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Some Properties of $(1, 2)^*$-Soft b-Connected Spaces

N.Revathi*[1] and K.Bageerathi†[2]

[1]Department of Mathematics, Rani Anna Govt. College, Tirunelveli, India.
[2]Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, India.

ABSTRACT

In this paper we introduce the concept of $(1, 2)^*$-sb-separated sets and $(1, 2)^*$-soft b-connected spaces and prove some properties related to these topics. Also we disscused the properties of $(1, 2)^*$-soft b- compactness in soft bitopological space .

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1 Introduction

Soft set theory was first introduced by Molodtsov[5] in 1999 as a general mathematical tool for dealing uncertain fuzzy,not clearly defined objects.Shabir and Naz [9] initiated the study of the soft topological spaces. In 1963, J.C. Kelly [4], first initiated the concept of bitopological spaces. After then many authors studied some of basic concepts and properties of bitopological space. In 1996, Andrijevic [1] introduced a new class of open sets in a topological space called b- open sets. Metin Akdag and Alkan Ozkan [11]are defined soft b- open sets and soft b- continuous map studied their properties. In the year

*Corresponding author: N.Revathi. Email: revmurugan83@gmail.com
†mcrsm5678@rediff.mail

2014,Basavaraj M.Ittanagi [2] initiated the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of parameters.As well known, connectivity occupies very important role in topology. Many authors have presented different kinds of connectivity in general,fuzzy and soft topological spaces. The purpose of this paper is to introduce the concepts of $(1,2)^*$-soft b- connectedness and $(1,2)^*$-soft b-compactness and a detailed study of some of its properties.

2 Preliminaries

For basic notations and definitions not given here, the readers can refer [3]-[9].Throughout this paper, X is an initial universe, E is the set of parameters, $P(X)$ is the power set of X and A is a nonempty subset of E.

Definition 2.1. A soft set F_A on the universe X is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E\}$, where $f_A : E \to P(X)$ such that $f_A(x) = \phi$ if $x \in A$. Here f_A is called approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary, some of them may be empty , some may have non empty intersection. The set of all soft sets over X will be denoted by $SS_E(X)$.

Definition 2.2. [3] Let $F_A \widetilde{\in} SS_E(X)$.The soft power set of $F_A$is defined by $\widetilde{P}(A) = \left\{F_{A_i} : F_{A_i} \widetilde{\subseteq} F_A, i \in I \subseteq N\right\}$ and its cardinality is defined by $\left|\widetilde{P}(A)\right| = 2^{\sum_{x\in E}|f_A(x)|}$, where $|f_A(x)|$ is the cardinality of $f_A(x)$.

Example 2.3. [3] Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$ then $\widetilde{X} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_2\})\}$.
The possible soft subsets are
$F_{E_1} = \{(e_1, \{x_1\})\}$, $F_{E_2} = \{(e_1, \{x_2\})\}$, $F_{E_3} = \{(e_1, \{x_1, x_2\})\}$,
$F_{E_4} = \{(e_2, \{x_1\})\}$, $F_{E_5} = \{(e_2, \{x_2\})\}$, $F_{E_6} = \{(e_2, \{x_1, x_2\})\}$,
$F_{E_7} = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$, $F_{E_8} = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$,
$F_{E_9} = \{(e_1, \{x_1\}), (e_2, \{x_1, x_2\})\}$, $F_{E_{10}} = \{(e_1, \{x_2\}), (e_2, \{x_1\})\}$,
$F_{E_{11}} = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$, $F_{E_{12}} = \{(e_1, \{x_2\}), (e_2, \{x_1, x_2\})\}$,
$F_{E_{13}} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1\})\}$, $F_{E_{14}} = \{(e_1, \{x_1, x_2\}), (e_2, \{x_2\})\}$,
$F_{E_{15}} = \phi$, $F_{E_{16}} = \widetilde{X}$.

Definition 2.4. [9] Let $\widetilde{\tau}$ be the collection of soft sets over X, then $\widetilde{\tau}$ is said to be a Soft Topology on $\widetilde{X}$ if

(i) ϕ, $\widetilde{X}$ belongs to $\widetilde{\tau}$.

(ii) The soft union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

(iii) The soft intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet $(\widetilde{X}, \widetilde{\tau}, E)$ is called a Soft Topological Space over X.

Definition 2.5. [9] Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a soft topological space over X. Then the members of $\widetilde{\tau}$ are said to be soft open sets over X.

Definition 2.6. [10] Let $\widetilde{X}$ be a non-empty soft set on the universe X with a parameter set E and $\widetilde{\tau}_1$, $\widetilde{\tau}_2$ are two different soft topologies on $\widetilde{X}$. Then $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is called a soft bitopological space.

