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Further results on odd mean labeling of some subdivision graphs

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ABSTRACT

Let $G(V,E)$ be a graph with p vertices and q edges. A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0,1,2,\ldots,2q-1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1,3,5,\ldots,2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph. In this paper, we have studied an odd meanness property of the subdivision of the slanting ladder SL_n for all $n \geq 2$, $C_n \odot K_1$ for $n \geq 3$, the grid $P_m \times P_n$ for $m,n \geq 2$, $C_m@C_n$ for $m,n \geq 3$ and $P_{2m} \odot nK_1$ for all $m,n \geq 1$..

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1 Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [3].
Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. The slanting ladder SL_n is a graph obtained from two paths $u_1u_2u_3 \dots u_n$ and $v_1v_2v_3 \dots v_n$ by joining each u_i with $v_{i+1}, 1 \leq i \leq n-1$. If m number of pendant vertices are attached at each vertex of G, then the resultant graph obtained from G is the graph $G \odot mK_1$. When $m = 1$, $G \odot K_1$ is the corona of G. Let G_1 and G_2 be any two graphs with P_1 and P_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has P_1P_2 vertices which are $\{(u, v)/u \in G_1, v \in G_2\}$. The edge set of $G_1 \times G_2$ is obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The product $P_m \times P_n$ is called a planar grid. The graph $C_m@C_n$ is obtained by identifying an edge of C_m with an edge of C_n. $K_{1,m}$ is called a star graph and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{n,n}$ is often denoted by $B(n)$.
A graph which can be obtained from a given graph by breaking up each edge into one or more segments by inserting intermediate vertices between its two ends. If each edge of a graph G is broken into two by exactly one vertex, then the resultant graph is taken as $S(G)$.
The graceful labelings of graphs was first introduced by Rosa in 1961 [1] and R. B. Gnanajothi introduced odd graceful graphs [2]. The concept mean labeling was first introduced and studied by S. Somasundaram and R. ponraj [7]. Further some more results on mean graphs are discussed in [5, 6, 9, 10]. The concept of odd mean labeling was introduced and studied by K. Manickam and M. Marudai [4]. Also, some new results on odd mean graphs are discussed in [8, 11, 12, 13].
A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, \dots, 2q-1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \dots, 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph [4]. An odd mean labeling of $B_{3,3}$ is given in Figure 1.

Figure 1. An odd mean labeling of $B_{3,3}$.

2 Main Results

Theorem 2.1. *The subdivision graph of slanting ladder SL_n is an odd mean graph for $n \geq 2$.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices on the paths of length $n-1$. Let x_i, y_i and z_i be the vertices subdivided the edges u_iu_{i+1}, v_iv_{i+1} and u_iv_{i+1} respectively for each $i, 1 \leq i \leq n-1$.

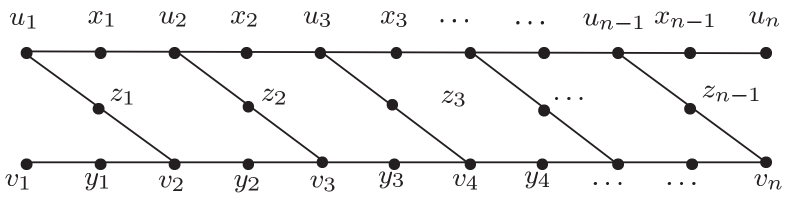

Figure 2. A subdivision of the graph SL_n.

Case (i). n is even and $n \geq 4$.
Define $f : V(S(SL_n)) \to \{0, 1, 2, 3, \ldots, 2q-1 = 12n-13\}$ as follows:

$$f(u_i) = \begin{cases} 12i-4, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 12i-8, & 1 \leq i \leq n-2 \text{ and } i \text{ is even} \\ 12n-13, & i = n \end{cases}$$

$$f(v_i) = \begin{cases} 0, & i = 1 \\ 12i-20, & 2 \leq i \leq n \text{ and } i \text{ is even} \\ 12i-16, & 2 \leq i \leq n \text{ and } i \text{ is odd} \end{cases}$$

$$f(x_i) = \begin{cases} 12i-2, & 1 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 12i+6, & 1 \leq i \leq n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 2, & i = 1 \\ 12i-6, & 2 \leq i \leq n-1 \text{ and } i \text{ is even} \\ 12i-14, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd} \end{cases}$$

$$\text{and } f(z_i) = \begin{cases} 6, & i = 1 \\ 12i-10, & 2 \leq i \leq n-1. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(u_ix_i) = \begin{cases} 12i-3, & 1 \le i \le n-1 \text{ and } i \text{ is odd} \\ 12i-1, & 1 \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(x_iu_{i+1}) = \begin{cases} 12i+1, & 1 \le i \le n-2 \text{ and } i \text{ is odd} \\ 12i+7, & 1 \le i \le n-2 \text{ and } i \text{ is even} \\ 12n-13, & i = n-1 \end{cases}$$

$$f^*(v_iy_i) = \begin{cases} 1, & i = 1 \\ 12i-13, & 2 \le i \le n-1 \text{ and } i \text{ is even} \\ 12i-15, & 2 \le i \le n-1 \text{ and } i \text{ is odd} \end{cases}$$

$$f^*(y_iv_{i+1}) = \begin{cases} 3, & i = 1 \\ 12i-5, & 2 \le i \le n-1 \text{ and } i \text{ is even} \\ 12i-11, & 2 \le i \le n-1 \text{ and } i \text{ is odd} \end{cases}$$

$$f^*(u_iz_i) = \begin{cases} 7, & i = 1 \\ 12i-9, & 2 \le i \le n-1 \text{ and } i \text{ is even} \\ 12i-7, & 2 \le i \le n-1 \text{ and } i \text{ is odd} \end{cases}$$

$$\text{and } f^*(z_iv_{i+1}) = \begin{cases} 5, & i = 1 \\ 12i-9, & 2 \le i \le n-1 \text{ and } i \text{ is odd} \\ 12i-7, & 2 \le i \le n-1 \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S(SL_n)$, for $n \ge 4$. Hence, $S(SL_n)$ is an odd mean graph. For example, an odd mean labeling of $S(SL_6)$ is shown in Figure 3.

