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3-difference cordial labeling of some cycle related graphs

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ABSTRACT

Let G be a (p, q) graph. Let k be an integer with $2 \leq k \leq p$ and f from $V(G)$ to the set $\{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. The function f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x $(x \in \{1, 2 \ldots, k\})$, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate the 3-difference cordial labeling of wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, and double wheel.

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1 Introduction

Graphs considered here are finite and simple. Recently Ponraj, Maria Adaickalam and Kala [3] have introduced the k-difference cordial labeling of graphs. In [3], they investigate the k-difference cordial labeling behavior of star, m copies of star etc. Also they discussed the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$. In [4, 5], Ponraj and Maria Adaickalam studied the 3-difference cordial labeling behavior of union of graphs with the star, union of graphs with splitting graph of star, union of graphs with subdivided star, union of graphs with bistar, $P_n \cup P_n$, $(C_n \odot K_1) \cup (C_n \odot K_1)$, $F_n \cup F_n$, mC_4, $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, splitting graph of a star, double fan DF_n and some other graphs. In this paper we investigate 3-difference cordial labeling of wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, and double wheel. Terms not defined here follows from Harary [2] and Gallian [1].

2 3-Difference cordial labeling

Definition 2.1. Let G be a (p, q) graph. Let f from $V(G)$ to $\{1, 2, \dots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. The map f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ denote the number of edges labelled with 1 and not labelled with 1, respectively. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

Theorem 2.1. If $n \equiv 0, 1 \pmod 3$, then the wheel W_n is 3-difference cordial.

Proof. Let $n = 3t + r$ where $0 \le r < 3$ and $r \ne 2$. Let $W_n = C_n + K_1$ where C_n is the cycle $u_1 u_2 \dots u_n u_1$ and $V(K_1) = \{u\}$. Assign the label 1 to the central vertex u. Then assign the labels 2,3,1 to the vertices u_1, u_2, u_3 respectively. Then we assign the labels 2,3,1 to the next three vertices u_4, u_5, u_6 resepctively. Continuing this way to assign the next six vertices and so on. Each time we have labeled three vertices. If $r = 0$ then we have labeled all the vertices. Otherwise, assign the label 2 to the last vertex. Note that in this process the vertex u_n received the label 1 or 2 according as $n \equiv 0 \pmod 3$ or $n \equiv 1 \pmod 3$. Clearly $e_f(0) = e_f(1) = n$ and the vertex condition is given in table 1.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{n}{3} + 1$	$\frac{n}{3}$	$\frac{n}{3}$
$n \equiv 1 \pmod 3$	$\frac{n+2}{3}$	$\frac{n+2}{3}$	$\frac{n-1}{3}$

Table 1:

Hence f is a 3-difference cordial labeling. □

Next we investigate the helm graph. The helm H_n is the graph obtained from the wheel by attaching the pendent edge at each vertex of the cycle C_n.

Theorem 2.2. Helms are 3-difference cordial.

Proof. Let $W_n = C_n + K_1$ be the wheel where C_n is the cycle $u_1u_2 \dots u_nu_1$ and $V(K_1) = \{u\}$. Let $V(H_n) = V(W_n) \cup \{v_i : 1 \le i \le n\}$ and $E(W_n) \cup \{u_iv_i : 1 \le i \le n\}$. Note that H_n has $2n+1$ vertices and $3n$ edges.
Case 1. $n \equiv 0 \pmod 3$.
Subcase 1a. $n \equiv 0 \pmod 6$.
Let $n = 6t$. Assign the labels 2,3,1,2,2,1 to the first six vertices $u_1, u_2, u_3, u_4, u_5, u_6$ of the cycle C_n. Then assign the labels 2,3,1,2,2,1 to the next six vertices $u_7, u_8 \dots u_{12}$ respectively. Proceeding like this, until we reach vertex u_n. Note that the vertex u_n received the label 1. Now our attention turn to the vertices v_i $(1 \le i \le n)$. Assign the labels 2,3,1,3,3,1 to the pendent vertices $v_1, v_2, v_3, v_4, v_5, v_6$ respectively. Then assign the labels 2,3,1,2,2,1 to the next six pendent vertices $v_7, v_8 \dots v_{12}$ respectively. Continuing this way, until we reach the vertex v_n. It is easy to verify that the vertex v_n received the label 1. Finally assign the label 2 to the vertex u.
Subcase 1b. $n \equiv 3 \pmod 6$.
As in subcase 1a, assign the label to the vertices u, u_i, v_i $(1 \le i \le n-3)$. Finally assign the labels 2,3,1 and 2,3,1 to the vertices u_{n-2}, u_{n-1}, u_n and v_{n-2}, v_{n-1}, v_n respectively. We now give the edge and vertex condition of the labeling for subcase 1 and 2. $v_f(1) = v_f(3) = \frac{2n}{3}$ and $v_f(2) = \frac{2n}{3} + 1$

