Journal of Algorithms and Computation

journal homepage: http://jac.ut.ac.ir

Optimizing Insurance Contract for a two-level two-period Supply Chain

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ABSTRACT

We can apply any method for organizing a supply chain, but contracting is more viable. Among many contracts that does so, the Insurance contract is more efficient. The problem is tuning the contract's parameters (for a two-level two-period supply chain with one supplier and one retailer) to achieve the optimum point where there is more gain for everyone separately, and the predictable risks all have been covered. The insurance contract covers every predictable risk that the downstream is facing. Instead, the retailer gives the supplier some money as a side payment (Premium). So, it has two main parameters, first, the fraction (β) of every predictable loss by the retailer, which the supplier must pay, and second, the side payment (M), which the retailer must pay. We will find the best one-supplier one-retailors β for a

ARTICLE INFO

Article history:
Research paper
Received 08 April 2022
Received in revised form 24, May 2022
Accepted 30 May 2022
Available online 01, June 2022

Keyword: Stochastic programming, Probabilistic programming, Insurance contract , Scenario reduction algorithm , Supply chain organization , Supply chain management.

AMS subject Classification: 90C15.

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1 Abstract continued

supply chain with two main sale periods. But for reaching the optimum state of all the insurance contract's possible forms, we designed some mathematical models based on scenarios. Then we optimize these stochastic models to find the best contract possible for 1000 initial scenarios. Every scenario indicates one possible number for price and one for demand in each period (generating 4000 possible numbers for the independent demand and price). We needed to know the maximum possible profit for the whole supply chain. First, we designed a centralized supply chain where the maximum profit is possible, then a decentralized supply chain to know the minimum of what's possible. When we have the ends of our range, we can design the final model with an insurance contract applied. In this model, we insert the β and M into the model as the insurance contract does. First, we reduce the number of scenarios to 20 with a novel method. Then we find the optimum point by solving the final model for each β. The result was better at β=0.25. In the next step, by supposing an equal negotiating power for sides, we split the extra money into two equivalent sizes, and the M amount was measured as half of the extra money that the contract can reach. The extra money was at 0.9893 of our range means the contract earns 99.97% of what is possible.

2 Introduction

A supply chain is a system with agents working on it; it's evident that when there is harmony in decision making, the system works better, and there is more profit for the whole system. A centralized supply chain is a chain in which one central agent decides about every issue, and there is maximum profit possible for the entire supply chain. We call the maximum profit the endpoint of the potential profit range. A decentralized supply chain is a chain in which every agent decides separately and independently from other agents' choices.

In this case, the minimum profit is possible for the whole system, and we call the profit amount the start point of the likely range. So, we have a range for the possible profit the whole supply chain can make.

This is showing a range for all possibilities of profit that the whole system is going to achieve at the end of two periods, as it is obvious when there is no coworking and they (the agent) are paying no attention to each other at all, the decentralize case happens.

The profit is minimum in the opposite side when the harmony is in the highest amount and all decisions are all made by one central agent the profit is maximum but it's not practical. What's real is the contracted one, when we use the insurance contract as an organizing instrument, we can achieve nearly the centralize case's profit just like the figure. Agreements and other coordination methods for a supply chain are workable in this range, and the goal is to reach what's possible in real because the centralized case is not entirely practical. The insurance contract is one of the coordination's methods that reach nearly the maximum potential profit, but the problem is how can we get a better point in this range. Like any other contract, an insurance contract has its condition. And it has three main content.

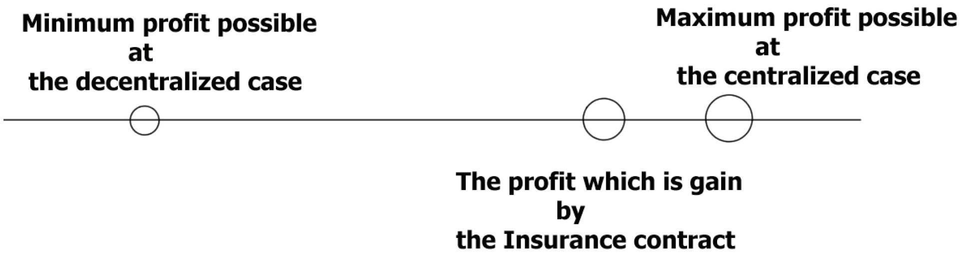

Figure 1: Range of the whole supply chain's possible profit

2.1 An insurance contract, in theory

In this contract, the upstream (here is one supplier) ensures the downstream (here is one retailer) pays a fraction of the retailer's predictable loss; instead, the retailer gives some side payment to the supplier and warranty that the supplier gains profit as much as before signing the contract did. In this manner, no one loos, and all the predictable risks are covered. Predictable risk means the risks caused by mismatching the order quantity from market demand, such as shortage risk, keeping risk, and salvage risk, not the loss caused by earthquake, fire, tornado, or any other disaster.

Table 1: definitions and notation

Decentralized SC	A supply chain with separated levels in which every agent decides independently
Centralized SC	It is a supply chain that one central agent runs. It has the most profit, and one central agent makes every decision
Contracted SC	A supply chain in which the levels are dependent, making decisions in a contracted manner.
α	Insurance contract's parameter. Retailer's share of losses caused by mismatching its forecasted demand (order quantity) from the actual demand
β	Insurance contract's parameter. Supplier's share of losses caused by mismatching retailer's forecasted demand (order quantity) from the market demand
M or Premium	Insurance contract's parameter. The side payment that retailer gives to supplier for sharing risks

A supply chain can move from a decentralized model to a centralized one by contract. The contract must increase everyone's profit, but in the centralized case, despite a more significant whole gain, there is no warranty for not decreasing the firm's profit. So, when an insurance contract has been signed, the sides know that there is no reduction in their profit.
We consider a supply chain with one supplier and one retailer, where the retailer faces the stochastic independent demand and price.