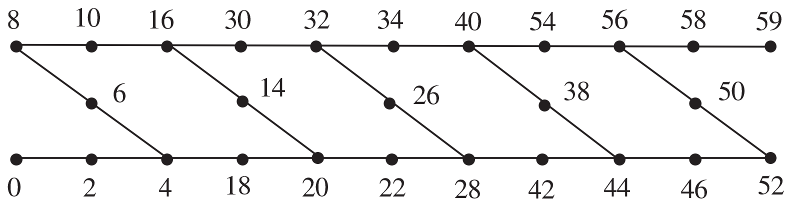

Figure 3. An odd mean labeling of $S(SL_6)$.

When $n = 2$, an odd mean labeling of the graph is given below.

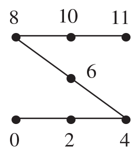

Figure 4. An odd mean labeling of $S(SL_2)$.

Case (ii). n is odd and $n \ge 5$.

Define $f : V(S(SL_n)) \rightarrow \{0, 1, 2, \ldots, 2q-1 = 12n-13\}$ as follows:

$$f(u_i) = \begin{cases} 8, & i = 1 \\ 24, & i = 2 \\ 12i-8, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 12i-4, & 3 \leq i \leq n-1 \text{ and } i \text{ is even} \\ 12n-13, & i = n \end{cases}$$

$$f(v_i) = \begin{cases} 0, & i = 1 \\ 4, & i = 2 \\ 12, & i = 3 \\ 12i-16, & 4 \leq i \leq n \text{ and } i \text{ is even} \\ 12i-20, & 1 \leq i \leq n \text{ and } i \text{ is odd} \end{cases}$$

$$f(x_i) = \begin{cases} 10, & i = 1 \\ 12i-2, & 2 \leq i \leq n-1 \text{ and } i \text{ is even} \\ 12i+6, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd} \end{cases}$$

$$f(y_i) = \begin{cases} 2, & i = 1 \\ 18, & i = 2 \\ 12i-6, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 12i-14, & 3 \leq i \leq n-1 \text{ and } i \text{ is even} \end{cases}$$

$$\text{and } f(z_i) = \begin{cases} 6, & i = 1 \\ 12i-10, & 2 \leq i \leq n-1. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_ix_i) = \begin{cases} 9, & i = 1 \\ 23, & i = 2 \\ 12i - 1, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 12i - 3, & 3 \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(x_iu_{i+1}) = \begin{cases} 17, & i = 1 \\ 12i + 1, & 2 \le i \le n-2 \text{ and } i \text{ is even} \\ 12i + 7, & 2 \le i \le n-2 \text{ and } i \text{ is odd} \\ 12n - 13, & i = n-1 \end{cases}$$

$$f^*(v_iy_i) = \begin{cases} 1, & i = 1 \\ 11, & i = 2 \\ 21, & i = 3 \\ 12i - 15, & 4 \le i \le n-1 \text{ and } i \text{ is even} \\ 12i - 13, & 4 \le i \le n-1 \text{ and } i \text{ is odd} \end{cases}$$

$$f^*(y_iv_{i+1}) = \begin{cases} 3, & i = 1 \\ 15, & i = 2 \\ 12i - 5, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 12i - 11, & 3 \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(u_iz_i) = \begin{cases} 7, & i = 1 \\ 19, & i = 2 \\ 12i - 9, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 12i - 7, & 3 \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

$$\text{and } f^*(z_iv_{i+1}) = \begin{cases} 5, & i = 1 \\ 13, & i = 2 \\ 12i - 7, & 3 \le i \le n-1 \text{ and } i \text{ is odd} \\ 12i - 9, & 3 \le i \le n-1 \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S(SL_n)$, for $n \geq 5$. Hence, $S(SL_n)$ is an odd mean graph. For example, an odd mean labeling of $S(SL_9)$ is shown in Figure 5.

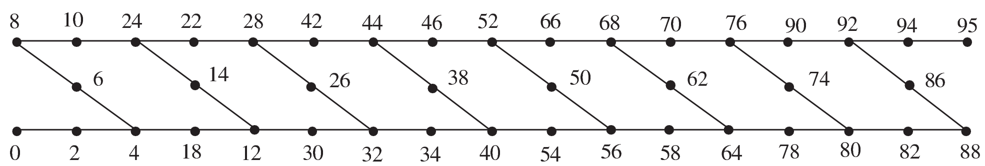

Figure 5. An odd mean labeling of $S(SL_9)$.

When $n = 3$, an odd mean labeling of the graph is given below.

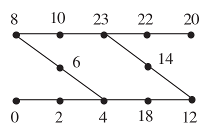

Figure 6. An odd mean labeling of $S(SL_3)$.

Hence, $S(SL_n)$ is an odd mean graph, for $n \geq 2$. □

Theorem 2.2. *The graph $S(C_n \odot K_1)$ is an odd mean graph.*

Proof. Let $u_1, v_1, u_2, v_2, \ldots, u_n$ and v_n be the vertices on the cycle and $u_i y_i x_i$ be the path on 3 vertices attached at each u_i.

Case (i). n is even.