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 6$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 3 \pmod 6$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

Table 2:

Case 2. $n \equiv 1 \pmod 3$.
Subcase 2a. $n \equiv 4 \pmod 6$.
Fix the labels 2,2,3,3 to the vertices u_1, u_2, u_3, u_4 respectively. Then assign the labels 2,1,3,2,2,3 to the next six vertices $u_5, u_6 \dots u_{10}$ respectively. Assign the labels 2,1,3,2,2,3 to the next six vertices $u_{11}, u_{12} \dots u_{16}$ respectively. Continuing this way assign the label to the next six vertices and so on. Next fix the labels 1,1,1,3 to the vertices v_1, v_2, v_3, v_4 respectively. Then assign the labels 2,1,3,1,1,3 to the next six vertices $v_5, v_6 \dots v_{10}$ respectively. Assign the labels 2,1,3,1,1,3 to the next six vertices $v_{11}, v_{12} \dots v_{16}$ respectively. Continuing this way assign the label to the next six vertices and so on. Finally assign the label 2 to the vertex u. The vertex condition of this labeling is $v_f(1) = v_f(2) = v_f(3) = \frac{2n+1}{3}$ and the edge condition is given in table 3.

Subcase 2b. $n \equiv 1 \pmod 6$.
As in subcase 2a, assign the label to the vertices u, u_i, v_i $(1 \le i \le n-3)$. Finally assign the labels 2,1,3 and 2,1,3 to the vertices u_{n-2}, u_{n-1}, u_n and v_{n-2}, v_{n-1}, v_n respectively.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 4 \pmod 6$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 1 \pmod 6$	$\frac{3n+1}{2}$	$\frac{3n-1}{2}$

Table 3:

Case 3. $n \equiv 2 \pmod 3$.
Subcase 3a. $n \equiv 5 \pmod 6$.
First fix the labels 1,3,2,2,3 to the vertices u_1, u_2, u_3, u_4, u_5 respectively. Then assign the labels 2,3,1,2,2,3 to the next six vertices $u_6, u_7 \ldots u_{11}$ respectively. Then assign the labels 2,3,1,2,2,3 to the next six vertices of the cycle. Proceeding like this, assign the label to the next six vertices and so on. Clearly in this process the last vertex u_n received the label 3. Next fix the labels 2,1,1,1,3 to the vertices v_1, v_2, v_3, v_4, v_5 respectively. Then assign the labels 2,3,1,1,1,3 to the next six vertices $v_6, v_7 \ldots v_{11}$ respectively and assign the labels 2,3,1,1,1,3 to the next six vertices. Continuing this way we assign the label to the next six vertices and so on. It is easy to verify that that the last vertex v_n received the label 3. Finally assign the label 2 to the vertex u.
Subcase 3b. $n \equiv 2 \pmod 6$.
As in subcase 3a, assign the label to the vertices u, u_i, v_i $(1 \le i \le n-2)$. Finally assign the labels 2,3 and 2,3 to the vertices u_{n-1}, u_n and v_{n-1}, v_n respectively.
In both subcases the vertex is $v_f(1) = v_f(2) = \frac{2n+2}{3}$, $v_f(3) = \frac{2n-1}{3}$ and edge condition is in table 4.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 2 \pmod 6$	$\frac{3n}{2}$	$\frac{3n}{2}$
$n \equiv 5 \pmod 6$	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$

Table 4:

□

Next is the flower graph. A flower is the graph obtained from a helm H_n by joining each pendent vertices to the central vertex of the helm. It is denoted by Fl_n.

