Journal of Algorithms and Computation
journal homepage: http://jac.ut.ac.ir

Further results on total mean cordial labeling of graphs

R. Ponraj, *1 and S. Sathish Narayanan†2

1,2 Department of Mathematics, Sri Paramakalyani College,Alwarkurichi-627 412, India.

ABSTRACT

A graph $G = (V, E)$ with p vertices and q edges is said to be a total mean cordial graph if there exists a function $f : V(G) \to \{0, 1, 2\}$ such that $f(xy) = \left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, $xy \in E(G)$, and the total number of 0, 1 and 2 are balanced. That is $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. In this paper, we investigate the total mean cordial labeling of C_n^2, ladder L_n, book B_m and some more graphs.

ARTICLE INFO

Article history:
Received 28, September 2015
Received in revised form 20, January 2016
Accepted 8, March 2016
Available online 18, March 2016

Keyword: cycle, path, union of graphs, star, ladder

AMS subject Classification: Primary $05C78$,

1 Introduction

Graphs considered here are finite, undirected and simple. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. The order and size of a graph G are respectively denoted by p and q. Origin of graph labeling is graceful labeling which was introduced by Rosa [8] in 1967. In 1980, Cahit [4] introduced the cordial labeling of graphs. Cordial labeling behavior of several graphs were studied in [10]. Somasundaram and Ponraj introduced the notion of mean labeling of graphs in [11]. Also in [1, 2, 3, 9], mean labeling behavior of several graphs were studied. Motivated by this labeling,

*Corresponding Author:R. Ponraj, Email:*ponrajmaths@gmail.com*;
†E-mail: *sathishrvss@gmail.com*

Ponraj, Ramasamy and Sathish Narayanan [7] introduced the concept of total mean cordial labeling of graphs and studied about the total mean cordial labeling behavior of path, cycle, wheel and some more standard graphs.
In this paper we investigate $C_n^{(2)}$, ladder L_n, book B_m and some more graphs. If x is any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [6] and Gallian [5].

2 Total mean cordial labeling

Definition 2.1. A total mean cordial labeling of a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, 2\}$ such that $f(xy) = \left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, $xy \in E(G)$, and the total number of 0, 1 and 2 are balanced. That is $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. If there exists a total mean cordial labeling on a graph G, we will call G is total mean cordial.

The following results are frequently used in the subsequent section.

Theorem 2.1. [7] Any path P_n is total mean cordial.

The above theorem 2.1 is used in theorem 3.1. So we recall the structure of the total mean cordial labeling g of a path.
Let $P_m : v_1, v_2, \ldots, v_m$ be the path. When $m = 3t$. Assign the label 2 to the vertices $v_1, v_2, \ldots, v_t$ and assign the integer 1 to the vertices $v_{t+1}, v_{t+2}, \ldots, v_{2t}$. Finally assign the label 0 to the vertices $v_{2t+1}, v_{2t+2}, \ldots, v_{3t}$.
In the case of $m = 3t + 1$, assign the label to the vertices v_i $(1 \leq i \leq 3t)$ as in the case of $m = 3t$. Then assign 0 to the vertex v_{3t+1}.
For $m = 3t + 2$, we define a map $g : V(P_m) \to \{0, 1, 2\}$ by

$$\begin{array}{rcl} g(v_i) & = & 0, \quad 1 \leq i \leq t+1 \\ g(v_{t+1+i}) & = & 1, \quad 1 \leq i \leq t \\ g(v_{2t+1+i}) & = & 2, \quad 1 \leq i \leq t \\ g(v_{3t+2}) & = & 1. \end{array}$$

Theorem 2.2. [7] The cycle C_n is total mean cordial if and only if $n \neq 3$.

The theorem 3.2 is based on theorem 2.2. So we recall the total mean cordial labeling of a cycle C_n. Let $C_n : u_1u_2 \ldots u_nu_1$ be the cycle. If $n = 3$, then we have $ev_f(0) = ev_f(1) = ev_f(2) = 2$. But this is an impossible one. Assume $n > 3$. When $n \equiv 0$ $(mod\ 3)$, let

$n = 3t$, $t \in \mathbb{Z}^+$. It is easy to see that C_6 is total mean cordial. Take $t \geq 3$. Define $f : V(C_n) \to \{0, 1, 2\}$ by

$$\begin{aligned} f(u_i) &= 0, \quad 1 \leq i \leq t \\ f(u_{t+i}) &= 2, \quad 1 \leq i \leq t \\ f(u_{2t+i}) &= 1, \quad 1 \leq i \leq t-2. \end{aligned}$$

$f(u_{3t-1}) = 0$ and $f(u_{3t}) = 1$. When $n \equiv 1, 2$ $(mod\ 3)$, The labeling g defined in theorem 2.1 satisfy total mean cordial condition of C_n.