Define $f : V(S(C_n \odot K_1)) \to \{0, 1, 2, \ldots, 2q - 1 = 8n - 1\}$ as follows:

$$f(u_i) = \begin{cases} 8i - 6, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 8i - 2, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 8i + 4, & 1 \leq i \leq \frac{n}{2} - 1 \text{ and } i \text{ is odd} \\ 8i, & 1 \leq i \leq \frac{n}{2} - 1 \text{ and } i \text{ is even} \\ 8i + 8, & \frac{n}{2} \leq i \leq n - 2 \text{ and } i \text{ is odd} \\ 8i + 4, & \frac{n}{2} \leq i \leq n - 1 \text{ and } i \text{ is even} \\ 0, & i = n \\ 8n - 1, & i = n - 1 \end{cases}$$

$$f(x_i) = \begin{cases} 8i - 2, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 8i - 6, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(y_i) = \begin{cases} 8i - 4, & 1 \leq i \leq \frac{n}{2} \text{ and } i \text{ is odd} \\ 8i - 8, & 1 \leq i \leq \frac{n}{2} \text{ and } i \text{ is even} \\ 8i, & \frac{n}{2} + 1 \leq i \leq n - 1 \text{ and } i \text{ is odd} \\ 8i - 4, & \frac{n}{2} + 1 \leq i \leq n - 1 \text{ and } i \text{ is even} \\ 8n - 4, & i = n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(u_i v_i) = \begin{cases} 8i - 1, & 1 \leq i \leq \frac{n}{2} - 1 \\ 8i + 1, & \frac{n}{2} \leq i \leq n - 1 \\ 4n - 1, & i = n \end{cases}$$

$$f^*(v_i u_{i+1}) = \begin{cases} 8i + 5, & 1 \leq i \leq \frac{n}{2} - 1 \text{ and } i \text{ is odd} \\ 8i + 1, & 1 \leq i \leq \frac{n}{2} - 1 \text{ and } i \text{ is even} \\ 8i + 7, & \frac{n}{2} \leq i \leq n - 1 \text{ and } i \text{ is odd} \\ 8i + 3, & \frac{n}{2} \leq i \leq n - 1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(u_i y_i) = \begin{cases} 8i - 5, & 1 \leq i \leq \frac{n}{2} \\ 8i - 3, & \frac{n}{2} + 1 \leq i \leq n \end{cases}$$

$$f^*(y_i x_i) = \begin{cases} 8i - 3, & 1 \leq i \leq \frac{n}{2} \text{ and } i \text{ is odd} \\ 8i - 7, & 1 \leq i \leq \frac{n}{2} \text{ and } i \text{ is even} \\ 8i - 1, & \frac{n}{2} + 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 8i - 5, & \frac{n}{2} + 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

and $f^*(v_n u_1) = 1$.

Thus, f is an odd mean labeling of $S(C_n \odot K_1)$. For example, an odd mean labeling of $S(C_8 \odot K_1)$ is shown in Figure 7.

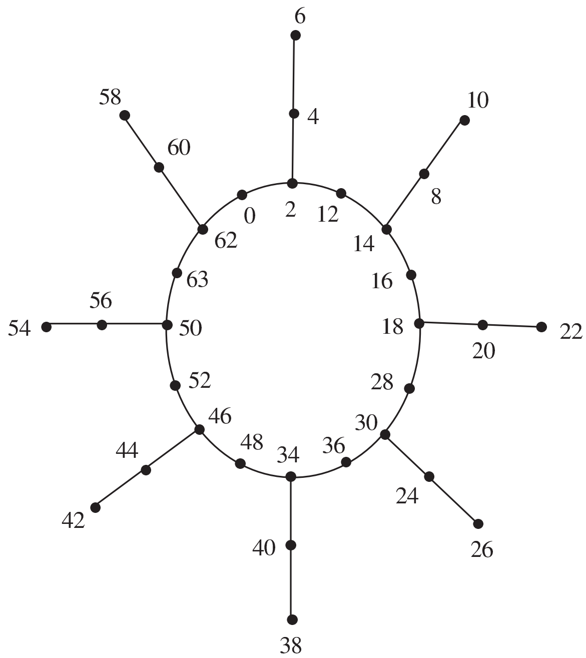

Figure 7. An odd mean labeling of $S(C_8 \odot K_1)$.

Case(ii). n is odd, $n \equiv 1(mod\ 4)$.
Define $f : V(S(C_n \odot K_1)) \rightarrow \{0, 1, 2, \ldots, 2q-1 = 8n-1\}$ as follows:

$$f(u_i) = \begin{cases} 8i-4, & 1 \leq i \leq n \text{ and } i \text{ is even} \\ 8i-8, & 1 \leq i \leq \frac{n-3}{2} \text{ and } i \text{ is odd} \\ 8i, & \frac{n-1}{2} \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 8n-4, & i = n \end{cases}$$

$$f(x_i) = \begin{cases} 8i-8, & 1 \leq i \leq \frac{n-1}{2} \text{ and } i \text{ is even} \\ 8i, & \frac{n+3}{2} \leq i \leq n \text{ and } i \text{ is even} \\ 8i-4, & 1 \leq i \leq \frac{n-3}{2} \text{ and } i \text{ is odd} \\ 8i-8, & i = \frac{n+1}{2} \\ 8i-4, & \frac{n+3}{2} \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 8n-1, & i = n \end{cases}$$

$$f(y_i) = \begin{cases} 8i-10, & 1 \leq i \leq \frac{n-1}{2} \text{ and } i \text{ is even} \\ 8i-6, & \frac{n+3}{2} \leq i \leq n \text{ and } i \text{ is even} \\ 8i-6, & 1 \leq i \leq \frac{n+1}{2} \text{ and } i \text{ is odd} \\ 8i-10, & \frac{n+3}{2} \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 8n-2, & i = n \end{cases}$$

$$\text{and } f(v_i) = \begin{cases} 8i+2, & 1 \leq i \leq \frac{n-3}{2} \text{ and } i \text{ is odd} \\ 8i-2, & \frac{n+1}{2} \leq i \leq n-2 \text{ and } i \text{ is odd} \\ 8i-2, & 1 \leq i \leq \frac{n-1}{2} \text{ and } i \text{ is even} \\ 8i+2, & \frac{n+3}{2} \leq i \leq n-1 \text{ and } i \text{ is even} \\ 8n-10, & i = n. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_iv_i) = \begin{cases} 8i-3, & 1 \le i \le \frac{n-1}{2} \\ 8i-1, & \frac{n+1}{2} \le i \le n-1 \\ 8n-7, & i = n \end{cases}$$