2.2 Paper Structure

In this section, we told what's necessary to understand the upcoming sections. In the next section, we review the literature on contracts that coordinate the supply chain and the literature about what's done in the supply chain management by stochastic programming. Then we define the problem with all aspects and have a schematic model of a two-period supply chain in the problem definition section. Then the mathematical models based on scenarios are developed, three stochastic models of decentralized, centralized, and insured supply chains. In the next step, we generate 1000 scenarios for price and demand in each period (4000 random numbers in whole). We reduce the number of scenarios to 20 by a novel and fast method without any change in the properties of the initial scenarios. After lowering the number of the scenario to 20, we optimize the objective function of our three models for these 20 scenarios (solving the models). We find the best β for the insurance contract by solving the final model for each β. in the next step; we split the extra money earned by the contract into the agreement's sides. At last, we discuss the limitation of the insurance contract. We have managerial insights and a conclusion in the end.

3 Literature review

This literature review brings together all of the past works done in this field and is necessary to know. The fundamental matter of inventory management is to assure product availability for the final consumer at the lowest possible cost while subjected to various specific conditions. In that sense, inventory management plays a prominent role in answering crucial questions such as when to order, how much to request, and how much to keep as safety stock [18]. The literature delivers different inventory control plans related to mathematical models to minimize the total inventory management cost. Such models can be separated into two main groups: deterministic models, which study that all parameters are formerly known, and probabilistic models, in which one or more parameters are considered uncertain. The reflection of uncertain parameters cracks these models more supporters to a real-world problem at the cost of becoming more interesting in terms of real fitness due to regular severe computational necessities. The literature delivers many inventory control strategies seeing demand ambiguity, such as the classical systems (R,Q),(R,S),(R,s,S),(s,S)and(s,Q). In these systems R,Q,s and S denote the evaluation period, the fixed order quantity, ordering point, and the target inventory level, one-to-one.

3.1 Coordinating Supply Chain with Contract & Negotiating on excess Profit

A supply chain contains different members with usually disagreeing objectives. If a supply chain member does not act according to the overall supply chain's optimum solution, coordination problems arise. If one can find a contract so that each member works wisely according to the supply chain's ideal solution, this contract is said to coordinate the supply chain [25]

[22] established an outline for choosing reliable suppliers and order distribution, which increases the supply chain's income concerning the risk reduction tactics and organization between the buyer and the supplier. Since the centralized decision-making assembly is not practical in real problems, additional agreements are applied to advance decentralized assembly decisions to raise SCs' total profitability [2].

A game-theoretical method is used in most of them, and different games are well-thought-out, such as Stackelberg, Nash bargaining, and/or Rubinstein bargaining games.

In most manufacturing and service administrations, supply chain planning (SCP) can be considered the front line of business roles, from discovering raw materials to satisfying customer demands. SCP can be classified into strategic, tactical, and operational decisions conferring on the time limit taken into justification. Today's multifaceted business situation is considered by enormous ambiguity, everyday disruption, and substantial inconsistency, so keeping a practical and workable supply chain becomes the primary contest for many businesses. A supply chain working in such an aggressive setting has to survive with scheduling parameters such as cost, demand, and supply that have essential ambiguity. Additionally, major natural or man-made disruptions can aggravate a supply chain, such as earthquakes, floods, terrorist attacks, and economic crises. So, SCP is often made in the existence of ambiguity, for which stochastic programming is a practical tool to assist in the attainment of SCP choices.

3.2 Scenario-based Stochastic programming for Inventory control in an SC

Conferring to [4], one alternative for relaxing the proposition of having a summarized model when facing stochastic demands is to use two-stage stochastic programming as a modeling outline.

The two-stage assembly is well-matched with the earlier stated inventory plans when used to model the control parameters (i.e.,R,S,s, and Q) as the first-stage variables in each scheme, on behalf of the decisions that should be made before the uncertainties are exposed. One of the significant rewards of the two-stage stochastic programming pattern is that the stochastic parameters can be displayed without adopting any limiting hypothesis for the stochastic phenomenon if an isolated set of scenarios can approximate

it. Yet, an inherent trade-off between the excellence of this estimate and computational necessities must be detected correctly.
Improved estimates naturally need a massive number of scenarios and numerous repetitions to be solved. Then, it becomes apparent the need for computational approaches that can solve resultant large-scale deterministic corresponding problems professionally. The appropriateness of two-stage stochastic programming as an outline for lecturing inventory management complications has been proved in the papers done by [8],[4] , and [6].
Up to now, the only papers originating in the literature and using stochastic programming functional to inventory management were those performed by [8],[4] , and [6]. Built on the constant system (s,S) [8] demonstrated a two-layer net wit hone manufacturer, one retailer, one item, ambiguity in the demand parameter, and pure lost sales analyzed the model in its centralized and decentralized forms. [4]projected a refill regulator and inventory model via two-stage stochastic programming, as periodic review (R,S) one item, and uncertain demand. In their model, there is no parameter representative of the initial inventory. Otherwise, they assume the limiting idea that the first order must be positioned at the start of the planning period, which needs the planning period to be insincerely amplified and the cost parameters to be measured at zero in the first few planning prospect periods so that (a not manageable) initial inventory can be gathered. Also, the model planned by [4] studies the pure lost sales occasion or the pure back-order situation with some modifications of the restraints. With the version of the model planned by [4], [6] projected a red blood cells inventory managing model that decreases the operative cost and perishability of blood multi-periods, multi-products, and ambiguous demand.
In the context of inventory control structures most of the literature stresses on strategic choices of single-echelon logistics nets. Papers such as [12] , [23] and [20] discourse on multi-echelon complications over stochastic programming. Those papers do not report inventory plans despite seeing inventory management and SC application together. Yet,[5],[26] report multi-echelon SC plans and inventory rules deprived of using the stochastic programming method. Lately,[8] projected an inventory plan for a two-echelon logistics net based on the (s, S) constant review system, seeing a single item with ambiguous demand using stochastic programming.
Stochastic programming, counting two-stage stochastic programming (2SSP), has been used to model inventory supply schemes. [8] proposed 2SSP models for a multi-period renewal problem, using a safety-stock-based plan to examine one retailer and one manufacturer supply chain.[4] established a 2SSP model for a refill control system using periodic review for a single layer logistic net, then applied Sample Average Approximation to get approximated ideal results.[21] considered a single-item single echelon invention scheduling problem as stochastic programming typical with an accidental restraint and non-stationary demand for a perishable product with a static lifetime. [1] planned an estimated dynamic programming model to see the stochastic supply and demand for the platelet inventory problem.
The most general in this arena is the two-stage stochastic programming based on scenario preparation in the modeling methods to deal with ambiguity within mathematical programming. Yet, it is naturally vital to instrument solution strategies that overwhelm the