3 Main Results

The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

Theorem 3.1. Let G be a (p, q) total mean cordial graph and $n \neq 3$ then $G \cup P_n$ is total mean cordial.

Proof. Let $v_1, v_2, \ldots, v_p$ be the vertices of G and $P_n : u_1, u_2, \ldots, u_n$ be the path. Let f be a total mean cordial labeling of G and g be the total mean cordial labeling of P_n as defined in theorem 2.1. The induced total mean cordial labeling h by f and g of $G \cup P_n$ as follows:

Case 1. $n \leq 8$.

Subcase 1. $p + q \equiv 0 \pmod 3$.

Let $p + q = 3t$. Assign the labels to the vertices of P_n $(n \leq 8)$ as in theorem 2.1. One can easily check that the labeling f together with g forms a total mean cordial labeling h of $G \cup P_n$.

Subcase 2. $p + q \equiv 1 \pmod 3$.

Put $p + q = 3t + 1$. Suppose $ev_f(0) = ev_f(1) = t$, $ev_f(2) = t + 1$ then the following table 1 establishes that $G \cup P_n$ $(n \leq 8)$ is total mean cordial.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
P_1	0								$t+1$	t	$t+1$
P_2	0	1							$t+1$	$t+2$	$t+1$
P_4	0	0	2	1					$t+3$	$t+2$	$t+3$
P_5	0	0	1	2	1				$t+3$	$t+3$	$t+4$
P_6	0	0	2	0	1	2			$t+4$	$t+4$	$t+4$
P_7	0	0	0	1	1	2	1		$t+5$	$t+5$	$t+4$
P_8	0	0	0	1	1	2	2	1	$t+5$	$t+5$	$t+6$

Table 1:

If $ev_f(0) = ev_f(2) = t$, $ev_f(1) = t+1$ then the following table 2 shows the total mean cordial condition of $G \cup P_n$ $(n \leq 8)$.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
P_1	0								$t+1$	$t+1$	t
P_2	0	2							$t+1$	$t+2$	$t+1$
P_4	0	0	2	2					$t+3$	$t+2$	$t+3$
P_5	0	0	2	2	1				$t+3$	$t+3$	$t+4$
P_6	0	0	2	2	1	0			$t+4$	$t+4$	$t+4$
P_7	0	0	0	2	2	1	1		$t+5$	$t+5$	$t+4$
P_8	0	0	0	2	1	2	1	0	$t+6$	$t+5$	$t+5$

Table 2:

If $ev_f(1) = ev_f(2) = t$, $ev_f(0) = t+1$ then the following table 3 establishes that $G \cup P_n$ $(n \leq 8)$ is total mean cordial.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
P_1	1								$t+1$	t	t
P_2	1	2							$t+1$	$t+1$	$t+2$
P_4	0	2	2	0					$t+2$	$t+2$	$t+3$
P_5	0	0	1	2	1				$t+4$	$t+3$	$t+3$
P_6	0	0	1	1	2	2			$t+4$	$t+4$	$t+4$
P_7	0	0	1	2	2	1	0		$t+5$	$t+4$	$t+5$
P_8	0	0	0	1	1	2	2	1	$t+6$	$t+5$	$t+5$

Table 3:

Subcase 3. $p+q \equiv 2 \pmod 3$.
Put $p+q = 3t+2$. Suppose $ev_f(0) = ev_f(1) = t+1$, $ev_f(2) = t$ then the following table 4 establishes that $G \cup P_n$ $(n \leq 8)$ is total mean cordial.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
P_1	2								$t+1$	$t+1$	$t+1$
P_2	0	2							$t+2$	$t+2$	$t+1$
P_4	0	2	2	0					$t+3$	$t+3$	$t+3$
P_5	0	0	2	2	1				$t+4$	$t+3$	$t+4$
P_6	0	0	1	2	1	2			$t+4$	$t+4$	$t+5$
P_7	0	0	2	1	2	1	0		$t+5$	$t+5$	$t+5$
P_8	0	0	0	1	1	2	2	1	$t+6$	$t+6$	$t+5$