Definition 2.7. [10] Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space and $F_A \mathrel{\widetilde{\subseteq}} \widetilde{X}$. Then F_A is called $\widetilde{\tau}_{1,2}$-open if $F_A = F_B \mathrel{\widetilde{\cup}} F_C$, where $F_B \mathrel{\widetilde{\in}} \widetilde{\tau}_1$ and $F_C \mathrel{\widetilde{\in}} \widetilde{\tau}_2$. The complement of $\widetilde{\tau}_{1,2}$-open set is called $\widetilde{\tau}_{1,2}$-closed.

Definition 2.8. [7] Let $\widetilde{X}$ be a soft bitopological space and $F_A \mathrel{\widetilde{\subseteq}} \widetilde{X}$. Then F_A is called $(1,2)^*$-soft b-open set (briefly $(1,2)^*$-sb-open) if $F_A \mathrel{\widetilde{\subseteq}} \widetilde{\tau}_{1,2}$-int($\widetilde{\tau}_{1,2}$-cl($F_A$)) $\widetilde{\cup}$ $\widetilde{\tau}_{1,2}$-cl($\widetilde{\tau}_{1,2}$-int(F_A)).

Definition 2.9. [7] Let $\widetilde{X}$ be a soft bitopological space and F_A be a soft set over $\widetilde{X}$.
$(i)(1,2)^*$-soft b-closure (briefly $(1,2)^*$-sbcl(F_A)) of a set F_A in $\widetilde{X}$ is defined by
$(1,2)^*$-sbcl$(F_A) = \widetilde{\cap} \left\{ F_E \widetilde{\supseteq} F_A : F_E \ is \ a(1,2)^* - soft \ b - closed \ set \ in \widetilde{X} \right\}$.
$(ii)(1,2)^*$-soft b-interior (briefly $(1,2)^*$-sbint(F_A)) of a set F_A in $\widetilde{X}$ is defined by $(1,2)^*$-sbint$(F_A) = \widetilde{\cup} \left\{ F_B \widetilde{\subseteq} F_A : F_B \ is \ a(1,2)^* - soft \ b - open \ set \ in \widetilde{X} \right\}$.

Definition 2.10. [8] A soft mapping $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is said to be $(1,2)^*$-soft b-continuous (briefly $(1,2)^*$-sb-continuous) if the inverse image of each $\widetilde{\tau}_{1,2}$-open set of $\widetilde{Y}$ is $(1,2)^*$-soft b-open set in $\widetilde{X}$.

Definition 2.11. [8] A soft mapping $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is said to be $(1,2)^*$-soft b-irresolute (briefly $(1,2)^*$-sb-irresolute) if $\widetilde{f}^{-1}(F_A)$ is a $(1,2)^*$-soft b-closed set in $\widetilde{X}$, for every $(1,2)^*$-soft b-closed set F_A in $\widetilde{Y}$.

Theorem 2.12. *[8]* Every $(1,2)^*$- soft-continuous function is $(1,2)^*$-soft b-continuous.

Theorem 2.13. *[8]* Every $(1,2)^*$- soft b-irresolute mapping is $(1,2)^*$-soft b-continuous.

Definition 2.14. [8] A soft mapping $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is said to be $(1,2)^*$-soft b-open map (briefly $(1,2)^*$-sb-open) if the image of every $\widetilde{\tau}_{1,2}$-open set of $\widetilde{X}$ is $(1,2)^*$-soft b-open set in $\widetilde{Y}$.

Definition 2.15. [8] A soft mapping $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is said to be $(1,2)^*$-soft b-closed map (briefly $(1,2)^*$-sb-closed) if the image of every $\widetilde{\tau}_{1,2}$-closed set of $\widetilde{X}$ is $(1,2)^*$-soft b-closed set in $\widetilde{Y}$.

3 $(1,2)^*$-Soft b-connectedness

In this section we introduce the concept of $(1,2)^*$-soft b-seperated sets and $(1,2)^*$-soft b-connected space in soft bitopological space.Also we discuss some of the main results and properties.

Definition 3.1. Two non empty soft subsets F_{E_1} and F_{E_2} of $SS_E(X)$ are said to be soft disjoint if $F_{E_1} \widetilde{\cap} F_{E_2} = \phi$.

Definition 3.2. Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space. Two non-empty soft disjoint soft subsets F_{E_1} andF_{E_2} of $SS_E(X)$ are called $(1,2)^*$-soft separated sets over X if $(\widetilde{\tau}_{1,2} - cl(F_{E_1})) \widetilde{\cap} F_{E_2} = F_{E_1} \widetilde{\cap} (\widetilde{\tau}_{1,2} - cl(F_{E_2}))= \phi$.