computational difficulty for problems with many scenarios. Numerous solution strategies are projected in the literature. [26]You (2013) primarily considered the consequence of supply and demand uncertainties using a Multi-cut L-shaped decomposition method. You compared it with the L-shaped approach to catch the influence of biomass supply and technology ambiguity. [16] merged Lagrangian relaxation and L-shaped solution methods. The paper projected by [?] mutually studies three substantial causes of uncertainties: switchgrass return due to random weather conditions, demand for bioethanol, and bioenergy sale price. The work suggests a solution technique concerning the serial application of a revised Sample Average Approximation approach and Benders decomposition to solve the anticipated stochastic optimization model capably and effectually.
However, to the best of our knowledge, no issued work tries scenario reduction approaches, like those projected by [13]. These scenario reduction approaches are obliging when many scenarios are imminent, falling it to a reduced set of scenarios that denote a decent estimate of the first set of scenarios. These approaches have the power to get control of the computational difficulty allied with the design of any supply chain when using scenario-based two-stage stochastic programming models with numerous causes of ambiguity.
[7]offered a multi-period stochastic mixed 0–1 problem rising in tactical supply chain planning (TSCP). An identical deductive model was projected to signify the parameters' ambiguity in a multi-stage scenario tree. They recommended the novel risk-averse tactic's added value using stochastic dynamic programming for TSCP. Likewise,[17] established a two-stage stochastic programming model for the complete strategic arrangement of supply chains under demand and supply ambiguity with the petition of the wind turbine business. Academic and arithmetical findings more confirmed this anticipated model. Usually, reservations ascend with the reflection of sourcing in the supply chain. Among other processes, sourcing includes high unpredictability due to supply disruption. Hence, [15] explored supply disruption with stochastic programming to maximize the anticipated effectiveness under loss aversion and portray the sole optimal order quantities.
Supply chain network (SCN) reform gains significant attention in the supply chain framework due to its sympathy for physical formation. It is essential to deal with the SCN reform under supply chain planning. [10] established a multi-stage stochastic program (MSSP) with SCN reform. This paper lectured two main subjects: i) that building a suitable scenario tree to model current ambiguity in stochastic parameters is a thought-provoking task, and ii) even with an appropriate scenario tree, an MSSP can bring about a large-scale optimization problem so that commercial solvers may not be professionally employed to solve it. Still, this study contributes to the literature through many critical novelties.

3.3 Research-GAP

Most of the studies done by other researchers, which have the insurance contract as an organizing instrument, are for supply chains with one sale period. However, our study uses an insurance contract for coordinating a supply chain with two consecutive periods of sale. From another perspective, this paper probabilistically models two-period systems with one thousand scenarios for stochastic demand and price in two consecutive periods. It is like a model for all probabilities that would happen for two periods in the future. This paper's main contribution is the algorithm used to reduce the number of scenarios with no change in the problem's initial data's properties (one thousand initial scenarios' configuration). Besides these points, our study has two significant challenges that fill this field's gaps, making us write and publish it.

- An essential characteristic of scenario-based stochastic programming methods is to make a well-organized set of scenarios to model current ambiguity in an optimization problem. More significantly, the supply chain planning area should examine the scenario generation procedures concerning solidity and quality standards. Lately, scenario reduction approaches and sample average approximation methods have been established in supply chain planning under ambiguity, and this feature needs more consideration than this
- Numerous stochastic optimization problems in supply chain planning with the multi-period set can cause a Multi-Stage Stochastic Program (MSSP). Developing MSSPs and giving effective answer tactics to them is a challenging matter. To the best of our knowledge, there has been no paper mattering this issue until recently; just four essays did so. ([11], [9],[10] and [7]just have dealt with this issue)

4 Methodology

4.1 Problem definition

We study a two-period SC model when the info is symmetrical. At the start of the selling period, the vendor commands an order Q with the provider built on his prediction of market demand D. The market demand D is a constrained, positive value random variable with probability density function $f(x)$ and cumulative density function $F(x)$. Price and demand both are stochastic and behave like a Brownian motion. The provider products the invention with an item cost c and asks the retailer a distributing price w. The retailer charges the clients with a selling price p. The lack (shortage) cost is v per item, and the re-claim (salvage) worth of any unsold invention is s per item. To keep away mistakes, we accept that $0 < s < c < w < p$ and $0 < v < w$ for each period. π signifies stochastic profit and Π signifies the expected value of stochastic profit. Superscript $*$ signifies optimality; subscripts s, r, and sc denote one-to-one supplier, retailer, and supply chain. Subscripts i signify insurance agreement. Earlier in the vending periods, the provider and the vendor settle on an insurance agreement with two main parameters. The first parameter $\beta(\beta\epsilon[0, 1])$ is the provider's part of losses made by the deviation of the vendor's demand amount from the market demand. The retailer's share is $\alpha(\alpha = 1 - \beta)$. The Next parameter is the cross(side) payment M, from the vendor to the provider. It is vital to memo that the cross payment is autonomous with the demand amount and can be negative. When M is negative, the provider gives a cross payment to the vendor. This

