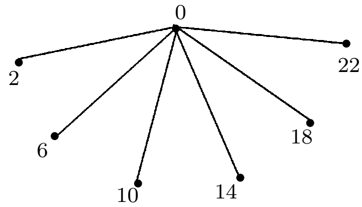

Figure 3

Theorem 2.4. *The graph $P_n \odot K_1, n(n \geq 1)$ is a skolem odd difference mean graph.*

Proof: Let $u_i(1 \leq i \leq n)$ be the vertices of P_n.
Let $v_i(1 \leq i \leq n)$ be the pendent vertices joined with $u_i(1 \leq i \leq n)$ by an edge.
Define $f : V(P_n \odot K_1) \rightarrow \{0, 1, 2, 3, 4, \ldots, p + 3q - 3 = 8n - 6\}$ as follows:
$f(u_{2i-1}) = 8(n - i) + 2$ for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$,
$f(u_{2i}) = 4 + 8(i - 1)$ for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$,
$f(v_{2i-1}) = 8(i - 1)$ for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$,
$f(v_{2i}) = 8(n - i) - 2$ for $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$.
For each vertex label f the induced edge label f^* is defined as follows:
$f^*(u_i u_{i+1}) = 4(n - i) - 1$ for $1 \leq i \leq n - 1$,
$f^*(u_i v_i) = 4(n - i) + 1$ for $1 \leq i \leq n$.
It can be verified that f is a skolem odd difference mean labeling of $P_n \odot K_1$.
Hence $P_n \odot K_1$ is a skolem odd difference mean graph. ■
For example, the skolem odd difference mean labeling of $P_5 \odot K_1$ is shown in Figure 4.

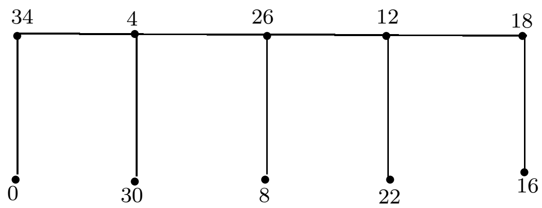

Figure 4

Theorem 2.5. *The coconut tree $T(n, m)$, obtained by identifying the central vertex of the star $K_{1,m}$ with a pendent vertex of a path P_n is a skloem odd difference mean graph.*

Proof: Let $v_0, v_1, v_2, \ldots, v_n$ be the vertices of a path, having path length $n(n \geq 1)$ and $u_1, u_2, \ldots, u_m$ be the pendent vertices being adjacent with v_0.
$E(T(n, m)) = \{v_0 u_i, v_0 v_1, v_j v_{j+1} : 1 \leq i \leq m,\ 1 \leq j \leq n - 1\}$.
Define $f : V(T(n, m)) \rightarrow \{0, 1, 2, 3, 4, \ldots, p + 3q - 3 = 4(m + n) - 2\}$ as follows:
$f(v_0) = 0$,
$f(u_i) = 4(m + n - i) + 2$ for $1 \leq i \leq m$,
$f(v_j) = 4n - 2j$ for $1 \leq j \leq n$ and j is odd,
$f(v_j) = 2j$ for $1 \leq j \leq n$ and j is even.
For each vertex label f the induced edge label f^* is defined as follows:

$f^*(v_0u_i) = 2(m+n-i)+1$ for $1 \leq i \leq m$,
$f^*(v_0v_1) = 2n-1$,
$f^*(v_jv_{j+1}) = 2(n-j)-1$ for $1 \leq j \leq n-1$.
It can be verified that f is a skolem odd difference mean labeling of $T(n,m)$. Hence $T(n,m)$ is a skolem odd difference mean graph. ■
For example,the skolem odd difference mean labeling of $T(4,6)$ is shown in Figure 5.