Table 4:

Suppose $ev_f(0) = ev_f(2) = t+1$, $ev_f(1) = t$ then with the help of the following table 5, we can understand that $G \cup P_n$ $(n \leq 8)$ is total mean cordial.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
P_1	1								$t+1$	$t+1$	$t+1$
P_2	0	1							$t+2$	$t+2$	$t+1$
P_4	0	2	1	0					$t+3$	$t+3$	$t+3$
P_5	0	1	2	1	0				$t+3$	$t+4$	$t+4$
P_6	0	0	1	1	2	1			$t+4$	$t+5$	$t+4$
P_7	0	0	2	1	1	2	0		$t+5$	$t+5$	$t+5$
P_8	0	0	0	1	1	2	2	1	$t+6$	$t+5$	$t+6$

Table 5:

Suppose $ev_f(1) = ev_f(2) = t+1$, $ev_f(0) = t$ then the following table 6 establishes that $G \cup P_n$ $(n \leq 8)$ is total mean cordial.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	$ev_h(0)$	$ev_h(1)$	$ev_h(2)$
P_1	0								$t+1$	$t+1$	$t+1$
P_2	0	2							$t+1$	$t+2$	$t+2$
P_4	0	0	1	2					$t+3$	$t+3$	$t+3$
P_5	0	0	1	2	1				$t+3$	$t+4$	$t+4$
P_6	0	0	1	2	1	0			$t+4$	$t+5$	$t+4$
P_7	0	0	0	1	1	2	2		$t+5$	$t+5$	$t+5$
P_8	0	0	0	1	1	2	2	1	$t+5$	$t+6$	$t+6$

Table 6:

Case 2. $n \geq 9$.
Now we define a vertex labeling $h : V(G \cup P_n) \rightarrow \{0, 1, 2\}$ by $h(v_i) = f(v_i)$, $1 \leq i \leq p$ and $h(u_i) = g(u_i)$, $1 \leq i \leq n$.
Subcase 1. $p + q \equiv 0 \pmod 3$ and $n \equiv 0 \pmod 3$.
Let $p+q = 3t_1$ and $n = 3t_2$. In this case $ev_h(0) = t_1 + 2t_2 - 1$, $ev_h(1) = ev_h(2) = t_1 + 2t_2$.
Subcase 2. $p + q \equiv 0 \pmod 3$ and $n \equiv 1 \pmod 3$.
Let $p + q = 3t_1$ and $n = 3t_2 + 1$. Here $ev_h(0) = t_1 + 2t_2 + 1$, $ev_h(1) = ev_h(2) = t_1 + 2t_2$.
Subcase 3. $p + q \equiv 0 \pmod 3$ and $n \equiv 2 \pmod 3$.
Let $p + q = 3t_1$ and $n = 3t_2 + 2$. In this case $ev_h(0) = ev_h(1) = ev_h(2) = t_1 + 2t_2 + 1$.
Subcase 4. $p + q \equiv 1 \pmod 3$ and $n \equiv 0 \pmod 3$.
Let $p + q = 3t_1 + 1$ and $n = 3t_2$. If $ev_f(0) = t_1 + 1$, $ev_f(1) = ev_f(2) = t_1$ then $ev_h(0) = ev_h(1) = ev_h(2) = t_1 + 2t_2$. If $ev_f(1) = t_1 + 1$, $ev_f(0) = ev_f(2) = t_1$ then relabel the vertex u_{t+2} by 0. Here also $ev_h(0) = ev_h(1) = ev_h(2) = t_1 + 2t_2$. If $ev_f(2) = t_1 + 1$, $ev_f(0) = ev_f(1) = t_1$ then relabel the vertices u_{2t}, u_{2t+2} by 2, 0 respectively. Hence $ev_h(0) = ev_h(1) = ev_h(2) = t_1 + 2t_2$.
Subcase 5. $p + q \equiv 1 \pmod 3$ and $n \equiv 1 \pmod 3$.