Definition 3.3. Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space. Two non-empty soft disjoint soft subsets F_{E_1} andF_{E_2} of $SS_E(X)$ are called $(1,2)^*$-soft b-separated ($(1,2)^*$-sb-separated) sets over X if $((1,2)^*$-sbcl$(F_{E_1})) \widetilde{\cap} F_{E_2} = F_{E_1} \widetilde{\cap} ((1,2)^*$-sbcl$(F_{E_2}))= \phi$.

Remark 3.4. From the fact that $(1,2)^*$-sbcl$(F_E) \widetilde{\subset} (\widetilde{\tau}_{1,2} - cl(F_E))$, for every soft subset F_E of $\widetilde{X}$, every $(1,2)^*$-soft separated set is $(1,2)^*$-soft b-separated. But the converse may not be true .

Definition 3.5. A $(1,2)^*$-soft b-separation ($(1,2)^*$-sb-separation) of a soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is a pair of $(1,2)^*$-soft b-separated sets F_{E_1} andF_{E_2} whose soft union is $\widetilde{X}$.

Definition 3.6. Let$(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space.Then $\widetilde{X}$ is called $(1,2)^*$-soft b-connected space if $\widetilde{X}$ cannot be expressed as the soft union of two $(1,2)^*$-soft b-seperated sets.

Remark 3.7. (i) In a soft bitopological space soft empty set is trivially $(1,2)^*$-soft b-connected set.

(ii) In a soft bitopological space every soft singleton set is $(1,2)^*$-soft b-connected since it cannot be expressed as a soft union of tow non-empty $(1,2)^*$-soft b-seperated sets.

Theorem 3.8. Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space.Then the following statements are equivalent:

(i) $\widetilde{X}$ is a $(1,2)^*$-soft b-connected space.

(ii) $\widetilde{X}$ and ϕ are the only $(1,2)^*$-soft b-clopen sets in $\widetilde{X}$.

(iii) $\widetilde{X}$ cannot be expressed as the union of two disjoint non-empty $(1,2)^*$-soft b-open sets.

(iv) $\widetilde{X}$ cannot be expressed as the union of two disjoint non-empty $(1,2)^*$-soft b-closed sets.

Proof. $(i) \to (ii)$ Let $\widetilde{X}$ be $(1,2)^*$-soft b-connected space. Let F_E be non-empty proper subset of $\widetilde{X}$ that is $(1,2)^*$-soft b-clopen . Then $\widetilde{X \setminus} F_E$ is a non-empty $(1,2)^*$-soft b-clopen set and $\widetilde{X} = F_E \widetilde{\cup} (\widetilde{X \setminus} F_E)$. This is a contradiction to $\widetilde{X}$ is a $(1,2)^*$-soft b-connected

space. Therefore $\widetilde{X}$ and ϕ are the only $(1,2)^*$-soft b-clopen sets in $\widetilde{X}$.
$(ii) \rightarrow (iii)$ Assume that $\widetilde{X}$ and ϕ are the only $(1,2)^*$-soft b-clopen sets in $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$. suppose (iii) is false. Then $\widetilde{X} = F_{E_1} \widetilde{\cup} F_{E_2}$ where F_{E_1} and F_{E_2} are disjoint non-empty $(1,2)^*$-soft b-open sets. Then $F_{E_2} = \widetilde{X} \widetilde{\setminus} F_{E_1}$ is $(1,2)^*$-soft b- closed and non-empty. Thus F_{E_2} is a non-empty proper $(1,2)^*$-soft b-clopen sets in $\widetilde{X}$, which contradicts (ii).
$(iii) \rightarrow (iv)$ Assume $\widetilde{X}$ cannot be expressed as the union of two disjoint non-empty $(1,2)^*$-soft b-open sets.Suppose (iv) false. Then $\widetilde{X} = F_{E_1} \widetilde{\cup} F_{E_2}$ where F_{E_1} and F_{E_2} are disjoint non-empty $(1,2)^*$-soft b-closed sets. Then $F_{E_1} = \widetilde{X} \widetilde{\setminus} F_{E_2}$ and $F_{E_2} = \widetilde{X} \widetilde{\setminus} F_{E_1}$ are disjoint non-empty $(1,2)^*$-soft b-open sets in $\widetilde{X}$. Thus $\widetilde{X}$ is the soft union of two soft disjoint non-empty $(1,2)^*$-soft b-open sets . This contradicts (iii).
$(iv) \rightarrow (i)$ Suppose $\widetilde{X}$ is not $(1,2)^*$-soft b-connected space. Then $\widetilde{X} = F_{E_1} \widetilde{\cup} F_{E_2}$ where F_{E_1} and F_{E_2} are disjoint non-empty $(1,2)^*$-soft b-open sets. Then $F_{E_1} = \widetilde{X} \widetilde{\setminus} F_{E_2}$ and $F_{E_2} = \widetilde{X} \widetilde{\setminus} F_{E_1}$ are disjoint non-empty $(1,2)^*$-soft b-closed sets in $\widetilde{X}$. This is a contradiction to (iv). □

Proposition 3.9. Every $(1,2)^*$-soft b-connected space is $(1,2)^*$-soft connected.