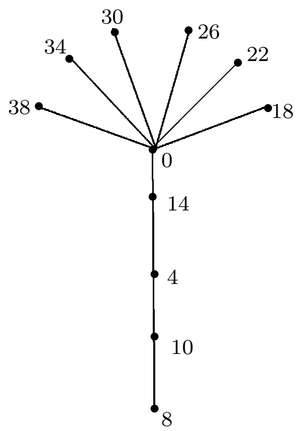

Figure 5

Theorem 2.6. *The graph $B_{m,n}(m,n \geq 1)$ is a skolem odd difference mean graph.*

Proof: Let $V(B_{m,n}) = \{u, v, u_i, v_j : 1 \leq i \leq m,\ 1 \leq j \leq n\}$.
Then $E(B_{m,n}) = \{uv, uu_i, vv_j : 1 \leq i \leq m,\ 1 \leq j \leq n\}$.
Define $f : V(B_{m,n}) \to \{0,1,2,3,4,\ldots,p+3q-3 = 4(m+n)+2\}$ as follows:
$f(u) = 0$,
$f(v) = 2$,
$f(u_i) = 4(m+n-i)+6$ for $1 \leq i \leq m$,
$f(v_j) = 4(n-j)+8$ for $1 \leq j \leq n$,
For each vertex label f the induced edge label f^* is defined as follows:
$f^*(uv) = 1$,
$f^*(uu_i) = 2(m+n-i)+3$ for $1 \leq i \leq m$,
$f^*(vv_j) = 2(n-j)+3$ for $1 \leq j \leq n$.
It can be verified that f is a skolem odd difference mean labeling of $B_{m,n}$. Hence $B_{m,n}$ is a skolem odd difference mean graph. ■
The skolem odd difference mean labeling of $B_{5,7}$ is shown in Figure 6.

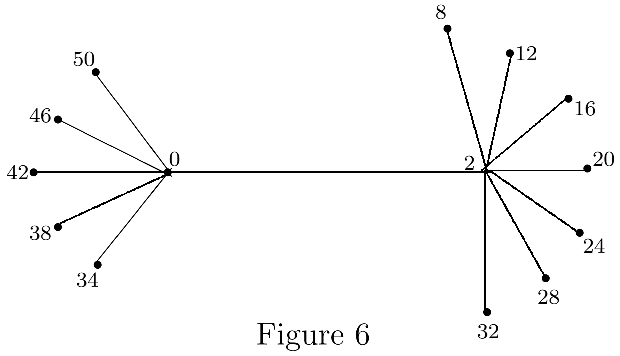

Figure 6

Theorem 2.7. *The caterpillar $S(n_1, n_2, \ldots, n_m)$ is a skloem odd difference mean graph.*

Proof: Let $V(S(n_1, n_2, \ldots, n_m)) = \{v_j, u_i^j : 1 \leq i \leq n_j,\ 1 \leq j \leq m\}$.
Let $E(S(n_1, n_2, \ldots, n_m)) = \{v_j v_{j+1} : 1 \leq j \leq m-1$ and $v_j u_i^j : 1 \leq i \leq n_j$, $1 \leq j \leq m\}$.
Define $f : V(S(n_1, n_2, \ldots, n_m)) \rightarrow$
$\{0, 1, 2, 3, 4, \ldots, p + 3q - 3 = 4(m + n_1 + n_2 + \ldots + n_m) - 6\}$ as follows:
$f(v_j) = 4(m + n_1 + n_2 + \ldots + n_m) - 4(n_2 + n_4 + \ldots + n_{j-1}) - 2(j + 2)$ for $1 \leq j \leq m$ and j is odd,
$f(v_j) = 4(n_1 + n_3 + \ldots + n_{j-1}) + 2(j - 2)$ for $1 \leq j \leq m$ and j is even,
$f(u_i^j) = 4(n_1 + n_3 + \ldots + n_{j-2}) + 2(2i + j) - 6$ for $1 \leq i \leq n_j,\ 1 \leq j \leq m$ and j is odd,
$f(u_i^j) = 4(m + n_1 + n_2 + \ldots + n_m) - 4(n_2 + n_4 + \ldots + n_{j-2}) - 2(j + 3) - 4(i - 1)$ for $1 \leq i \leq n_j,\ 1 \leq j \leq m$ and j is even,
Let $e_j = v_j v_{j+1}$ for $1 \leq j \leq m - 1$ and $e_i^j = v_j u_i^j$ for $1 \leq i \leq n_j,\ 1 \leq j \leq m$.
For each vertex label f the induced edge label f^* is defined as follows:
$f^*(e_j) = 2(m + n_{j+1} + \ldots + n_m) - 2j - 1$ for $1 \leq j \leq m - 1$,
$f^*(e_i^j) = 2(m + n_j + n_{j+1} + \ldots + n_m) - 2(i + j) + 1$ for $1 \leq i \leq n_j,\ 1 \leq j \leq m$.
It can be verified that f is a skolem odd difference mean labeling of $S(n_1, n_2, \ldots, n_m)$.
Hence $S(n_1, n_2, \ldots, n_m)$ is a skolem odd difference mean graph. ■
An example for skolem odd difference mean labeling of $S(4, 2, 3, 2)$ is shown in Figure 7.