Let $p+q=3t_1+1$ and $n=3t_2+1$. If $ev_f(0)=t_1+1$, $ev_f(1)=ev_f(2)=t_1$ then relabel the vertex u_1 by 2. Hence $ev_h(0)=t_1+2t_2$, $ev_h(1)=ev_h(2)=t_1+2t_2+1$. Suppose $ev_f(1)=t_1+1$, $ev_f(0)=ev_f(2)=t_1$ then $ev_h(0)=ev_h(1)=t_1+2t_2+1$, $ev_h(2)=t_1+2t_2$. If $ev_f(2)=t_1+1$, $ev_f(0)=ev_f(1)=t_1$ then $ev_h(0)=ev_h(2)=t_1+2t_2+1$, $ev_h(1)=t_1+2t_2$.
Subcase 6. $p+q\equiv 1 \pmod 3$ and $n\equiv 2 \pmod 3$.
Let $p+q=3t_1+1$ and $n=3t_2+2$. If $ev_f(0)=t_1+1$, $ev_f(1)=ev_f(2)=t_1$ Here $ev_h(0)=t_1+2t_2+2$, $ev_h(1)=ev_h(2)=t_1+2t_2+1$. Suppose $ev_f(1)=t_1+1$, $ev_f(0)=ev_f(2)=t_1$ then $ev_h(0)=ev_h(2)=t_1+2t_2+1$, $ev_h(1)=t_1+2t_2+2$. If $ev_f(2)=t_1+1$, $ev_f(0)=ev_f(1)=t_1$ then $ev_h(0)=ev_h(1)=t_1+2t_2+1$, $ev_h(2)=t_1+2t_2+2$.
Subcase 7. $p+q\equiv 2 \pmod 3$ and $n\equiv 0 \pmod 3$.
Let $p+q=3t_1+2$ and $n=3t_2$. If $ev_f(0)=t_1$, $ev_f(1)=ev_f(2)=t_1+1$ then relabel the vertex u_{t+2} by 0. In this case $ev_h(2)=t_1+2t_2+1$, $ev_h(0)=ev_h(1)=t_1+2t_2$. Suppose $ev_f(1)=t_1$, $ev_f(0)=ev_f(2)=t_1+1$ then $ev_h(0)=ev_h(1)=t_1+2t_2$, $ev_h(2)=t_1+2t_2+1$. If $ev_f(2)=t_1$, $ev_f(0)=ev_f(1)=t_1+1$ then $ev_h(0)=ev_h(2)=t_1+2t_2$, $ev_h(1)=t_1+2t_2+1$.
Subcase 8. $p+q\equiv 2 \pmod 3$ and $n\equiv 1 \pmod 3$.
Let $p+q=3t_1+2$ and $n=3t_2+1$. If $ev_f(0)=t_1$, $ev_f(1)=ev_f(2)=t_1+1$ then $ev_h(0)=ev_h(1)=ev_h(2)=t_1+2t_2+1$. Suppose $ev_f(1)=t_1$, $ev_f(0)=ev_f(2)=t_1+1$ then relabel the vertices u_1, u_{t+3} by 1, 0 respectively. In this case $ev_h(0)=ev_h(1)=ev_h(2)=t_1+2t_2+1$. If $ev_f(2)=t_1$, $ev_f(0)=ev_f(1)=t_1+1$ then relabel the vertices u_1, u_{t+3} by 2, 0 respectively. Here also $ev_h(0)=ev_h(1)=ev_h(2)=t_1+2t_2+1$.
Subcase 9. $p+q\equiv 2 \pmod 3$ and $n\equiv 2 \pmod 3$.
Let $p+q=3t_1+2$ and $n=3t_2+2$. If $ev_f(0)=t_1$, $ev_f(1)=ev_f(2)=t_1+1$ then $ev_h(0)=t_1+2t_2+1$, $ev_h(1)=ev_h(2)=t_1+2t_2+2$. Suppose $ev_f(1)=t_1$, $ev_f(0)=ev_f(2)=t_1+1$ then $ev_h(0)=ev_h(2)=t_1+2t_2+2$, $ev_h(1)=t_1+2t_2+1$. If $ev_f(2)=t_1$, $ev_f(0)=ev_f(1)=t_1+1$ then $ev_h(0)=ev_h(1)=t_1+2t_2+2$, $ev_h(1)=t_1+2t_2+1$. □

The graph $C_n^{(t)}$ denotes the one point union of t copies of the cycle $C_n: u_1, u_2, \dots, u_n, u_1$.

Theorem 3.2. The graph $C_n^{(2)}$ is total mean cordial.