Proof. Let F_E be a $(1,2)^*$-soft b-connected set in the soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$. Then there does not exist a $(1,2)^*$-soft b-separation of F_E . Since every $\widetilde{\tau}_{1,2}$-open set is a $(1,2)^*$-soft b-open set, there does not exist a $(1,2)^*$-soft separation of F_E . Hence F_E is a $(1,2)^*$-soft connected set in in the soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$. □

The converse is not true as shown in the following example.

Example 3.10. $(1,2)^*$-soft connectedness does not imply $(1,2)^*$-soft b-connectedness. Let $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a soft bitopological space, where $\widetilde{X}$ and its soft subsets are considered as in Example 2.3 .
Let $\widetilde{\tau}_1 = \left\{\widetilde{X}, \phi, F_{E_1}, F_{E_7}, F_{E_{13}}\right\}$ and $\widetilde{\tau}_2 = \left\{\widetilde{X}, \phi, F_{E_4}, F_{E_{10}}\right\}$. Then $\widetilde{\tau_{1,2}}$-open sets are $\left\{\widetilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_7}, F_{E_{10}}, F_{E_{13}}\right\}$ and $\widetilde{\tau_{1,2}}$-closed sets are $\left\{\widetilde{X}, \phi, F_{E_{12}}, F_{E_{14}}, F_{E_{11}}, F_{E_8}, F_{E_5}\right\}$. It is clear that $\widetilde{X}$is $(1,2)^*$-soft-connected since the only $(1,2)^*$-soft-clopen sets are ϕ and $\widetilde{X}$. Also $(1,2)^*$-soft b-open sets are $\left\{\widetilde{X}, \phi, F_{E_1}, F_{E_4}, F_{E_6}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{10}}, F_{E_{12}}, F_{E_{13}}\right\}$ and $(1,2)^*$-soft b-closed sets are $\left\{\widetilde{X}, \phi, F_{E_{12}}, F_{E_{14}}, F_{E_3}, F_{E_{11}}, F_{E_{10}}, F_{E_2}, F_{E_8}, F_{E_1}, F_{E_5}\right\}$.
Let $\widetilde{X} = F_{E_1} \widetilde{\cup} F_{E_{12}}$,then $(1,2)^*$-sbcl$(F_{E_1}) = F_{E_1}$, $(1,2)^*$-sbcl$(F_{E_{12}}) = F_{E_2}$ and $(1,2)^*$-sbcl$(F_{E_1}) \widetilde{\cap} F_{E_{12}} = \phi$, $(1,2)^*$-sbcl$(F_{E_{12}}) \widetilde{\cap} F_{E_1} = \phi$. Hence $\widetilde{X}$ can be expressed as a union of two $(1,2)^*$-soft b-seperated sets F_{E_1} and $F_{E_{12}}$. Therefore $\widetilde{X}$ is not $(1,2)^*$-soft b-connected.

Example 3.11. $(1,2)^*$-soft b-connectivity is not a hereditary property.
Consider the soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2)$,where $\widetilde{X}$ and its soft subsets are considered as in Example 2.3 . Let $\widetilde{\tau}_1 = \left\{\widetilde{X}, \phi, F_{E_1}, F_{E_7}\right\}$ and $\widetilde{\tau}_2 = \left\{\widetilde{X}, \phi, F_{E_3}\right\}$. Then $\widetilde{\tau_{1,2}}$-open

set $= \left\{\widetilde{X}, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_{13}}\right\}$ .Also $(1,2)^*$-soft b-open sets are $\left\{\widetilde{X}, \phi, F_{E_1}, F_{E_3}, F_{E_7}, F_{E_8}, F_{E_9}, F_{E_{13}}, F_{E_{14}},\right\}$ and $(1,2)^*$-soft b-closed sets are $\left\{\widetilde{X}, \phi, F_{E_{12}}, F_{E_6}, F_{E_{11}}, F_{E_{10}}, F_{E_2}, F_{E_5}, F_{E_4},\right\}$.It is clear that $\widetilde{X}$ is $(1,2)^*$-soft b-connected, since the only $(1,2)^*$-soft b-clopen sets are ϕ and $\widetilde{X}$.
Let $Y = \{x_1\} \ \widetilde{\subseteq}\ X$ and $E = \{e_1, e_2\}$. Then $\widetilde{Y} = \{(e_1, \{x_1\}), (e_2, \{x_1\})\} = F_{E_7}$.
Let $\widetilde{\sigma_1} = \left\{\widetilde{Y}, \phi, F_{E_1}\right\}$ and $\widetilde{\sigma_2} = \left\{\widetilde{Y}, \phi, F_{E_4}\right\}$. Then $\widetilde{\sigma_{1,2}}$-open set $= \left\{\widetilde{Y}, \phi, F_{E_1}, F_{E_4}\right\}$. Also $(1,2)^*$-soft b-clopen sets are $\left\{\widetilde{Y}, \phi, F_{E_1}, F_{E_4}\right\}$ .Clearly $\widetilde{Y}$ is not $(1,2)^*$-soft b-connected, since F_{E_1}, F_{E_4} are $(1,2)^*$-soft b-clopen sets other than ϕ and $\widetilde{Y}$.