Theorem 2.3. The flower graph Fl_n is 3-difference cordial.

Proof. Take the vertex and edge set of the helm as in theorem 2.2.
Case 1. $n \equiv 0 \pmod 3$.

Assign the labels 1,3,2 to the first three vertices u_1, u_2, u_3 respectively of the cycle C_n. Then assign the labels 1,3,2 to the next three vertices u_4, u_5, u_6 respectively to the cycle. Proceeding in this way, assign the labels 1,3,2 to the next three vertices of the cycle and so on. Clearly in this process the last vertex u_n received the label 2. Now consider the vertices v_i. Assign the labels 2,3,1 to the vertices v_1, v_2, v_3 respectively. Next we assign the labels 2,3,1 to the vertices v_4, v_5, v_6 respectively. Continue in this pattern assign the labels to the next three vertices respectively and so on. It is easy to verify that in this process the vertex v_n received the label 1. Finally assign the label 3 to the central vertex vertex u. The fact that this labeling is a 3-difference cordial follows from $e_f(0) = e_f(1) = 2n$ and $v_f(1) = v_f(2) = \frac{2n}{3}$, $v_f(3) = \frac{2n+3}{3}$.

Case 2. $n \equiv 1 \pmod 3$.

Assign the labels to the vertices u, u_i, v_i $(1 \le i \le n-1)$ as in case 1. Next assign the labels 1,2 to the vertices u_n, v_n respectively. Clearly in this case, $e_f(0) = e_f(1) = 2n$ and $v_f(1) = v_f(2) = v_f(3) = \frac{2n+1}{3}$.

Case 3. $n \equiv 2 \pmod 3$.

Let $n = 3t + 2$. Assign the labels 1,2,3 to the vertices u_1, u_2, u_3 respectively. Next we assign the labels 1,2,3 to the next three vertices u_4, u_5, u_6 respectively. Continuing this process, until we reach the vertex u_{3t}. Note that in this process the vertex u_{3t} received the label 3. Then assign the labels 1,2 to the vertices u_{3t+1}, u_{3t+2} respectively. Now our attention turn to the vertices v_i. Fix the labels 3,1 to the vertices v_1, v_2 respectively. Next we assign the labels 2,3,1 to the next three vertices v_3, v_4, v_5 respectively. Next assign the labels 2,3,1 to the next three vertices v_6, v_7, v_8 respectively. Proceeding like this, we assign the label to the next three vertices and so on. Clearly the vertex v_n received the label 1. Finally assign the label 3 to u. The vertex and edge condition of this labeling is given below $e_f(0) = e_f(1) = 2n$ and $v_f(1) = v_f(3) = \frac{2n+2}{3}$, $v_f(2) = \frac{2n-1}{3}$. □

Illustration 1. A 3-difference cordial labeling of the flower graph Fl_8 is in FIGURE 1.

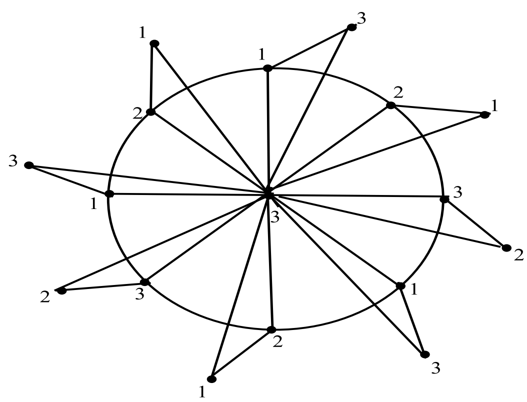

Figure 1:

The sunflower graph S_n is obtained by taking a wheel $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2 \dots u_nu_1$, $V(K_1) = \{u\}$ and new vertices $v_1, v_2 \dots v_n$ where v_i is join by the vertices $u_i, u_{i+1} \pmod{n}$.