$$f^*(v_iu_{i+1}) = \begin{cases} 8i+3, & 1 \le i \le \frac{n-3}{2} \text{ and } i \text{ is odd} \\ 8i+1, & \frac{n+1}{2} \le i \le n-2 \text{ and } i \text{ is odd} \\ 8i-1, & 1 \le i \le \frac{n-5}{2} \text{ and } i \text{ is even} \\ 8i+3, & i = \frac{n-1}{2} \\ 8i+5, & \frac{n+3}{2} \le i \le n-3 \\ 8n-5, & i = n-1 \end{cases}$$

$$f^*(v_nu_1) = 4n-5$$

$$f^*(u_iy_i) = \begin{cases} 8i-7, & 1 \le i \le \frac{n-1}{2} \\ 8i-3, & i = \frac{n+1}{2} \\ 8i-5, & \frac{n+3}{2} \le i \le n-1 \\ 8n-3, & i = n \end{cases}$$

$$\text{and } f^*(y_ix_i) = \begin{cases} 8i-5, & 1 \le i \le \frac{n-3}{2} \text{ and } i \text{ is odd} \\ 8i-7, & \frac{n+1}{2} \le i \le n-2 \text{ and } i \text{ is odd} \\ 8n-1, & i = n \\ 8i-9, & 1 \le i \le \frac{n-1}{2} \text{ and } i \text{ is even} \\ 8i-3, & \frac{n+3}{2} \le i \le n-1 \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S(C_n \odot K_1)$.
Case(iii). n is odd, $n \equiv 3(mod\ 4)$.
Define $f : V(S(C_n \odot K_1)) \to \{0, 1, 2, \ldots, 2q-1 = 8n-1\}$ as follows:

$$f(u_i) = \begin{cases} 8i-8, & 1 \le i \le \frac{n-1}{2} \text{ and } i \text{ is odd} \\ 8i-4, & \frac{n+3}{2} \le i \le n \text{ and } i \text{ is odd} \\ 8i-4, & 1 \le i \le \frac{n-3}{2} \text{ and } i \text{ is even} \\ 8i, & \frac{n+1}{2} \le i \le n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 8i+2, & 1 \le i \le n-2 \text{ and } i \text{ is odd} \\ 8n-6, & i = n \\ 8i-2, & 1 \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f(x_i) = \begin{cases} 8i-4, & 1 \le i \le \frac{n-1}{2} \text{ and } i \text{ is odd} \\ 8i, & \frac{n+3}{2} \le i \le n-1 \text{ and } i \text{ is odd} \\ 8i-8, & 1 \le i \le \frac{n+1}{2} \text{ and } i \text{ is even} \\ 8i-4, & \frac{n+5}{2} \le i \le n-1 \text{ and } i \text{ is even} \\ 8n-1, & i = n \end{cases}$$

$$\text{and } f(y_i) = \begin{cases} 8i-6, & 1 \le i \le n-2 \text{ and } i \text{ is odd} \\ 8n-2, & i = n \\ 8i-10, & 1 \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_iv_i) = \begin{cases} 8i-3, & 1 \le i \le \frac{n-1}{2} \\ 8i-1, & \frac{n+1}{2} \le i \le n-1 \\ 8n-5, & i = n \end{cases}$$

$$f^*(v_iu_{i+1}) = \begin{cases} 8i+3, & 1 \le i \le \frac{n-5}{2} \text{ and } i \text{ is odd} \\ 8i+5, & \frac{n-1}{2} \le i \le n-2 \text{ and } i \text{ is odd} \\ 8i-1, & 1 \le i \le \frac{n-3}{2} \text{ and } i \text{ is even} \\ 8i+1, & \frac{n+1}{2} \le i \le n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(v_nu_1) = 4n-3$$

$$f^*(u_iy_i) = \begin{cases} 8i-7, & 1 \le i \le \frac{n-1}{2} \\ 8i-5, & \frac{n+1}{2} \le i \le n-1 \\ 8n-3, & i = n \end{cases}$$

$$\text{and } f^*(y_ix_i) = \begin{cases} 8i-5, & 1 \le i \le \frac{n-1}{2} \text{ and } i \text{ is odd} \\ 8i-3, & \frac{n+3}{2} \le i \le n-1 \text{ and } i \text{ is odd} \\ 8n-1, & i = n \\ 8i-9, & 1 \le i \le \frac{n+1}{2} \text{ and } i \text{ is even} \\ 8i-7, & \frac{n+5}{2} \le i \le n-1 \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S(C_n \odot K_1)$. For example, an odd mean labeling of $S(C_{11} \odot K_1)$ is shown in Figure 8.

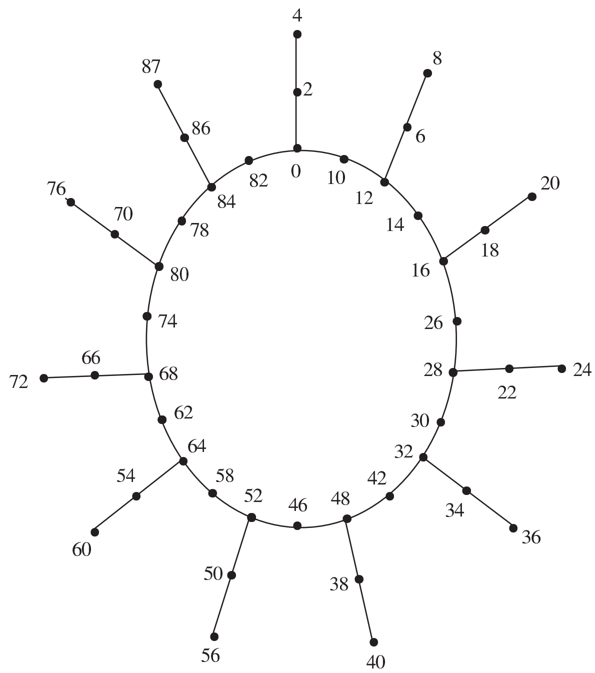

Figure 8. An odd mean labeling of $S(C_{11} \odot K_1)$.