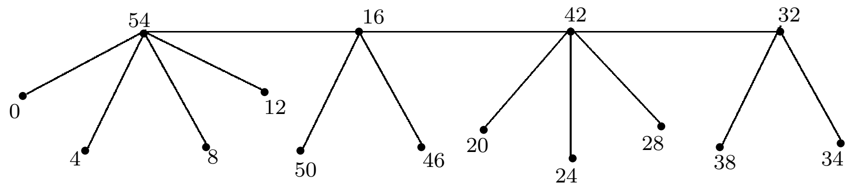

Figure 7

Theorem 2.8. *The graph $P_m@P_n$ is a skolem odd difference mean graph.*

Proof: Let $V(P_m@P_n) = \{v_j, u_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$.
Let $E(P_m@P_n) = \{v_t v_{t+1},\ u_i^j u_{i+1}^j : 1 \leq t \leq m - 1,\ 1 \leq i \leq n - 1,\ 1 \leq j \leq m\}$ with $v_j = u_n^j$ $1 \leq j \leq m$.

Define $f : V(P_m@P_n) = \{0, 1, 2, 3, \ldots, p + 3q - 3 = 4mn - 6\}$
If n is odd, define

$$f(v_j) = \begin{cases} 2(nj-1) & \text{for } 1 \le j \le m \text{ and } j \text{ is odd} \\ 4(mn-1) - 2n(j-1) & \text{for } 1 \le j \le m \text{ and } j \text{ is even} \end{cases} \quad \text{and}$$

If n is even, define

$$f(v_j) = \begin{cases} 2n(2m-j) - 2 & \text{for } 1 \le j \le m \text{ and } j \text{ is odd} \\ 2n(j-1) & \text{for } 1 \le j \le m \text{ and } j \text{ is even} \end{cases}$$

$f(u_i^j) = 2(n(j-1) + i - 1)$ for $1 \le i \le n,\ 1 \le j \le m$ and i is odd, j is odd,
$f(u_i^j) = 2n(2m - j + 1) - 2(i + 1)$ for $1 \le i \le n,\ 1 \le j \le m$ and i is even, j is odd,
$f(u_i^j) = 4(mn - 1) + 2(i - nj)$ for $1 \le i \le n,\ 1 \le j \le m$ and i is odd, j is even,
$f(u_i^j) = 2(nj - i)$ for $1 \le i \le n,\ 1 \le j \le m$ and i is even, j is even.
Let $e_j = v_j v_{j+1}$ for $1 \le j \le m - 1$ and $e_i^j = u_i^j u_{i+1}^j$ for $1 \le i \le n - 1,\ 1 \le j \le m$. For each vertex label f the induced edge label f^* is defined as follows:
$f^*(e_j) = 2n(m - j) - 1$ for $1 \le j \le m - 1$,
$f^*(e_i^j) = 2n(m - j + 1) - 2i - 1$ for $1 \le i \le n - 1,\ 1 \le j \le m$ and j is odd,
$f^*(e_i^j) = 2n(m - j) + 2i - 1$ for $1 \le i \le n - 1,\ 1 \le j \le m$ and j is even.
It can be verified that f is a skolem odd difference mean labeling of $P_m@P_n$. Hence $P_m@P_n$ is a skolem odd difference mean graph. ■
The skolem odd difference mean labeling of $P_4@P_4$ is shown in Figure 8.