Theorem 2.4. The sunflower graph S_n is 3-difference cordial.

Proof. **Case 1.** $n \equiv 0 \pmod 3$.
Assign the labels 1,3,2 to the vertices u_1, u_2, u_3 respectively. Next we assign the labels 1,3,2 to the next three vertices u_4, u_5, u_6 respectively. In this sequence, assign all the vertices of the cycle C_n. Clearly the last vertex u_n of the cycle received the label 2. Next we move to the vertices v_i. Assign the labels to the vertices $v_i \, (1 \le i \le n)$ in the same technique as in $u_i \, (1 \le i \le n)$. That is assign the labels 1,3,2 to the vertices v_1, v_2, v_3 and 1,3,2 to the vertices v_4v_5, v_6 respectively. Proceeding like this, assign the next three vertices and so on. Finally assign the label 2 to the central vertex u.
Case 2. $n \equiv 1 \pmod 3$.
Fix the labels 1,3,1,3 to the vertices u_1, u_2, u_3, u_4 respectively. Now we assign the labels 1,3,2 to the next three vertices u_5, u_6, u_7 respectively. Then assign the labels 1,3,2 to the next three vertices u_8, u_9, u_{10} respectively. Continuing this way, assign the label to the next three vertices and so on. Clearly in this process the vertex u_n received the label 2. Now our attention turn to the vertices v_i. Fix the label 2 to the vertex v_1. Then assign the labels 3,1,2 to the next three vertices v_2, v_3, v_4 respectively. Next we assign the labels 3,1,2 to the next three vertices v_5, v_6, v_7 respectively. Proceeding like this way, until we reach the vertex v_n. Note that 2 is the label of the last vertex v_n. Finally assign the label 2 to the central vertex.
Case 3. $n \equiv 2 \pmod 3$.
In this case fix the labels 1,3 to the vertices u_1 and u_2 respectively. Then assign the labels 1,3,2 to the next three vertices u_3, u_4, u_5 respectively. Now we assign the labels 1,3,2 to the next three vertices u_6, u_7, u_8 respectively. Continuing this pattern, until we reach the vertex u_n. It is obvious that, the label of the last vertex u_n is 2. Next our attention move to v_i. Fix the labels 3,2 to the vertices v_1, v_2 respectively. Then we assign the labels 1,3,2 to the next three vertices v_3, v_4, v_5 respectively. Now we assign the labels 1,3,2 to the next three vertices v_6, v_7, v_8 respectively. Proceeding like this we reach the vertex v_n. Clearly 2 is the label of the last vertex v_n. Finally assign the label 2 to the central vertex. The vertex and edge condition of this labeling is in table 5. In all the cases $e_f(0) = e_f(1) = 2n$. □

We now investigate the graph lotus inside a circle. The lotus inside a circle LC_n is a graph obtained from the cycle $C_n : u_1u_2 \dots u_nu_1$ and the star $K_{1,n}$ with central vertex u and the end vertices $v_1v_2 \dots v_n$ by joining each v_i to u_i and $u_{i+1} \pmod{n}$.

Theorem 2.5. The lotus inside a circle LC_n is 3-difference cordial.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n+3}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

Table 5:

Proof. **Case 1.** $n \equiv 0 \pmod 3$.
Assign the label 1 to the vertices $u_{3i-2}, v_{3i-2}\,(1 \leq i \leq \frac{n}{3})$. Then assign the label 3 to the vertices $u_{3i-1}, v_{3i-1}\,(1 \leq i \leq \frac{n}{3})$ and assign the label 2 to the vertices $u_{3i}, v_{3i}\,(1 \leq i \leq \frac{n}{3})$. Finally assign the label 2 to the central vertex u.
Case 2. $n \equiv 1 \pmod 3$.
First we fix the label 3 to the vertex v_1. Then assign the labels 1,1,3 to the next three vertices v_2, v_3, v_4 respectively. Now we assign the labels 1,1,3 to the next three vertices v_5, v_6, v_7 respectively. Continuing in this pattern, unitl reach the vertex v_n. Clearly v_n received the label 3. Now we move to the cycle vertices u_i. Fix the label 2 to the vertex u_1. Then assign the labels 2,2,3 to the next three vertices u_2, u_3, u_4 respectively. Then we assign the labels 2,2,3 to the next three vertices u_5, u_6, u_7 respectively. Proceeding like this, until we reach the last vertex u_n. Then u_n received the label 3. Finally assign the label 1 to the central vertex u.
Case 3. $n \equiv 2 \pmod 3$.
Fix the labels 2,3 to the vertices u_1 and u_2 respectively. Then we assign the labels 2,2,3 to the next three vertices u_3, u_4, u_5 respectively. We assign the labels 2,2,3 to the next three vertices u_6, u_7, u_8 respectively. Continuing this way, we reach a last cycle vertex u_n. Clearly u_n received the label 3. Now we move to the vertices $v_i\,(1 \leq i \leq n)$. Fix the labels 3,1 to the vertices v_1 and v_2 respectively. Then we assign the labels 1,1,3 to the next three vertices v_3, v_4, v_5 respectively. Next we assign the labels 1,1,3 to the next three vertices v_6, v_7, v_8 respectively. Proceeding like this, we assign the next three vertices and so on. Clearly 3 is the label of the last vertex v_n. Finally assign the label 1 to the central vertex u. Then f is a 3-difference cordial labeling follows from $e_f(0) = e_f(1) = 2n$ and the table 6.

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n+3}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$

Table 6:

□

Illustration 2. A 3-difference cordial labeling of lotus inside the circle LC_{11} is given in

FIGURE 2.

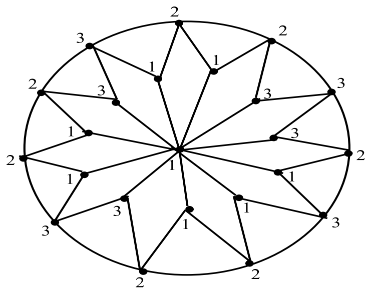

Figure 2:

Next investigation is about closed helm. Closed helm is the graph obtained from a helm by joining each pendent vertex to form a cycle.

Theorem 2.6. Closed helm CH_n is 3-difference cordial.

Proof. Let $V(CH_n) = \{u, u_i, v_i : 1 \leq i \leq n\}$ and $E(CH_n) = \{uu_i, u_iv_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1}, u_1u_n, v_1v_n : 1 \leq i \leq n-1\}$.
Case 1. $n \equiv 0 \pmod 3$.
Assign the labels 2,2,3 to the first three vertices u_1, u_2, u_3 respectively. Then assign the labels 2,2,3 to the next three vertices u_4, u_5, u_6 respectively. Proceeding like this we assign the next three vertices and so on. In this process, the last vertex u_n received the label 3. Next we move to the vertices v_i and u. Assign the labels 1,1,3 to the first three vertices v_1, v_2, v_3 respectively. Next we assign the labels 1,1,3 to the next three vertices v_4, v_5, v_6 respectively. Continuing this process until we reach the last vertex v_n. It is clear that 3 is the label of the last vertex v_n. Finally assign the label 1 to the central vertex u.
Case 2. $n \equiv 1 \pmod 3$.
Assign the labels to the vertices $u, v_i, u_i\,(1 \leq i \leq n-1)$ as in case 1. Next we assign the labels 2 and 3 to the vertices u_n and v_n respectively.
Case 3. $n \equiv 2 \pmod 3$.
As in case 2, assign the labels to the vertices $u, v_i, u_i\,(1 \leq i \leq n-1)$. Then we assign the labels 2 and 3 to the vertices u_n and v_n respectively.The fact that this labeling f is a 3-difference cordial labeling follows from the edge condition $e_f(0) = e_f(1) = 2n$ and the vertex condition given in table 7.