Proposition 3.12. Let F_E be $(1,2)^*$-soft b-connected set and F_{E_1} and F_{E_2} are $(1,2)^*$-soft b-seperated sets. If $F_E \ \widetilde{\subseteq}\ F_{E_1} \widetilde{\cup} F_{E_2}$ then either $F_E \ \widetilde{\subseteq}\ F_{E_1}$ or $F_E \ \widetilde{\subseteq}\ F_{E_2}$.

Proof. Let F_E be $(1,2)^*$-soft b-connected set and F_{E_1} and F_{E_2} are $(1,2)^*$-soft b-seperated sets such that $F_E \widetilde{\subseteq} F_{E_1} \ \widetilde{\cup}\ F_{E_2}$. Let $F_E \ \widetilde{\nsubseteq}\ F_{E_1}$ and $F_E \ \widetilde{\nsubseteq}\ F_{E_2}$. Suppose $G_E = F_{E_1} \ \widetilde{\cap}\ F_E \neq \phi$ and $H_E = F_{E_2} \ \widetilde{\cap}\ F_E \neq \phi$ then $F_E = G_E \ \widetilde{\cup}\ H_E$. Since $G_E \ \widetilde{\subseteq}\ F_{E_1}$, $((1,2)^*\text{-sbcl}(G_E)) \ \widetilde{\subseteq}\ ((1,2)^*\text{-sbcl}(F_{E_1}))$. Also $((1,2)^*\text{-sbcl}(F_{E_1})) \ \widetilde{\cap}\ F_{E_2} = \phi$ then $((1,2)^*\text{-sbcl}(G_E)) \ \widetilde{\cap}\ H_E = \phi$. Since $H_E \ \widetilde{\subseteq}\ F_{E_2}, ((1,2)^*\text{-sbcl}(H_E)) \ \widetilde{\subseteq}\ ((1,2)^*\text{-sbcl}(F_{E_2}))$. Also $((1,2)^*\text{-sbcl}(F_{E_2})) \ \widetilde{\cap}\ F_{E_1} = \phi$ then $((1,2)^*\text{-sbcl}(H_E)) \ \widetilde{\cap}\ G_E = \phi$. But $F_E = G_E \ \widetilde{\cup}\ H_E$, therefore F_E is not $(1,2)^*$-soft b-connected space.This is a contradiction.Then either $F_E \ \widetilde{\subseteq}\ F_{E_1}$ or $F_E \ \widetilde{\subseteq}\ F_{E_2}$. □

Theorem 3.13. If F_E be $(1,2)^*$-soft b-connected set and $F_E \ \widetilde{\subseteq}\ G_E \ \widetilde{\subseteq}\ ((1,2)^*\text{-sbcl}(F_E))$ then G_E is $(1,2)^*$-soft b-connected.

Proof. Suppose G_E is not $(1,2)^*$-soft b-connected then there exists two non-empty soft sets F_{E_1} and F_{E_2} such that $((1,2)^*\text{-sbcl}(F_{E_1})) \ \widetilde{\cap}\ F_{E_2} = F_{E_1} \ \widetilde{\cap}\ ((1,2)^*\text{-sbcl}(F_{E_2})) = \phi$ and $G_E = F_{E_1} \ \widetilde{\cup}\ F_{E_2}$ .Since $F_E \ \widetilde{\subseteq}\ G_E$ then either $F_E \ \widetilde{\subseteq}\ F_{E_1}$ or $F_E \ \widetilde{\subseteq}\ F_{E_2}$ .Suppose $F_E \ \widetilde{\subseteq}\ F_{E_1}$ then $((1,2)^*\text{-sbcl}(F_E)) \ \widetilde{\subseteq}\ ((1,2)^*\text{-sbcl}(F_{E_1}))$, thus $((1,2)^*\text{-sbcl}(F_E)) \ \widetilde{\cap}\ F_{E_2} = F_E \ \widetilde{\cap}\ ((1,2)^*\text{-sbcl}(F_{E_2})) = \phi$. But $F_{E_2} \ \widetilde{\subseteq} G_E \ \widetilde{\subseteq}\ ((1,2)^*\text{-sbcl}(F_E))$,thus $((1,2)^*\text{-sbcl}(F_E)) \ \widetilde{\cap} F_{E_2} = F_{E_2}$. Therefore $F_{E_2} = \phi$, which is a contradiction . If $F_E \ \widetilde{\subseteq}\ F_{E_2}$, then by the same way we can prove that $F_{E_1} = \phi$ .This is a contradiction.Thus G_E is $(1,2)^*$-soft b-connected. □