Proof. Let the two copies of the cycles be $u_1u_2\dots u_nu_1$ and $v_1v_2\dots v_nv_1$. Assume $u_i=v_j$. It is clear that $p+q=4n-1$.
Case 1. $n\le 8$.
FIGURE 1 shows that $C_n^{(2)}$ $(3\le n\le 8)$ is Total Mean Cordial.
Case 2. $n>8$.
Treat u_i as u_1, u_{i+1} as u_2 and so on. Similarly treat v_j as v_1, v_{j+1} as v_2 and so on. Assign the labels to the vertices of the two copies of the cycle C_n as in theorem 2.2.
Subcase 1. $n\equiv 0 \pmod 3$.
Let $n=3t$. In this case $ev_f(0)=4t-1$, $ev_f(1)=ev_f(2)=4t$.
Subcase 2. $n\equiv 1 \pmod 3$.

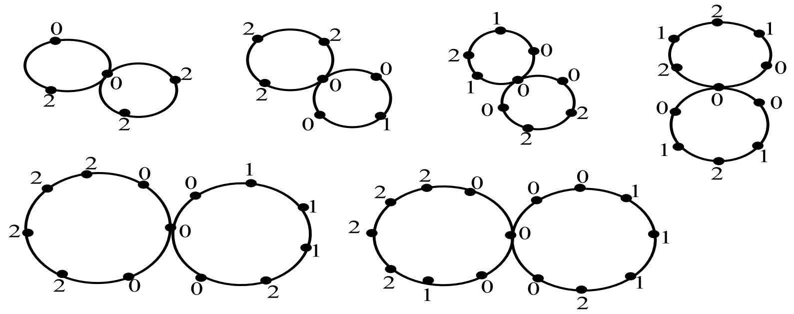

Figure 1:

Let $n = 3t + 1$. Relabel the vertices u_2, u_{t+2} and u_{t+4} by 2, 0, 0 respectively. Then we have $ev_f(0) = ev_f(1) = ev_f(2) = 4t + 1$.

Subcase 3. $n \equiv 2 \pmod 3$.

Let $n = 3t+2$. Relabel the vertex u_{t+2} by 0. Then $ev_f(0) = 4t+3$, $ev_f(1) = ev_f(2) = 4t+2$. Thus f is a total mean cordial labeling of C_n^2. □

The book B_m is the graph $S_m \times P_2$ where S_m is the star with $m + 1$ vertices.

Theorem 3.3. The book B_m is total mean cordial.

Proof. Let the vertex set of B_m be $V(B_m) = \{u, v, u_i, v_i : 1 \le i \le m\}$ and the edge set of B_m be $E(B_m) = \{uv, uu_i, vv_i, u_iv_i : 1 \le i \le m\}$. It is clear that $p + q = 5m + 3$.

Case 1. $m \equiv 0 \pmod 6$.

Let $m = 6t$ and $t > 0$. Define a function $f : V(B_m) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 2$,

$$\begin{array}{lcl} f(u_i) & = & 0, \quad 1 \le i \le 5t \\ f(u_{5t+i}) & = & 1, \quad 1 \le i \le t \\ f(v_i) & = & 2, \quad 1 \le i \le 4t \\ f(v_{4t+i}) & = & 1, \quad 1 \le i \le 2t. \end{array}$$

In this case, $ev_f(0) = ev_f(1) = ev_f(2) = 10t + 1$.

Case 2. $m \equiv 1 \pmod 6$.

Let $m = 6t - 5$ and $t > 0$. Define a map $f : V(B_m) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 2$,

$$\begin{array}{lcl} f(u_i) & = & 0, \quad 1 \le i \le 5t - 4 \\ f(u_{5t-4+i}) & = & 1, \quad 1 \le i \le t - 1 \\ f(v_i) & = & 2, \quad 1 \le i \le 4t - 3 \\ f(v_{4t-3+i}) & = & 1, \quad 1 \le i \le 2t - 2. \end{array}$$

In this case, $ev_f(0) = ev_f(2) = 10t - 7$, $ev_f(1) = 10t - 8$.

Case 3. $m \equiv 2 \pmod 6$.