□

The graph $(C_n \cup C_n) + K_1$ is called the double wheel. It is denoted by DW_n. Let $V(DW_n) = V(W_n) \cup \{v_i : 1 \leq i \leq n\}$ and edge set $E(DW_n) = E(W_n) \cup \{uv_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1v_n\}$

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}+1$	$\frac{2n+3}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

Table 7:

Theorem 2.7. The double wheel DW_n is 3-difference cordial.

Proof. **Case 1.** $n \equiv 0 \pmod 3$.
Assign the labels 1,1,3 to the first three vertices u_1, u_2, u_3 respectivly. Then assign the labels 2,2,3 to the next three vertices u_4, u_5, u_6 respectively. Next we assign the labels 1,1,3 to the next three vertices u_7, u_8, u_9 respectively and assign the labels 2,2,3 to the next three vertices u_{10}, u_{11}, u_{12} respectively. Continuing this process we assign the next three vertices and so on. Note that in this case the last vertex u_n received the label 3. Next our attention move to the vertices v_i. Assign the labels 2,2,3 to the first three vertices v_1, v_2, v_3 respectivly. Then assign the labels 1,1,3 to the next three vertices v_4, v_5, v_6 respectively. Then assign the labels 2,2,3 to the next three vertices v_7, v_8, v_9 respectively and we assign the labels 1,1,3 to the next three vertices v_{10}, v_{11}, v_{12} respectively. Proceeding like this we assign the next three vertices and so on. In this case 3 is the label of the last vertex v_n. Finally assign the label 2 to the central vertex u.
Case 2. $n \equiv 1 \pmod 3$.
Subcase 2a. $n \equiv 1 \pmod 6$.
Fix the label 1 to the vertex u_1. Then assign the labels 2,2,3,1,1,3 to the next six vertices $u_2, u_3, u_4, u_5, u_6, u_7$ respectively. Next we assign the labels 2,2,3,1,1,3 to the next six vertices $u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}$ respectively. Proceeding like this we assign the next six vertices and so on. In this case the last vertex u_n received the label 3 according as $n \equiv 4 \pmod 6$ and $n \equiv 1 \pmod 6$. Next we move to the vertices v_i. Fix the label 3 to the first vertex v_1. Then we assign the labels 1,1,3,2,2,3 to the next six vertices $v_2, v_3, v_4, v_5, v_6, v_7$ respectively. Next we assign the labels 1,1,3,2,2,3 to the next six vertices $v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}$ respectively. Continuing this way we reach the last vertex v_n. Finally assign the label 2 to the central vertex u.
Subcase 2b. $n \equiv 4 \pmod 6$.
Assign the label to the vertices u, u_i, v_i $(1 \leq i \leq n-3)$ as in subcase 2a. Finally assign the labels 2,2,3 respectively to the vertices u_{n-2}, u_{n-1}, u_n and 1,1,3 to the vertices v_{n-2}, v_{n-1}, v_n respectively. Obviously this labeling pattern is a 3-difference cordial labeling.
Case 3. $n \equiv 2 \pmod 3$.
Assign the labels to the vertices u, u_i, v_i $(1 \leq i \leq n-2)$ as in case 1. Then assign the labels 1,3 and 1,3 to the vertices u_{n-1}, u_n and v_{n-1}, v_n respectively. The edge condition for these three condition is $e_f(0) = e_f(1) = 2n$ and the vertex condition given in table 8. Hence f is a 3-difference cordial labeling. □

Values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{2n}{3}$	$\frac{2n}{3}+1$	$\frac{2n}{3}$
$n \equiv 1 \pmod 3$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod 3$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$

Table 8:

Illustration 3. A 3-difference cordial labeling of DW_8 is given in FIGURE 3.

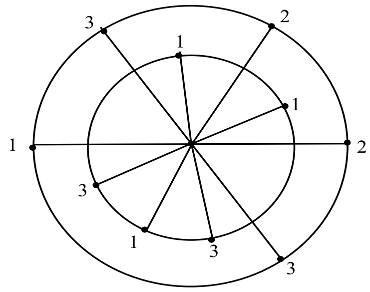

Figure 3:

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