□

Theorem 2.3. *The Grid $S(P_m \times P_n)$ is an odd mean graph for all $m \geq 2, n \geq 2$.*

Proof. Let $u_{i,j}, 1 \leq i \leq m$ and $1 \leq j \leq n$ be the vertices of the grid $P_m \times P_n$. Let $v_{i,j}$ be the vertex divides the edge $u_{i,j}u_{i,j+1}$ for each $1 \leq i \leq m$ and $1 \leq j \leq n-1$ and $w_{i,j}$ be the vertex divides the edge $u_{i,j}u_{i+1,j}$ for each $1 \leq i \leq m-1$ and $1 \leq j \leq n$.

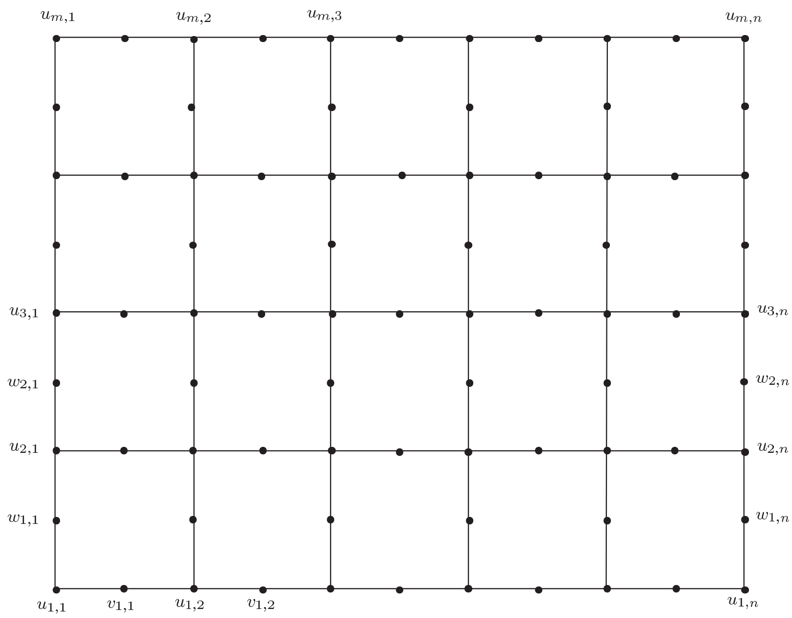

Figure 9. A subdivision of the graph $P_m \times P_n$.

Define $f : V(S(P_m \times P_n)) \to \{0, 1, 2, \ldots, 2q-1 = 8mn - 4m - 4n - 1\}$ as follows:
For $1 \leq j \leq n$,

$$f(u_{i,j}) = \begin{cases} 4(i-1)(2n-1) + 4j - 4, & 1 \leq i \leq m-1 \text{ and } i \text{ is odd,} \\ \\ 4(i-2)(2n-1) + 12n - 4j - 4, & 1 \leq i \leq m-1 \text{ and } i \text{ is even,} \end{cases}$$

$f(u_{m,j}) = 4(m-1)(2n-1) + 4j - 4, 1 \leq j \leq n-1,$
$f(u_{m,n}) = 8mn - 4m - 4n - 1.$

For $1 \leq j \leq n-1$,

$$f(v_{i,j}) = \begin{cases} 4j - 2, & i = 1 \\ 2(2i-3)(2n-1) + 8j - 4, & 3 \leq i \leq m \text{ and } i \text{ is odd} \\ 2(2i-1)(2n-1) - 8j, & 2 \leq i \leq m \text{ and } i \text{ is even,} \end{cases}$$

For $1 \leq j \leq n$,

$$f(w_{i,j}) = \begin{cases} 2(2i+1)(2n-1) - 8j + 4, & 1 \leq i \leq m-1 \text{ and } i \text{ is odd} \\ 2(2i-1)(2n-1) + 8(j-1), & 1 \leq i \leq m-1 \text{ and } i \text{ is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

For $1 \le j \le n-1$,

$$f^*(u_{i,j}v_{i,j}) = \begin{cases} 4j-3, & i=1 \\ (4i-5)(2n-1)+6j-4 & 3 \le i \le m \text{ and } i \text{ is odd} \\ (4i-5)(2n-1)+6n-6j-2, & 2 \le i \le m \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_{i,j}u_{i,j+1}) = \begin{cases} 4j-1, & i=1 \\ (4i-5)(2n-1)+6j-2, & 3 \le i \le m \text{ and } i \text{ is odd} \\ (4i-5)(2n-1)+6n-6j-4, & 2 \le i \le m \text{ and } i \text{ is even.} \end{cases}$$

For $1 \le j \le n$,

$$f^*(u_{i,j}w_{i,j}) = \begin{cases} (4i-1)(2n-1)-2j, & 1 \le i \le m-1 \text{ and } i \text{ is odd} \\ (4i-5)(2n-1)+6n+2j-6, & 1 \le i \le m-1 \text{ and } i \text{ is even,} \end{cases}$$

and

$$f^*(w_{i,j}u_{i+1,j}) = \begin{cases} (4i-1)(2n-1)+6n-6j & 1 \le i \le m-1 \text{ and } i \text{ is odd} \\ (4i-1)(2n-1)+6j-6 & 1 \le i \le m-1 \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S(P_m \times P_n)$. Hence, $S(P_m \times P_n)$ is an odd mean graph. For example, an odd mean labeling of $S(P_5 \times P_6)$ is shown in Figure 10.