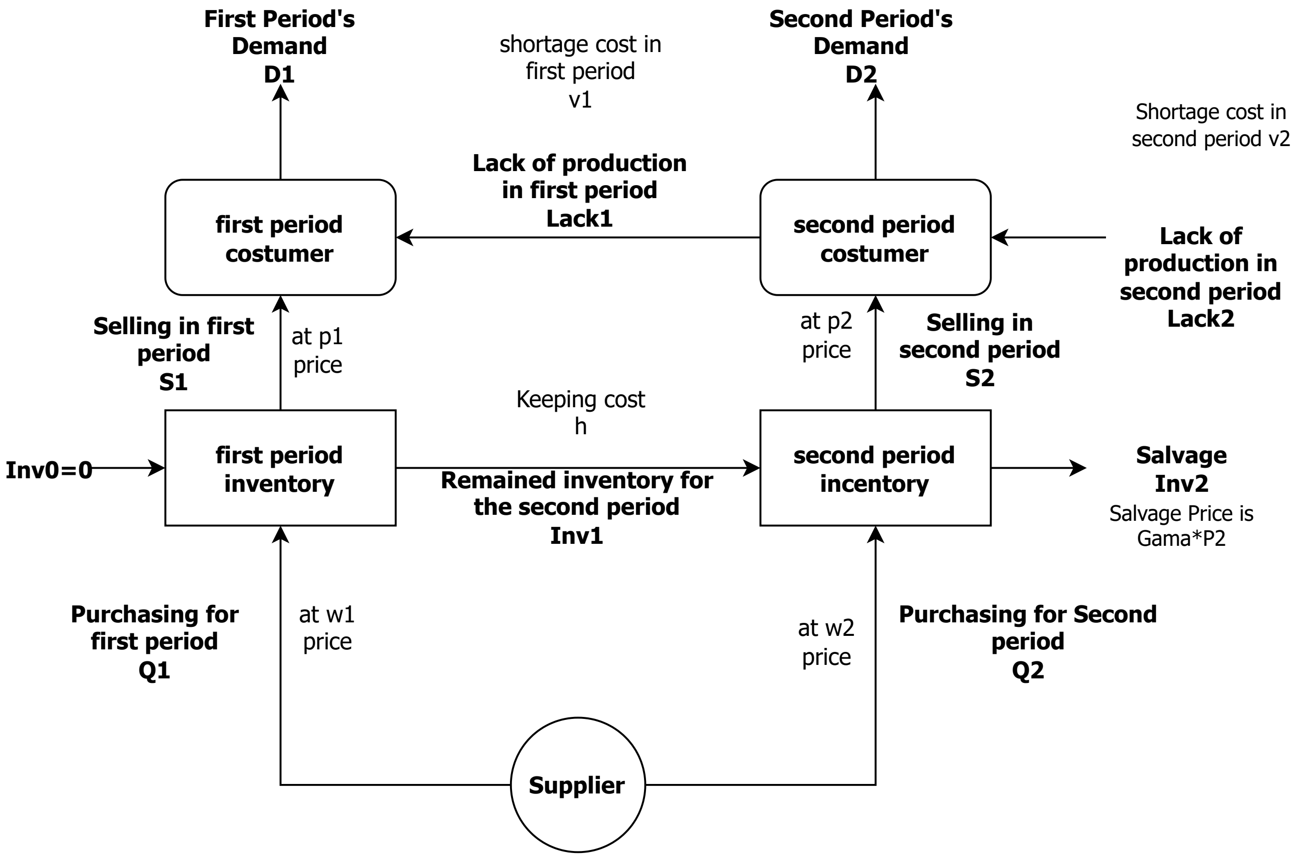

Figure 2: system schematic model

This figure shows the primary schematic model of paper, and it shows the flow of material throw the supply chain in two periods. The mathematical models' constraints came from here by counterbalancing items (good) on the four rectangles in the above figure.

insurance agreement specifies that the provider must share some or all of the vendor's losses, whereas the vendor gives a premium to the provider; the act of this SC is then enhanced.

This figure is designed to understand better where the models came from. The objective functions and all constraints are made by focusing on this schematic model for an SC with two periods of sale

Table 2: Definitions

Sets	
S	Scenarios Set with "s" as an Indices
I	Periods Sets, with "i" as an Indices
Parameters	
$D_{i,s}$	Product's Demand for the i_{th} period in s_{th} scenario
$P_{i,s}$	Retailing Price for the i_{th} period in s_{th} scenario
K_s	s_{th} Scenario's Happening Probability
w_i	The wholesale price in i_{th} period
c_i	Production cost in the i_{th} period
v_i	Retailer's Shortage cost in the i_{th} period
H	Keeping cost until the second period
γ	Salvage's ratio (a percentage of product's price)
Variables	
Q_i	Quantity of ordered production in the i_{th} period
$Lack_{i,s}$	Shortage amount for i_{th} period in s_{th} scenario
$Sell_{i,s}$	Retailer's sold amount for i_{th} period in s_{th} scenario
$Inv_{i,s}$	Retailer's inventory for the i_{th} period in s_{th} scenario
β	It is a number between 0 and 1. A percentage of the retailer's loss is caused by mismatching forecasted and the actual amount of demand, which the supplier gives to the retailer.
α	$\alpha = 1 - \beta$.The retailer's share of the loss is caused by mismatching forecasted and the market's demand.
M	It is premium or the money which the retailer gives to the supplier. It is independent of the volume of orders, and when it is negative, the supplier provides it to the retailer.
Π	Profit
Index	
I	Insurance Contract
C	Centralized
Dc	Decentralized
R	Retailer
S	Supplier
SC	Supply Chain

4.2 Model development

In this section, we designed three models; first, a two-level decentralized supply chain for two consecutive periods in which no one knows each other, and every decision made is independent of another's choices (to know about the minimum profit the supply chain system can make). Second, a centralized supply chain has been designed where only one

agent (central agent) has decided (to know about the maximum profit the supply chain system can make). The third is a supply chain with an insurance contract with two parameters β and M.
Before explaining, let's go straight to stochastic programming (mathematical models based on scenarios). Every mathematical model in the operation research field has an objective function subject to some constraints. Usually, we will optimize the objective function with the constraint's limitation. Here is no exception. We define and create the whole supply chain's profit function as an objective function subjected to 4 primary restrictions. "Inventory level and demand limitation in each period." These four constraints come from 4 counterbalances on item (good) in the inventory or at the customer's hand in each period (item counterbalance on four rectangles in figure 2).

4.2.1 Decentralized supply chain model

In this case, to create the whole system's profit function, we need to generate supplier and retailer's profit functions separately and then add them together. So, we first write the retailer's profit function as follows:

$$\Pi_r = -\sum_{i \epsilon I} w_i Q_i + \sum_{s \epsilon S} k_s \left(\sum_{i \epsilon I} P_{i,s} Sell_{i,s} - \sum_{i \epsilon I} v_i Lack_{i,s} - Inv_{1,s} h + Inv_{2,s} \gamma P_{2,s} \right) \quad (1)$$