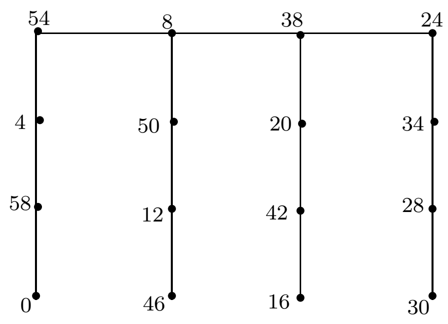

Figure 8

Theorem 2.9. *The graph $P_m@2P_n$ is a skolem odd difference mean graph.*

Proof: Let $V(P_m@2P_n) = \{v_j, u_{1,i}^j, u_{2,i}^j : 1 \le j \le m,\ 1 \le i \le n\}$ with $u_{1,n}^j = u_{2,n}^j = v_j$ for $1 \le j \le m$.
Then $E(P_m@2P_n) = \{v_j v_{j+1}, u_{1,i}^j u_{1,i+1}^j, u_{2,i}^j u_{2,i+1}^j : 1 \le j \le m - 1,\ 1 \le i \le n - 1\}$.
Define $f : V(P_m@2P_n) = \{0, 1, 2, 3, \ldots, p + 3q - 3 = 4m(2n - 1) - 6\}$ as follows:

If n is odd, define

$$f(v_j) = \begin{cases} 2j(2n-1)-2n & \text{for } 1 \leq j \leq m \text{ and } j \text{ is odd} \\ 4n(2m-j+1)-2(2m-j+7) & \text{for } 1 \leq j \leq m \text{ and } j \text{ is even} \end{cases}$$

and if n is even, then

$$f(v_j) = \begin{cases} 2(2n-1)(2m-j)+2(n-2) & \text{for } 1 \leq j \leq m \text{ and } j \text{ is odd} \\ 2j(2n-1)-2n & \text{for } 1 \leq j \leq m \text{ and } j \text{ is even} \end{cases}$$

$f(u^j_{1,i}) = 4n(j-1)-2(j-i)$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is odd, j is odd,
$f(u^j_{1,i}) = 4m(2n-1)-4n(j-1)+2(j-i-2)$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is even, j is odd,
$f(u^j_{1,i}) = 4m(2n-1)-2(2n-1)(j-2)-4n-2i$ for $1 \leq i \leq n,\ 1 \leq j \leq m$, i is odd, j is even,
$f(u^j_{1,i}) = 4n(j-1)+2(i-j)$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is even, j is even,
$f(u^j_{2,i}) = 2j(2n-1)-2i$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is odd, j is odd,
$f(u^j_{2,i}) = (2n-1)(4m-2j)+2(n+i-7)$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is even, j is odd,
$f(u^j_{2,i}) = (2n-1)(4m-2j)+2(i-2)$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is odd, j is even,
$f(u^j_{2,i}) = 2j(2n-1)-2i$ for $1 \leq i \leq n,\ 1 \leq j \leq m$ and i is even, j is even.
Let $e_j = v_j v_{j+1}$ for $1 \leq j \leq m-1$,
$e^j_1 = u^j_{1,i} u^j_{1,i+1}$ for $1 \leq i \leq n-1,\ 1 \leq j \leq m$,
$e^j_2 = u^j_{2,i} u^j_{2,i+1}$ for $1 \leq i \leq n-1$ and $1 \leq j \leq m$.
For each vertex label f the induced edge label f^* is defined as follows:
$f^*(e_j) = 2(2n-1)(m-j)-1$ for $1 \leq j \leq m-1$,
$f^*(e^j_1) = 2(2n-1)(m-j)+2(2n-i)-3$ for $1 \leq i \leq n-1,\ 1 \leq j \leq m$
$f^*(e^j_2) = 2(2n-1)(m-j)+2i-1$ for $1 \leq i \leq n-1,\ 1 \leq j \leq m$.
It can be verified that f is a skolem odd difference mean labeling of$P_m@2P_n$. Hence $P_m@2P_n$ is a skolem odd difference mean graph. ■
The skolem odd difference mean labeling of $P_3@2P_4$ is shown in Figure 9.