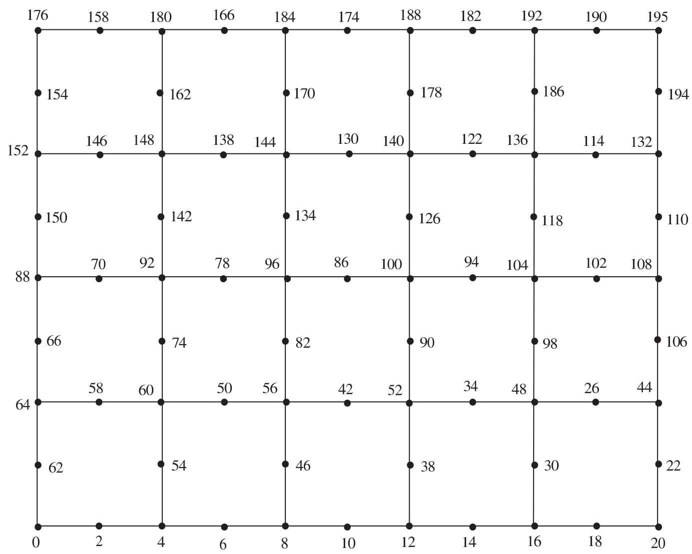

Figure 10. An odd mean labeling of $S(P_5 \times P_6)$.

□

Theorem 2.4. *The graph $S(C_m@C_n)$ is an odd mean graph for any positive integers $m, n \ge 3$.*

Proof. In $S(C_m@C_n), 2(m+n-2)$ vertices lies on the circle and one vertex lies on a chord. Let $v_1, v_2, \ldots, v_{2(m+n-2)}$ be the vertices on the cycle in $S(C_m@C_n)$

and $v_{2(m+n-2)}$ be the vertex having neighbours v_1 and $v_{2(m+n-2)-1}$ and also let $v_{2(m+n-2)+1}$ be the vertex having neighbours v_{2n-2} and $v_{2(m+n-2)}$.

Case (i). m and n are odd with $n \geq m$.

Define $f : V(S(C_m@C_n)) \to \{0, 1, 2, \dots, 2q-1 = 4m+4n-5\}$ as follows:

$$f(v_i) = \begin{cases} 4m+4n-8, & i = 1 \\ 2i-2, & 2 \leq i \leq m+2n-3 \text{ and } i \text{ is even} \\ 2i+2, & m+2n-2 \leq i \leq 2m+2n-4 \text{ and } i \text{ is even} \\ 2i-6, & 3 \leq i \leq m+n-1 \text{ and } i \text{ is odd} \\ 2i-2, & m+n \leq i \leq 2m+2n-4 \text{ and } i \text{ is odd} \\ 4m+4n-5, & i = 2m+2n-3. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2m+2n-3, & i = 1 \\ 2i-3, & 2 \leq i \leq m+n-1 \\ 2i-1, & m+n \leq i \leq m+2n-3 \\ 2i+1, & m+2n-2 \leq i \leq 2m+2n-5 \end{cases}$$

$$\begin{aligned} f^*(v_{2n-2}v_{2m+2n-3}) &= 2m+4n-5 \\ f^*(v_{2m+2n-4}v_1) &= 4m+4n-7 \text{ and} \\ f^*(v_{2m+2n-4}v_{2m+2n-3}) &= 4m+4n-5. \end{aligned}$$

Thus, f is an odd mean labeling of $S(C_m@C_n)$. Hence, $S(C_m@C_n)$ is an odd mean graph.

Case (ii). m is odd, n is even and $n \geq m+3$.

The labeling $f : V(S(C_m@C_n)) \to \{0, 1, 2, \dots, 2q-1 = 4m+4n-5\}$ as follows:

$$f(v_i) = \begin{cases} 4m+4n-8, & i = 1 \\ 2i-2, & 2 \leq i \leq m+n-1 \text{ and } i \text{ is even} \\ 2i+2, & m+n+1 \leq i \leq 2m+2n-4 \text{ and } i \text{ is even} \\ 2i-6, & 3 \leq i \leq m+2n-2 \text{ and } i \text{ is odd} \\ 2i-2, & m+2n \leq i \leq 2m+2n-4 \text{ and } i \text{ is odd} \\ 4m+4n-5, & i = 2m+2n-3. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2m+2n-3, & i = 1 \\ 2i-3, & 2 \leq i \leq m+n-1 \\ 2i-1, & m+n \leq i \leq m+2n-2 \\ 2i+1, & m+2n-1 \leq i \leq 2m+2n-5 \end{cases}$$

$$\begin{aligned} f^*(v_{2n-2}v_{2m+2n-3}) &= 2m+4n-3 \\ f^*(v_{2m+2n-4}v_1) &= 4m+4n-7 \text{ and} \\ f^*(v_{2m+2n-4}v_{2m+2n-3}) &= 4m+4n-5. \end{aligned}$$

Thus, f is an odd mean labeling of $S(C_m@C_n)$. Hence, $S(C_m@C_n)$ is an odd mean graph.

Case (iii). m is odd, $n = m + 1$.

Define $f : V(S(C_m@C_n)) \to \{0, 1, 2, \ldots, 2q - 1 = 4m + 4n - 5\}$ as follows:

$$f(v_i) = \begin{cases} 2i, & 1 \le i \le 2m + 2n - 5 \text{ and } i \text{ is odd} \\ 2(i-2), & 2 \le i \le 2n - 2 \text{ and } i \text{ is even} \\ 2i, & 2n \le i \le m + 2n - 5 \text{ and } i \text{ is even} \\ 2(i+2), & m + 2n - 4 \le i \le 2m + 2n - 6 \text{ and } i \text{ is even} \\ 4m + 4n - 5, & i = 2m + 2n - 4, \\ 4m + 4n - 6, & i = 2m + 2n - 3. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2i - 1, & 1 \le i \le 2n - 2 \\ 2i + 1, & 2n - 1 \le i \le m + 2n - 5 \\ 2i + 3, & m + 2n - 4 \le i \le 2m + 2n - 4 \end{cases}$$

$$f^*(v_{2m+2n-4} v_1) = 2m + 2n - 1 \text{ and}$$

$$f^*(v_{2n-2} v_{2m+2n-3}) = 2m + 4n - 7.$$

Thus, f is an odd mean labeling of $S(C_m@C_n)$. Hence, $S(C_m@C_n)$ is an odd mean graph. For example, an odd mean labeling of $S(C_9@C_{10})$ is shown in Figure 11.