With a little algebraic progress, it changed to:

$$\Pi_r = -\sum_{i \epsilon I} w_i Q_i + \sum_{s \epsilon S} k_s \left(\sum_{i \epsilon I} P_{i,s} Sell_{i,s} + P_{2,s} Inv_{2,s} \right) - \sum_{s \epsilon S} k_s \left(\sum_{i \epsilon I} v_i Lack_{i,s} + Inv_{1,s} h + Inv_{2,s} (1 - \gamma P \right. \quad (2)$$

$$\Pi_r = -\sum_{i \epsilon I} w_i Q_i + \sum_{s \epsilon S} k_s \left(\sum_{i \epsilon I} P_{i,s} Sell_{i,s} + P_{2,s} Inv_{2,s} \right) - U(Q_1, Q_2) \quad (3)$$

the term U(Q_1, Q_2) is the expected losses generated by the retailer's order quantity deviation from the market demand. Here is the summation of the shortage, keeping, and salvage costs.

$$U(Q_1, Q_2) = \sum_{s \epsilon S} k_s \left(\sum_{i \epsilon I} v_i Lack_{i,s} + Inv_{1,s} h + Inv_{2,s} (1 - \gamma P_{2,s}) \right) \quad (4)$$

The Model By adding the constraints to the profit function of the retailer, the complete model is as follows:

$$Max\pi_r = -\sum_{i\epsilon I} w_i Q_i + \sum_{s\epsilon S} k_s \left(\sum_{i\epsilon I} P_{i,s} Sell_{i,s} + P_{2,s} Inv_{2,s} \right) - U(Q_1, Q_2) \tag{5}$$

Subject to:

$$Inv_{1,s} = Q_1 - Sell_{1,s} \quad \forall s\epsilon S \tag{6}$$

$$Inv_{2,s} = Inv_{1,s} + Q_2 - Sell_{2,s} \quad \forall s\epsilon S \tag{7}$$

$$Sell_{1,s} + Lack_{1,s} = D_{1,s} \quad \forall s\epsilon S \tag{8}$$

$$Sell_{2,s} + Lack_{2,s} = Lack_{1,s} + D_{2,s} \quad \forall s\epsilon S \tag{9}$$

$$U(Q_1, Q_2) = \sum_{s\epsilon S} k_s \left(\sum_{i\epsilon I} v_i Lack_{i,s} + Inv_{1,s} h + Inv_{2,s}(1 - \gamma P_{2,s}) \right) \tag{10}$$

$$Q_1, Q_2, Inv_{1,s}, Inv_{2,s}, Sell_{1,s}, Sell_{2,s}, Lack_{1,s}, Lack_{2,s}, U(Q_1, Q_2) \geq 0 \quad \forall s\epsilon S \tag{11}$$

The objective function is the expected profit for Q_1 and Q_2 order amounts, and it has to be maximized. The term U in the objective function (discussed earlier) is noted as a constraint. The first two constraints (7&6) are derived from a counterbalance on inventory in two periods. The third and fourth constraints (9&8) balance demand with sold and lacked amounts in two periods. (for a better understanding, refer to figure 2)
As the supplier is not decided in this case, the profit function for the supplier is:

$$\pi_s^* = \sum_{i\epsilon I} (w_i - c_i) Q_i^* \tag{12}$$

By putting the resultant amount for Q_1^* and Q_2^* in the above equation, the maximum profit the supplier could make is π_s^*.
By adding the two amounts derived, the whole system's maximum profit is:

$$\pi_{sc}^* = \pi_s^* + \pi_r^* \tag{13}$$

4.2.2 Centralized supply chain model

In this case, agents decide together, and the goal is to maximize the whole system's profit (not the agent's profit separately). By adding the retailer and supplier's profits together, we have:

$$\pi_{sc} = \pi_s + \pi_r = \sum_{i\epsilon I}(w_i - c_i)Q_i - \sum_{i\epsilon I} w_i Q_i + \sum_{s\epsilon S} k_s(\sum_{i\epsilon I} P_{i,s}Sell_{i,s} + P_{2,s}Inv_{2,s}) - U(Q_1, Q_2) \tag{14}$$

$$= -\sum_{i\epsilon I} c_i Q_i + \sum_{s\epsilon S} k_s(\sum_{i\epsilon I} P_{i,s}Sell_{i,s} + P_{2,s}Inv_{2,s}) - U(Q_1, Q_2) \tag{15}$$

The Model By adding the inventory balancing and the demand balancing constraints to the above function, the mathematical model of the problem forms like this:

$$Max\pi_{sc} = -\sum_{i\epsilon I} c_i Q_i + \sum_{s\epsilon S} k_s(\sum_{i\epsilon I} P_{i,s}Sell_{i,s} + P_{2,s}Inv_{2,s}) - U(Q_1, Q_2) \tag{16}$$

Subject to:

$$Inv_{1,s} = Q_1 - Sell_{1,s} \quad \forall s\epsilon S \tag{17}$$

$$Inv_{2,s} = Inv_{1,s} + Q_2 - Sell_{2,s} \quad \forall s\epsilon S \tag{18}$$

$$Sell_{1,s} + Lack_{1,s} = D_{1,s} \quad \forall s\epsilon S \tag{19}$$

$$Sell_{2,s} + Lack_{2,s} = Lack_{1,s} + D_{2,s} \quad \forall s\epsilon S \tag{20}$$

$$U(Q_1, Q_2) = \sum_{s\epsilon S} k_s \left(\sum_{i\epsilon I} v_i Lack_{i,s} + Inv_{1,s}h + Inv_{2,s}(1 - \gamma P_{2,s}) \right) \tag{21}$$

$$Q_1, Q_2, Inv_{1,s}, Inv_{2,s}, Sell_{1,s}, Sell_{2,s}, Lack_{1,s}, Lack_{2,s}, U(Q_1, Q_2) \geq 0 \quad \forall s\epsilon S \tag{22}$$

As we are paying attention to this driven model, we now understand it is like the decentralized model when only one agent decides about orders' quantities. It is no more a retailer; the supplier produces and distributes the product itself. Therefore, the supplier makes the goods by costing c and selling them by $P_{i,s}$. with another perspective; it is like the retailer purchasing the products at a cheaper cost ($w \leq c$). Same as the decentralized case, this problem is linear programming (with scenarios indeed), and it can solve by optimal founder software. Here the whole system's profit is more than the decentralized and every contracted supply chain.
Most harmonies in co-working happen when the supply chain is centralized. In this case, agents completely trust each other. They let some agents' profit decrease in a situation where the system's profit increases (then they can split the extra profit so that every agent sees the rise in profit). The best and perfect contracts are those which tend the agents to decide like the centralized case. An agreement has to make the necessary trust between agents, so everyone is helping the whole as they decrease their risks and have no reduction in the final profit. (Profit at the end of splitting the extra amount that the contract made)