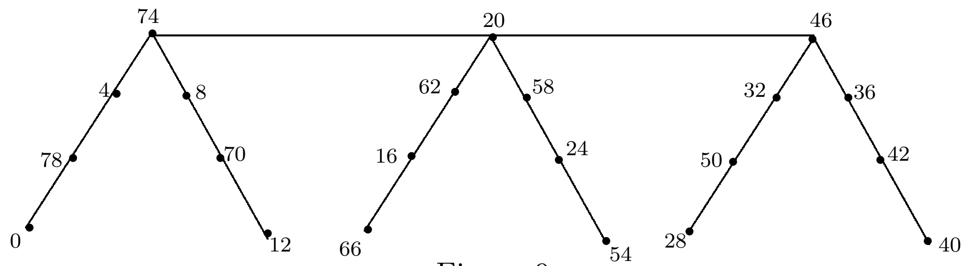

Figure 9

Theorem 2.10. *The complete graph $K_n, n > 3$ is not a skolem odd difference mean graph.*

Proof: In a complete graph K_n, the number of edges $q = \frac{n(n-1)}{2}$. Therefore $p + 3q - 3 = \frac{3n^2-n-6}{2}$.
To get $2q - 1 = n^2 - n - 1$ as edge label, the minimum vertex label is $2n^2 - 2n - 3$.
But $\frac{3n^2-n-6}{2} < 2n^2 - 2n - 3$ for all $n \geq 4$.
Therefore $2q - 1$ cannot occur as an edge label of K_n for $n \geq 4$.
Hence, K_n is not a skolem odd difference mean graph. ■

Theorem 2.11. *The graph $K_{2,n}$ is a skolem odd difference mean graph if $n \leq 2$.*

Proof: $K_{2,1} = P_3$ and $K_{2,2} = C_4$. Hence $K_{2,n}$ is a skolem odd difference mean graph for $n \leq 2$. ■

Theorem 2.12. *The graph $K_{2,n}(n \geq 3)$ is not a skolem odd difference mean graph.*

Proof: The graph $K_{2,n}$ has $n + 2$ vertices and $2n$ edges.
For $n \geq 3$, $p + 3q - 3 = 7n - 1$.
That is the maximum possible vertex label of $K_{2,n}$ is $7n - 1$.
Therefore it is not possible to get an edge with label $2q - 1 = 4n - 1$. ■
Hence $K_{2,n}(n \geq 3)$ is not a skolem odd difference mean labeling.

3 Skolem Even Vertex Odd Difference Mean Labeling

A graph G is said to be skolem even vertex odd difference mean if there exists a function $f : V(G) \to \{0, 2, 4, \ldots, p + 3q - 3\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q - 1\}$ defined by $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits skolem even vertex odd difference mean labeling is called skolem even vertex odd difference mean graph. That is, we call a skolem odd difference mean labeling f of a graph G as skolem even vertex odd difference mean labeling if all the vertex labels $f(v)$ of G are even.
■

Theorem 3.1. *The following graphs are even vertex odd difference mean.*

(i) $P_n(n \geq 1)$
(ii) $K_{1,n}(n \geq 1)$
(iii) $P_n \odot K_1(n \geq 1)$
(iv) The coconut tree $T(n, m)$, obtained by identifying the central vertex of the star $K_{1,m}$ with a pendent vertex of a path P_n
(v) $B(m, n)(m, n \geq 1)$
(vi) Caterpillar $S(n_1, n_2, \ldots, n_m)$
(vii) $P_m@P_n(m, n \geq 1)$

(viii) $P_m@2P_n(m, n \geq 1)$

Proof: The proof follows from Theorems 2.1, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 and 2.9. ■

Theorem 3.2. *The cycle $C_n, (n \geq 3)$ is not a skolem even vertex odd difference mean graph.*

Proof: Let f be a skolem even vertex odd difference mean labeling of C_n.
Here$p + 3q - 3 = 4n - 3$ is an odd number. Therefore the maximum possible vertex label in C_n is $4n - 4$. Hence, the edge label $2n - 1$ cannot occur. Thus C_n $(n \geq 3)$ is not a skolem even vertex odd difference mean graph.

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