Theorem 3.14. If F_E be $(1,2)^*$-soft b-connected set then $(1,2)^*\text{-sbcl}(F_E)$ is $(1,2)^*$-soft b-connected .

Proof. Suppose F_E is $(1,2)^*$-soft b-connected and $(1,2)^*\text{-sbcl}(F_E)$ is not $(1,2)^*$-soft b-connected. Then there exists two $(1,2)^*$-soft b-seperated sets F_{E_1} and F_{E_2} such that $(1,2)^*\text{-sbcl}(F_E) = F_{E_1} \ \widetilde{\cup}\ F_{E_2}$ .But $F_E \ \widetilde{\subseteq}\ (1,2)^*\text{-sbcl}(F_E)$ then $F_E \ \widetilde{\subseteq} F_{E_1} \ \widetilde{\cup} F_{E_2}$ and since F_E is $(1,2)^*$-soft b-connected set then either $F_E \ \widetilde{\subseteq}\ F_{E_1}$ or $F_E \ \widetilde{\subseteq} F_{E_2}$.
(i) If $F_E \ \widetilde{\subseteq}\ F_{E_1}$ then $((1,2)^*\text{-sbcl}(F_E)) \ \widetilde{\subseteq}\ ((1,2)^*\text{-sbcl}(F_{E_1}))$. But $(1,2)^*\text{-sbcl}(F_{E_1}) \ \widetilde{\cap}\ F_{E_2} = \phi$, hence $(1,2)^*$-soft $\text{bcl}(F_E) \ \widetilde{\cap} F_{E_2} = \phi$. Since $F_{E_2} \ \widetilde{\subseteq} (1,2)^*\text{-sbcl}(F_E)$, then$(1,2)^*$-soft $\text{bcl}(F_E) \ \widetilde{\cap} F_{E_2} = F_{E_2}$ hence $F_{E_2} = \phi$ which is a contradiction.
(ii) If $F_E \ \widetilde{\subseteq} F_{E_2}$ then the same way we can prove that $F_{E_1} = \phi$ which is a contradiction. Therefore $(1,2)^*\text{-sbcl}(F_E)$ is $(1,2)^*$-soft b-connected. □

Theorem 3.15. The soft union F_E of any family $\{F_{E_i} : i \in I\}$ of $(1,2)^*$-soft b-connected sets having a nonempty soft intersection is $(1,2)^*$-soft b-connected set.

Proof. Let F_E be a soft union of any family of $(1,2)^*$-soft b-connected sets having a non-empty soft intersection. Suppose that $F_E = F_{E_1} \widetilde{\cup} F_{E_2}$, where F_{E_1} and F_{E_2} form a $(1,2)^*$-soft b-separation of F_E. By hypothesis, we may choose a soft point $x_e \widetilde{\in} \widetilde{\cap}_{i\in I} F_{E_i}$. Then $x_e \widetilde{\in} F_{E_i}$ for all $i \in I$. If $x_e \widetilde{\in} F_E$, then either $x_e \widetilde{\in} F_{E_1}$ or $x_e \widetilde{\in} F_{E_2}$ but not both. Since, F_{E_1} and F_{E_2} are soft disjoint, we must have $F_{E_i} \widetilde{\subseteq} F_{E_1}$, since F_{E_i} is $(1,2)^*$-soft b-connected and it is true for all $i \in I$, and so $F_E \widetilde{\subseteq} F_{E_1}$. From this we obtain that $F_{E_2} = \phi$, which is a contradiction. Thus, there does not exist a $(1,2)^*$-soft b-separation of F_E. Therefore, F_E is $(1,2)^*$-soft b-connected set. $\square$

Theorem 3.16. (i) If $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is a $(1,2)^*$-soft b-continuous surjection and $\widetilde{X}$ is $(1,2)^*$-soft b-connected then $\widetilde{Y}$ is $(1,2)^*$-soft connected.

(ii) If $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is a $(1,2)^*$-soft b-irresolute surjection and $\widetilde{X}$ is $(1,2)^*$-soft b-connected then $\widetilde{Y}$ is $(1,2)^*$-soft b-connected.