4.2.3 Insured supply chain model

When a solution is proposed to splits the risk between the agents, the retailer orders more, and everyone sees an increase in profit. In this contract, the retailer proposes the supplier pay some (or all) of the losses caused by not matching the forecasted and actual happened demand. Then the retailer pays some amount (Premium) to the supplier and guarantees that the supplier has no losses compared to when there is no contract. It is worth mentioning that every arrangement can increase the whole system's profit just to the centralized case's profit because the most harmony happens when one central agent makes all the decisions. The insurance contract unifies the supply chain with two parameters ß and M. By inserting these parameters into the developed model (actually by signing the contract), the model has some changes to the following model: (consider that it is possible to summarize the objective function by using just one U, but it is better to understand when we do not factor U)

The Model for the retailer

$$Max\pi_r^i = -\sum_{i\epsilon I} w_i Q_i + \sum_{s\epsilon S} k_s \left(\sum_{i\epsilon I} P_{i,s} Sell_{i,s} + P_{2,s} Inv_{2,s} \right) - U(Q_1, Q_2) + \beta U(Q_1, Q_2) - M \tag{23}$$

Subject to:

$$Inv_{1,s} = Q_1 - Sell_{1,s} \quad \forall s\epsilon S \tag{24}$$

$$Inv_{2,s} = Inv_{1,s} + Q_2 - Sell_{2,s} \quad \forall s \epsilon S \tag{25}$$

$$Sell_{1,s} + Lack_{1,s} = D_{1,s} \quad \forall s \epsilon S \tag{26}$$

$$Sell_{2,s} + Lack_{2,s} = Lack_{1,s} + D_{2,s} \quad \forall s \epsilon S \tag{27}$$

$$U(Q_1, Q_2) = \sum_{s \epsilon S} k_s \left(\sum_{i \epsilon I} v_i Lack_{i,s} + Inv_{1,s} h + Inv_{2,s}(1 - \gamma P_{2,s}) \right) \tag{28}$$

$$\sum_{i \epsilon I} (w_i - c_i) Q_i - \beta U(Q_1, Q_2) \geq \Pi_s^* \tag{29}$$

$$\beta \leq 1 \tag{30}$$

$$Q_1, Q_2, Inv_{1,s}, Inv_{2,s}, Sell_{1,s}, Sell_{2,s}, Lack_{1,s}, Lack_{2,s}, U(Q_1, Q_2) \geq 0 \quad \forall s \epsilon S \tag{31}$$

In this model, the expected value for the retailer's profit in the contracted supply chain is the objective function. The first two constraints (25, 24) show inventory balancing. The third and fourth constraints (27, 26) demonstrate the counterbalance on demand. The fifth constraint (28) is just a part of the objective function (U). The sixth constraint (29) shows the retailer's warranty to the supplier about not reducing profit after signing the contract. The two final constraints (31, 30) show that ß is a fraction between zero and one. Therefore, the objective function of the supplier's profit is:

$$\pi_s^i = \sum_{i \epsilon I} (w_i - c_i) Q_i - \beta U(Q_1, Q_2) + M \tag{32}$$

the whole system profit is:

$$\pi_{sc} = \pi_s + \pi_r \tag{33}$$

The problem is finding the best ß and M, which maximizes the contract's effect, as there is no reduction in any agent's profit.

4.3 Validation

Profit in a centralized supply chain is more than any contract can achieve. One factor for validating any agreement is the amount of profit that has been made divided by the centralized case profit. Another way to validate a new contract with new parameters is by comparing it with a similar agreement. In this case, we can use the revenue sharing contract for comparison because it has the most likeliness with the insurance contract in many ways.

5 Numerical analysis

5.1 Scenario Making

Demand and price are random parameters that are assumed to be independent, and both follow the spontaneous process of Geometric Brownian Motion (GBM). We need to create scenarios based on this random process to identify their behavior in each period. GBM is a continuous stochastic process in which the logarithm of the variable follows Brownian motion. Therefore, it can predict demands and prices in two consecutive periods. ACCORDING TO THE FOLLOWING EQUATION, the GBM process of predicting the desired parameter (demand or price) is performed.

$$S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})t - \sigma \omega_t} \quad t = 1, 2, ..., N \tag{34}$$

In this equation, S is a parameter that we want to estimate its value in periods one to N. As it seems, its value in each period is determined based on its predicted value in the previous period. We have to have S0 to estimate the value of S in periods 1 through N. μ and σ are the mean and standard deviation of the growth rate, respectively, which affect the predicted parameter value during the planning period, and ω_t is a random process that follows the Brownian motion.
We first generate one thousand initial scenarios based on the Brownian distribution for demand and price. The initial demand (D_0) equals 50 (item), and the average and standard deviation of demand growth in each period (μ_D and σ_D), respectively. We considered it equal to 0.6 and 0.15. In the same way, we assumed the initial price value (P_0) to be 30,000 and the average and standard deviation of price growth in each period (μ_p and σ_P) to be 0 and 0.2, respectively. You can see the scenarios for demand and price in the following figures (figures 2 and 3).