Let $m = 6t - 4$ and $t > 0$. Define a function $f : V(B_m) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 2$,

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 5t-3 \\ f(u_{5t-3+i}) &= 1, \quad 1 \le i \le t-1 \\ f(v_i) &= 2, \quad 1 \le i \le 4t-3 \\ f(v_{4t-3+i}) &= 1, \quad 1 \le i \le 2t-1. \end{aligned}$$

Here, $ev_f(0) = 10t - 5$, $ev_f(1) = ev_f(2) = 10t - 6$.
Case 4. $m \equiv 3 \pmod 6$.
Let $m = 6t - 3$ and $t > 0$. When $t = 1$, the vertex labeling of B_3 given in FIGURE 2 is a total mean cordial labeling.

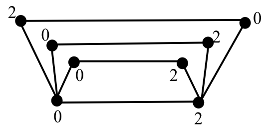

Figure 2:

Assume $t > 1$. Define a function $f : V(B_m) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 2$,

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 5t-3 \\ f(u_{5t-3+i}) &= 1, \quad 1 \le i \le t \\ f(v_i) &= 2, \quad 1 \le i \le 4t-1 \\ f(v_{4t-1+i}) &= 1, \quad 1 \le i \le 2t-3. \end{aligned}$$

and $f(v_{6t-3}) = 0$. In this case, $ev_f(0) = ev_f(1) = ev_f(2) = 10t - 4$.
Case 5. $m \equiv 4 \pmod 6$.
Let $m = 6t-2$ and $t > 0$. Define a function $f : V(B_m) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 2$,

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 5t-2 \\ f(u_{5t-2+i}) &= 1, \quad 1 \le i \le t \\ f(v_i) &= 2, \quad 1 \le i \le 4t-1 \\ f(v_{4t-1+i}) &= 1, \quad 1 \le i \le 2t-1. \end{aligned}$$

In this case, $ev_f(0) = 10t - 3$, $ev_f(1) = ev_f(2) = 10t - 2$.
Case 6. $m \equiv 5 \pmod 6$.
Let $m = 6t-1$ and $t > 0$. Define a function $f : V(B_m) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(v) = 2$,

$$\begin{aligned} f(u_i) &= 0, \quad 1 \le i \le 5t-1 \\ f(u_{5t-1+i}) &= 1, \quad 1 \le i \le t \\ f(v_i) &= 2, \quad 1 \le i \le 4t \\ f(v_{4t+i}) &= 1, \quad 1 \le i \le 2t-1. \end{aligned}$$

In this case, $ev_f(0) = ev_f(1) = 10t - 1$, $ev_f(2) = 10t$. Hence B_m is a total mean cordial graph. □

The graph $L_n = P_n \times P_2$ is called ladder.

Theorem 3.4. The ladder L_n is total mean cordial.

Proof. Let $V(L_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(L_n) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\}$. Clearly $p + q = 5n - 2$.
Case 1. $n \equiv 0 \pmod 6$.
Let $n = 6t$ and $t > 0$. Define a vertex labeling $f : V(L_n) \to \{0, 1, 2\}$ as follows:

$$\begin{array}{lcll} f(u_i) & = & 0, & 1 \leq i \leq 5t \\ f(u_{5t+i}) & = & 1, & 1 \leq i \leq t \\ f(v_i) & = & 2, & 1 \leq i \leq 5t \\ f(v_{5t+i}) & = & 1, & 1 \leq i \leq t. \end{array}$$

In this case, $ev_f(0) = ev_f(1) = 10t - 1$, $ev_f(2) = 10t$.
Case 2. $n \equiv 1 \pmod 6$.
Let $n = 6t + 1$ and $t > 0$. Define a vertex labeling $f : V(L_n) \to \{0, 1, 2\}$ as follows.

$$\begin{array}{lcll} f(u_i) & = & 0, & 1 \leq i \leq 5t+1 \\ f(u_{5t+1+i}) & = & 1, & 1 \leq i \leq t \\ f(v_i) & = & 2, & 1 \leq i \leq 5t-1 \\ f(v_{5t-1+i}) & = & 1, & 1 \leq i \leq t+1 \end{array}$$

and $f(v_{6t+1}) = 2$. Here, $ev_f(0) = ev_f(1) = ev_f(2) = 10t + 1$.
Case 3. $n \equiv 2 \pmod 6$.
Let $n = 6t - 4$ and $t > 0$. Define a vertex labeling $f : V(L_n) \to \{0, 1, 2\}$ as follows:

$$\begin{array}{lcll} f(u_i) & = & 0, & 1 \leq i \leq 5t-3 \\ f(u_{5t-3+i}) & = & 1, & 1 \leq i \leq t-1 \\ f(v_i) & = & 2, & 1 \leq i \leq 5t-4 \\ f(v_{5t-4+i}) & = & 1, & 1 \leq i \leq t. \end{array}$$