Proof. (i) Suppose $\widetilde{Y}$ is not $(1,2)^*$-soft connected. Let $\widetilde{Y} = F_E \widetilde{\cup} G_E$ where F_E and G_E are disjoint non-empty $\widetilde{\tau}_{1,2}$-open set in $\widetilde{Y}$. Since $\widetilde{f}$ is $(1,2)^*$-soft b-continuous and onto $\widetilde{X} = \widetilde{f}^{-1}(F_E) \widetilde{\cup} \widetilde{f}^{-1}(G_E)$ where $\widetilde{f}^{-1}(F_E)$ and $\widetilde{f}^{-1}(G_E)$ are disjoint non-empty $(1,2)^*$-soft b-open in $\widetilde{X}$. This contradicts the fact that $\widetilde{X}$ is $(1,2)^*$-soft b-connected . Hence $\widetilde{Y}$ is $(1,2)^*$-soft connected .

(ii) Suppose $\widetilde{Y}$ is not $(1,2)^*$-soft b- connected. Let $\widetilde{Y} = F_E \widetilde{\cup} G_E$ where F_E and G_E are disjoint non-empty $(1,2)^*$-soft b-open set in $\widetilde{Y}$. Since $\widetilde{f}$ is $(1,2)^*$-soft b-irresolute and onto $\widetilde{X} = \widetilde{f}^{-1}(F_E) \widetilde{\cup} \widetilde{f}^{-1}(G_E)$ where $\widetilde{f}^{-1}(F_E)$ and $\widetilde{f}^{-1}(G_E)$ are disjoint non-empty $(1,2)^*$-soft b-open in $\widetilde{X}$. This contradicts the fact that $\widetilde{X}$ is $(1,2)^*$-soft b-connected . Hence $\widetilde{Y}$ is $(1,2)^*$-soft b-connected .

$\square$

4 $(1,2)^*$-Soft b-Compactness

In this section $(1,2)^*$-soft b-compactness is defined and some of the charaterizations are proved.

Definition 4.1. A collection $\{F_{E_i} : i \in \Lambda\}$ of $(1,2)^*$-soft b-open sets in soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is called a $(1,2)^*$-soft b-open cover of a subset F_E if $F_E \widetilde{\subset} \widetilde{\bigcup} \{F_{E_i} : i \in \Lambda\}$.

Definition 4.2. A soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is called a $(1,2)^*$-soft b-compact if every $(1,2)^*$-soft b-open cover of $\widetilde{X}$ has a finite subcover.

Definition 4.3. A subset F_E of a soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said to be $(1,2)^*$-soft b-compact relative to $\widetilde{X}$, if for every collection $\{F_{E_i} : i \in \Lambda\}$ of $(1,2)^*$-soft b-open subsets of $\widetilde{X}$ such that $F_E \mathrel{\widetilde{\subset}} \widetilde{\bigcup}\{F_{E_i} : i \in \Lambda\}$ there exists a finite subset Λ_0 of Λ such that $F_E \mathrel{\widetilde{\subset}} \widetilde{\bigcup}\{F_{E_i} : i \in \Lambda_0\}$.

Definition 4.4. A subset F_E of a soft bitopological space $(\widetilde{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said to be $(1,2)^*$-soft b-compact if F_E is $(1,2)^*$-soft b-compact as a subspace of $\widetilde{X}$.

Theorem 4.5. Every $(1,2)^*$-soft b-closed subset of a $(1,2)^*$-soft b-compact space $\widetilde{X}$ is $(1,2)^*$-soft b-compact relative to $\widetilde{X}$.

Proof. Let F_E be a $(1,2)^*$-soft b-closed subset of $\widetilde{X}$. Then $F_E^{\widetilde{C}}$ is $(1,2)^*$-soft b-open set in $\widetilde{X}$. Let $S = \{G_{E_i} : i \in \Lambda\}$ be a cover of F_E by $(1,2)^*$-soft b-open subsets in $\widetilde{X}$. Then $S \mathrel{\widetilde{\cup}} F_E^{\widetilde{C}}$ is a $(1,2)^*$-soft b-open cover for $\widetilde{X}$. Since $\widetilde{X}$ is a $(1,2)^*$-soft b- compact, it has a finite sub cover say $\widetilde{X} = G_{E_1} \widetilde{\cup} G_{E_2} \widetilde{\cup} .... \widetilde{\cup} G_{E_n} \widetilde{\cup} F_E^{\widetilde{C}}$, $G_{E_i} \widetilde{\in} S$. But F_E and $F_E^{\widetilde{C}}$ are disjoint. Hence $F_E \widetilde{\subset} G_{E_1} \widetilde{\cup} G_{E_2} \widetilde{\cup} .... \widetilde{\cup} G_{E_n} \widetilde{\in} S$. Thus we have shown that any $(1,2)^*$-soft b-open cover of $\widetilde{S}$ of F_E contains a finite sub cover. Therefore F_E is $(1,2)^*$-soft b-compact relative to $\widetilde{X}$. $\square$

Theorem 4.6. A $(1,2)^*$-soft b-continuous image of a $(1,2)^*$-soft b-compact space is $(1,2)^*$-soft compact.