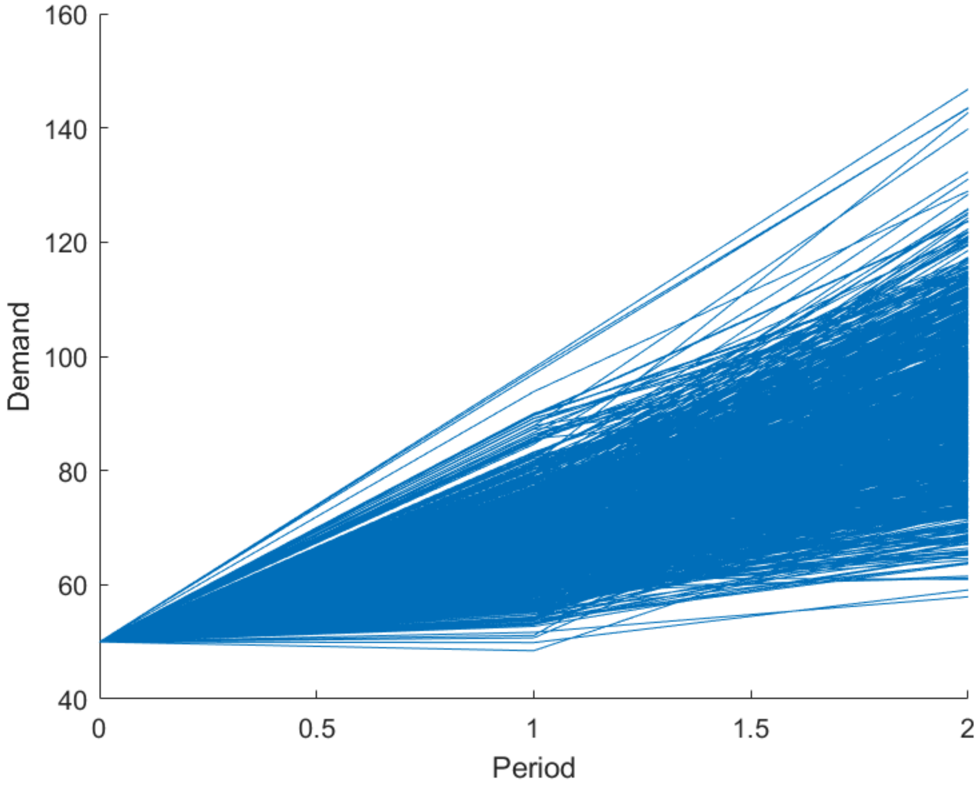

Figure 3: initial 1000 scenarios for demand in 2 periods
One thousand scenarios tree in two period for demand

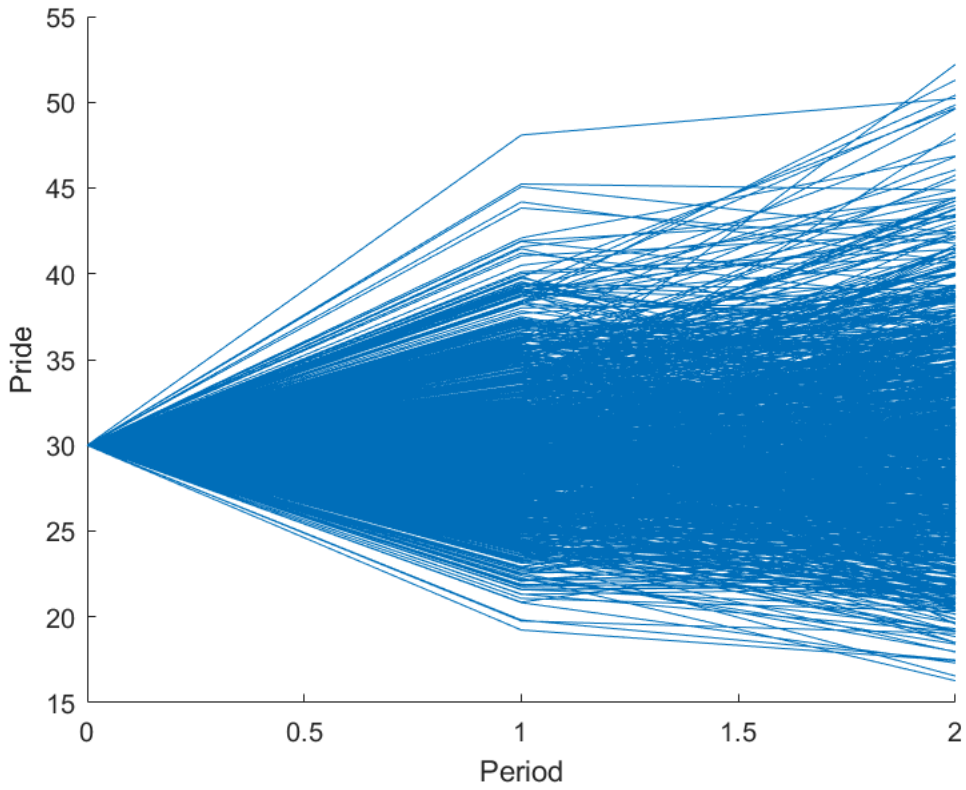

Figure 4: initial 1000 scenarios for price in 2 periods
One thousand scenarios tree in two period for price

Since working with many scenarios is very difficult and takes much time (to solve the model). It is necessary to reduce the generated scenarios for each of the two random parameters by using a scenario reduction method. We use the [13] method called Fast Forward Scenario Reduction.

5.2 Scenario Reduction

The volume of calculations to solve scenario-based optimization models depends on the number of scenarios. Therefore, it is necessary to reduce the set of main scenarios so that the characteristics of the potential problem do not change harshly. The number of decreased scenarios depends on the type and nature of the optimization problem and should be less than a quarter of the generated scenarios [13].
The primary idea of reducing the scenario is to eliminate low-probability and close-up scenarios. Therefore, scenario reduction algorithms identify a subset of scenarios and calculate the probabilities for the new scenarios so that the probabilities of the reduced scenarios are added to the nearest scenario in terms of probability distance. The scenario

reduction algorithm reduces the batch scenarios using the Kantrovich distance matrix. For example, each scenario consists of twenty-four hours for the next day's hour-by-hour scenarios, and deleting a scenario is equivalent to deleting a twenty-four-hour string. The Kantrovich distance is the probability distance between two different sets of scenarios, and the small space between the two scenarios indicates two identical possible processes. The Kantrovich distance ensures that the maximum probability scenarios are reduced without tolerable error. The probability of all deleted scenarios is considered zero. The new probability of the preserved scenario is equal to the sum of the prior probabilities and the probabilities of the closest deleted scenarios.
To build and reduce the scenarios. Using this method, we converted the number of scenarios to 20 final scenarios, shown for demand and price in the following figures (figures 5 & 6), respectively.