In this case, $ev_f(0) = ev_f(1) = 10t - 7$, $ev_f(2) = 10t - 8$.
Case 4. $n \equiv 3 \pmod 6$.
Let $n = 6t - 3$ and $t > 0$. Define a map $f : V(L_n) \to \{0, 1, 2\}$ by

$$\begin{array}{lcll} f(u_i) & = & 0, & 1 \leq i \leq 5t-2 \\ f(u_{5t-2+i}) & = & 1, & 1 \leq i \leq t-1 \\ f(v_i) & = & 2, & 1 \leq i \leq 5t-3 \\ f(v_{5t-3+i}) & = & 1, & 1 \leq i \leq t. \end{array}$$

In this case, $ev_f(0) = 10t - 5$, $ev_f(1) = ev_f(2) = 10t - 6$.

Case 5. $n \equiv 4 \pmod 6$.
Let $n = 6t - 2$ and $t > 0$. When $t = 1$, the graph L_4 given in FIGURE 3 is total mean cordial.

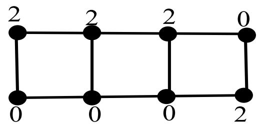

Figure 3:

Define a function $f : V(L_n) \to \{0, 1, 2\}$ by

$$\begin{array}{lcl} f(u_i) & = & 0, \quad 1 \le i \le 5t-2 \\ f(u_{5t-2+i}) & = & 1, \quad 1 \le i \le t \\ f(v_i) & = & 2, \quad 1 \le i \le 5t-3 \\ f(v_{5t-3+i}) & = & 1, \quad 1 \le i \le t-1 \end{array}$$

$f(v_{6t-3}) = 0$ and $f(v_{6t-2}) = 2$. In this case, $ev_f(0) = ev_f(1) = ev_f(2) = 10t - 4$.
Case 6. $n \equiv 5 \pmod 6$.
Let $n = 6t - 1$ and $t > 0$. Define a map $f : V(L_n) \to \{0, 1, 2\}$ by

$$\begin{array}{lcl} f(u_i) & = & 0, \quad 1 \le i \le 5t-1 \\ f(u_{5t-1+i}) & = & 1, \quad 1 \le i \le t \\ f(v_i) & = & 2, \quad 1 \le i \le 5t-1 \\ f(v_{5t-1+i}) & = & 1, \quad 1 \le i \le t. \end{array}$$

Here, $ev_f(0) = 10t - 3$, $ev_f(1) = ev_f(2) = 10t - 2$. Hence L_n is total mean cordial. $\square$

Acknowledgement. *The authors are sincerely thankful to the referees for their careful reading and constructive comments on earlier version of this manuscript, which resulted in better presentation of this article.*

References

[1] C. Barrientos and E. Krop, Mean graphs, *AKCE Int. J. Graphs Comb.,* **(1), 11** (2014) 13-26.

[2] C. Barrientos, M. E. Abdel-Aal, S. Minion, D. Williams, The mean labeling of some crowns, *Journal of Algorithms and Computation*, **45** (2014), 43 - 54.

[3] C. Barrientos, Mean trees, *Bull. Inst. Combin. Appl.,* to appear.

[4] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars combin.,* **23** (1987) 201-207.

[5] J. A. Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **18** (2015) # Ds6.

[6] F. Harary, Graph theory, *Narosa Publishing house*, New Delhi (2001).

[7] R. Ponraj, A. M. S. Ramasamy and S. Sathish Narayanan, Total mean cordial labeling of graphs, *International J.Math. Combin.* **4**(2014), 56-68.

[8] A. Rosa, On certain valuations of the vertices of a graph, *Theory of Graphs (Internat. Symposium, Rome, July 1966)*, Gordon and Breach, N. Y. and Dunod Paris (1967) 349-355.

[9] M. A. Seoud and M. A Salim, On mean graphs, *Ars Combin.,* **115** (2014) 13-34.

[10] M. A. Seoud and A. E. I. Abdel Maqsoud, On cordial and balanced labelings of graphs, *J. Egyptian Math. Soc.,* **7**(1999) 127-135.

[11] S. Somasundaram and R. Ponraj, Mean labelings of graphs, *National Academy Science letter*, **26**(2003), 210-213.