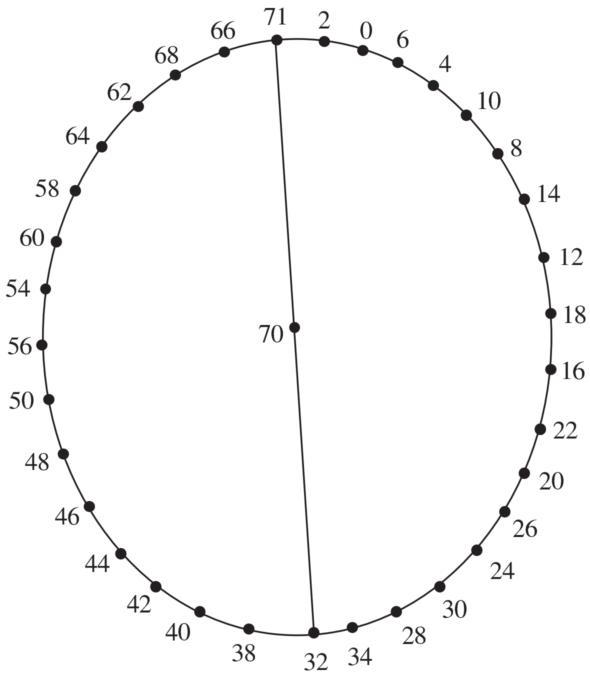

Figure 11. An odd mean labeling of $S(C_9@C_{10})$.

Case (iv). m is even and $n \ge m + 1$ is odd.

Define $f : V(S(C_m@C_n)) \to \{0, 1, 2, \ldots, 2q-1 = 4m+4n-5\}$ as follows:

$$f(v_i) = \begin{cases} 2i-2, & 1 \le i \le m+n-2 \text{ and } i \text{ is odd} \\ 2i+2, & m+n \le i \le 2m+2n-5 \text{ and } i \text{ is odd} \\ 2i-2, & 2 \le i \le m+2n-4 \text{ and } i \text{ is even} \\ 2i+2, & m+2n-3 \le i \le 2m+2n-4 \text{ and } i \text{ is even} \\ 4m+4n-5, & i = 2m+2n-3. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 2i-1, & 1 \le i \le m+n-2 \\ 2i+1, & m+n-1 \le i \le m+2n-4 \\ 2i+3, & m+2n-3 \le i \le 2m+2n-4 \end{cases}$$

$$f^*(v_{2m+2n-4}v_1) = 2m+2n-3 \text{ and}$$

$$f^*(v_{2n-2}v_{2m+2n-3}) = 2m+4n-5.$$

Thus, f is an odd mean labeling of $S(C_m@C_n)$. Hence, $S(C_m@C_n)$ is an odd mean graph.

Case (v). m is even and $n \ge m+2$ is even.

Define $f : V(S(C_m@C_n)) \to \{0, 1, 2, \ldots, 2q-1 = 4m+4n-5\}$ as follows:

$$f(v_i) = \begin{cases} 2i-2, & 1 \le i \le m+2n-3 \text{ and } i \text{ is odd} \\ 2i+2, & m+2n-2 \le i \le 2m+2n-5 \text{ and } i \text{ is odd} \\ 2i-2, & 2 \le i \le m+n-2 \text{ and } i \text{ is even} \\ 2i+2, & m+n-1 \le i \le 2m+2n-4 \text{ and } i \text{ is even} \\ 4m+4n-5, & i = 2m+2n-3. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 2i-1, & 1 \le i \le m+n-2 \\ 2i+1, & m+n-1 \le i \le m+2n-3 \\ 2i+3, & m+2n-2 \le i \le 2m+2n-4 \end{cases}$$

$$f^*(v_{2m+2n-4}v_1) = 2m+2n-3 \text{ and}$$

$$f^*(v_{2n-2}v_{2m+2n-3}) = 2m+4n-3.$$

Thus, f is an odd mean labeling of $S(C_m@C_n)$. Hence, $S(C_m@C_n)$ is an odd mean graph. For example an odd mean labeling of $S(C_4@C_6)$ is shown in Figure 12.

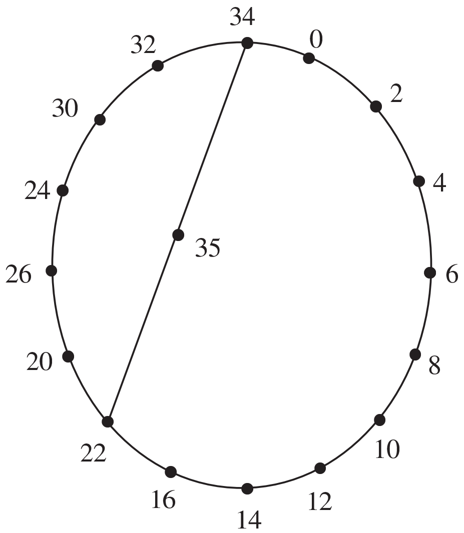

Figure 12. An odd mean labeling of $S(C_4@C_6)$

Case (vi). m is even and $n = m$.
Define $f : V(S(C_m@C_n)) \to \{0, 1, 2, \ldots, 2q - 1 = 8m - 5\}$ as follows:

$$f(v_i) = \begin{cases} 2i - 2, & 1 \leq i \leq m - 1 \text{ and } i \text{ is odd} \\ 2i + 2, & m + 1 \leq i \leq 2m - 3 \text{ and } i \text{ is odd} \\ 12m - 2i - 7, & i = 2m - 1 \\ 12m - 2i - 6, & 2m + 1 \leq i \leq 4m - 5 \text{ and } i \text{ is odd} \\ 2i - 2, & 2 \leq i \leq 2m - 2 \text{ and } i \text{ is even} \\ 12m - 2i - 6, & 2m \leq i \leq 3m - 2 \text{ and } i \text{ is even} \\ 12m - 2i - 10, & 3m \leq i \leq 4m - 4 \text{ and } i \text{ is even} \\ 4m, & i = 4m - 3. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 2i - 1, & 1 \leq i \leq m - 1 \\ 2i + 1, & m \leq i \leq 2m - 3 \\ 6m - 5, & i = 2m - 2 \\ 12m - 2i - 7, & 2m - 1 \leq i \leq 3m - 2 \\ 12m - 2i - 9, & 3m - 1 \leq i \leq 4m - 4 \end{cases}$$

$$f^*(v_{4m-4} v_1) = 2m - 1 \text{ and}$$
$$f^*(v_{2m-2} v_{4m-3}) = 4m - 3.$$

Thus, f is an odd mean labeling of $S(C_m@C_n)$. Hence, $S(C_m@C_n)$ is an odd mean graph. $\square$

Theorem 2.5. *The graph $S(P_{2m} \odot nK_1)$ is an odd mean graph for all $m, n \geq 1$.*

Proof. Let $v_1, v_2, \ldots, v_{2m}$ be the vertices of the path of length $2m - 1$ and $u_{i,1}, u_{i,2}, \ldots, u_{i,n}$ be the pendant vertices attached at $v_i, 1 \leq i \leq 2m$ in the graph

$P_{2m} \odot nK_1$. Each edge $v_iv_{i+1}, 1 \leq i \leq 2m-1$, is subdivided by a vertex $x_{i,1}$ and each pendant edge $v_iu_{i,j}, 1 \leq i \leq 2m, 1 \leq j \leq n$ is subdivided by a vertex $y_{i,j}$. Define $f : V(S(P_{2m} \odot nK_1)) \to \{0, 1, 2, \ldots, 2q-1 = 8nm + 8m - 5\}$ as follows:

$$f(v_i) = \begin{cases} 4(n+1)(i-1), & i \text{ is odd and } 1 \leq i \leq 2m-1 \\ 4[(n+1)i-1], & i \text{ is even and } 1 \leq i \leq 2m-1 \\ 8nm+8m-5, & i = 2m \end{cases}$$

$$f(x_{i,1}) = \begin{cases} 4[(n+1)i+n-1]+2, & i \text{ is odd and } 1 \leq i \leq 2m-1 \\ 4[(n+1)i-1]+2, & i \text{ is even and } 1 \leq i \leq 2m-1 \end{cases}$$

$$f(y_{i,j}) = \begin{cases} 4(n+1)(i-1)+8(j-1)+2, & i \text{ is odd, } 1 \leq i \leq 2m \\ & \text{and } 1 \leq j \leq n \\ 4[(n+1)(i-2)+1]+8(j-1)+2, & i \text{ is even, } 1 \leq i \leq 2m \\ & \text{and } 1 \leq j \leq n \end{cases}$$

$$\text{and } f(u_{i,j}) = \begin{cases} 4[(n+1)(i-1)+1]+8(j-1), & i \text{ is odd, } 1 \leq i \leq 2m \\ & \text{and } 1 \leq j \leq n \\ 4[(n+1)(i-2)+2]+8(j-1), & i \text{ is even, } 1 \leq i \leq 2m \\ & \text{and } 1 \leq j \leq n. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(v_iy_{i,j}) = 4(n+1)(i-1)+4j-3, \quad 1 \leq i \leq 2m \text{ and } 1 \leq j \leq n$$

$$f^*(y_{i,j}u_{i,j}) = \begin{cases} 4(n+1)(i-1)+8j-5, & i \text{ is odd, } 1 \leq i \leq 2m \\ & \text{and } 1 \leq j \leq n \\ 4(n+1)(i-2)+8j-1, & i \text{ is even, } 1 \leq i \leq 2m \\ & \text{and } 1 \leq j \leq n \end{cases}$$

$$f^*(v_ix_{i,1}) = 4(n+1)i-3, \quad 1 \leq i \leq 2m-1$$

$$\text{and } f^*(x_{i,1}v_{i+1}) = \begin{cases} 4(n+1)i+11, & i \text{ is odd and } 1 \leq i \leq 2m-1 \\ 4(n+1)i-1, & i \text{ is even and } 1 \leq i \leq 2m-1 \end{cases}$$

Thus, f is an odd mean labeling of $S(P_{2m} \odot nK_1)$. Hence, $S(P_{2m} \odot nK_1)$ is an odd mean graph. For example, an odd mean labeling of $S(P_4 \odot 3K_1)$ is shown in Figure 13.

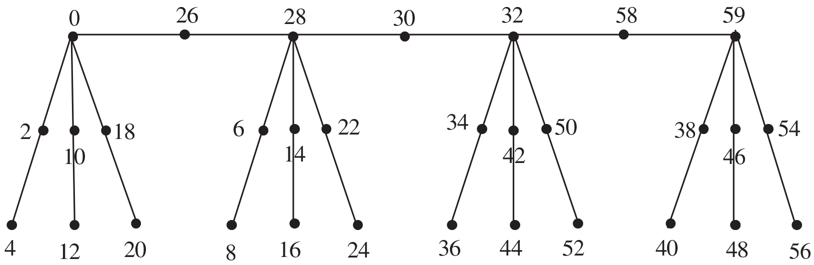

Figure 13. An odd mean labeling of $S(P_4 \odot 3K_1)$.

□

Corollary 2.6. *The subdivision of the bistar graph $B(n)$ is an odd mean graph for $n \geq 1$.*

Proof. By taking $m = 1$, in Theorem 2.5, the result follows. □

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