Proof. Let $\{F_{E_i} : i \in \Lambda\}$ be an $\widetilde{\tau}_{1,2}$-open cover of $\widetilde{Y}$. Then $\{f^{-1}(F_{E_i}) : i \in \Lambda\}$ is a $(1,2)^*$-soft b-open cover of $\widetilde{X}$. Since $\widetilde{X}$ is $(1,2)^*$-soft b-compact , it has a finite subcover say, $\{f^{-1}(F_{E_1}), f^{-1}(F_{E_2}), ...f^{-1}(F_{E_n})\}$. Since $\widetilde{f}$ is onto, $\{F_{E_1}, F_{E_2}, ...F_{E_n}\}$ is a $\widetilde{\tau}_{1,2}$-open cover of $\widetilde{Y}$ and hence $\widetilde{Y}$ is $(1,2)^*$-soft compact. $\square$

Theorem 4.7. If a map $\widetilde{f} : \widetilde{X} \to \widetilde{Y}$ is $(1,2)^*$-soft b-irresolute and a subset F_E of $\widetilde{X}$ is $(1,2)^*$-soft b-compact relative to $\widetilde{X}$ then the image $\widetilde{f}(F_E)$ is is $(1,2)^*$-soft b-compact relative to $\widetilde{Y}$.

Proof. Let $\{F_{E_i} : i \in \Lambda\}$ be a collection of $(1,2)^*$-soft b-open sets in $\widetilde{Y}$ such that $\widetilde{f}(F_E) \mathrel{\widetilde{\subset}} \widetilde{\bigcup}\ \{F_{E_i} : i \in \Lambda\}$. Then $F_E \mathrel{\widetilde{\subset}} \widetilde{\bigcup}\left\{\widetilde{f}^{-1}(F_{E_i}) : i \in \Lambda\right\}$ where $f^{-1}(F_{E_i})$ is a $(1,2)^*$-soft b-open in $\widetilde{X}$ for each i. Since F_E of $\widetilde{X}$ is $(1,2)^*$-soft b-compact relative to $\widetilde{X}$, there exists a finite sub collection $\{F_{E_1}, F_{E_2}, ...F_{E_n}\}$ such that $F_E \widetilde{\subset} \bigcup\left\{\widetilde{f}^{-1}(F_{E_i}) : i = 1\ to\ n\right\}$ that is, $\widetilde{f}(F_E) \mathrel{\widetilde{\subset}} \widetilde{\bigcup}\{F_{E_i} : i = 1\ to\ n\}$. Hence $\widetilde{f}(F_E)$ is is $(1,2)^*$-soft b-compact relative to $\widetilde{Y}$. $\square$

5 Conclusion

$(1,2)^*$-soft b-connected sets and $(1,2)^*$-soft b-separated sets in the soft bitopological space have been introduced. Also we have introduced the $(1,2)^*$-soft b-compactness and we hope that the findings in this paper will help researchers to enhance and promote the further study on Soft bitopology to carry out a general framework for their applications in practical life.

References

[1] Andrijievic, D. *On b-open sets*, Mat. Vesnik. **48** (1996), 59-64.

[2] Basavaraj M. Ittanagi , Soft Bitopological Spaces, Comp.and Math.with App.107, No.(2014).

[3] Cagman N. , Karatas.S , Enginoglu.S : *Soft topology* , Comp.and Math.with App. 62(1), 351-358 (2011).

[4] Kelly J.C, *Bitopological Spaces*,proc.London Math.Soc 13(3) 71-83(1963).

[5] Molodtsov D.A, : *Soft set theory- first results*, Comp.and Math.with App.37(4-5),19-31(1999).

[6] Peygan.E,Samadi.B and Tayebi.A, *On Soft Connectedness*, arXiv:1202. 1668v1 [math.GN],8 Feb 2012.

[7] Revathi.N, Bageerathi.K, *On Soft B- open Sets in Soft Bitopological Spaces*, International Journel of Applied Research 2015; 1 (11): 615-623.

[8] Revathi.N, Bageerathi.K, $(1,2)^*$*-soft b-continuous and* $(1,2)^*$*-soft b-closed map*, communicated.

[9] Shabir M., and Naz M. : *On Soft topological spaces*. Comp.and Math.with App.61, 1786 1799(2011).

[10] Senel G., Cagman N.: *Soft Closed sets on Soft bitopological Spaces*. Journel of New Results in Science, pp 57-66(2014).