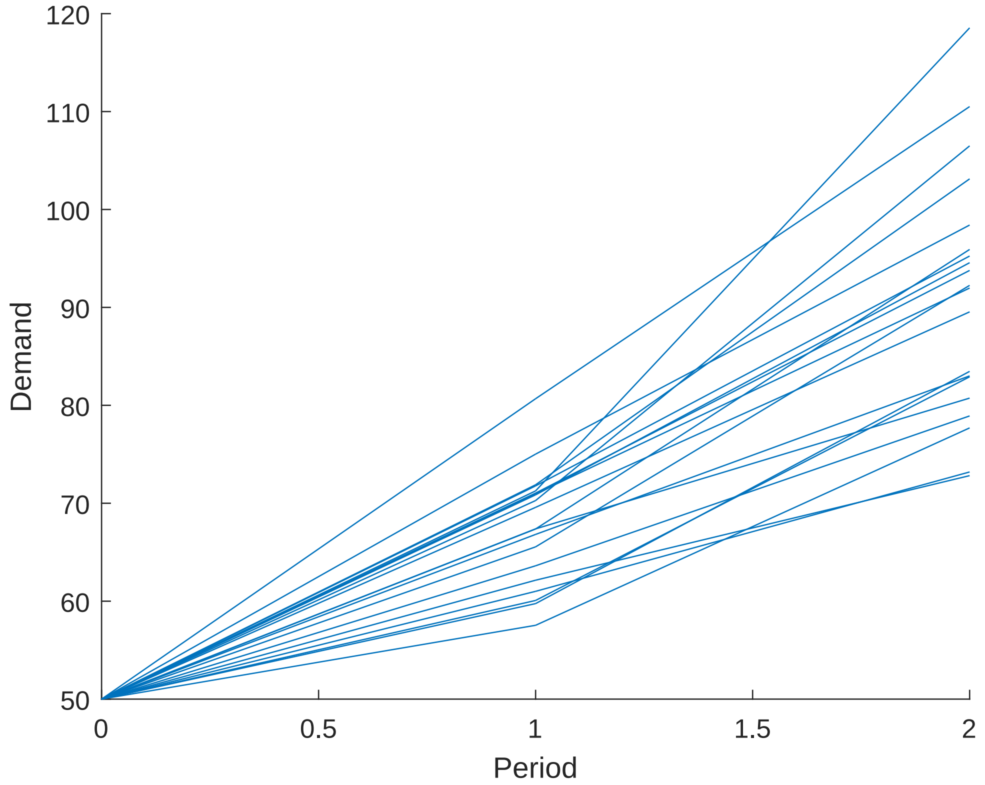

Figure 5: 20 final scenarios for demand in 2 periods
Reducted (20) scenarios tree in two period for demand

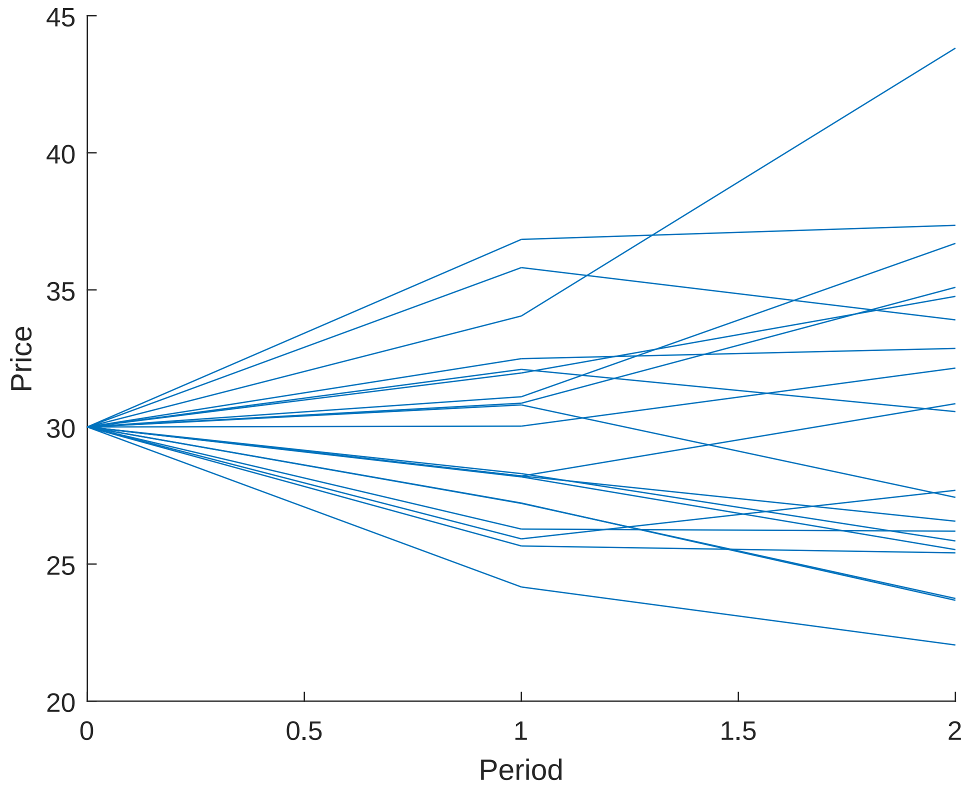

Figure 6: 20 final scenarios for price in 2 periods
Reducted (20) scenarios tree in two period for price

Now there are twenty different scenarios, each with a specific probability. Table 3 summarizes the results of the scenario-making and reducing process

Table 3: final scenarios configuration

SCENARIO'S NUMBER	FIRST PERIOD'S PRICE	SECOND PERIOD'S PRICE	FIRST PERIOD'S DEMAND	SECOND PERIOD'S DEMAND	SCENARIO'S PROBABILITY
1	37000	37000	71	92	0.07
2	32000	33000	61	73	0.054
3	27000	24000	67	83	0.034
4	26000	25000	72	103	0.052
5	26000	26000	62	73	0.071
6	27000	24000	58	78	0.041
7	24000	22000	72	95	0.038
8	28000	31000	67	96	0.051
9	32000	35000	70	106	0.039
10	28000	27000	67	81	0.038
11	26000	28000	70	90	0.053
12	28000	26000	81	111	0.051
13	32000	31000	60	83	0.086
14	30000	32000	71	119	0.036
15	31000	35000	66	92	0.054
16	31000	27000	71	95	0.073
17	36000	34000	75	98	0.038
18	28000	26000	71	94	0.051
19	31000	37000	64	79	0.042
20	34000	44000	60	83	0.028

5.3 Solving Models (Optimizing objective function for these 20 scenarios)

First, we must add value to the known parameters:

Table 4: amounting the known parameters

Assumed amount	Parameters
6000	c_1
6500	c_2
15000	w_1
15500	w_2
2500	v_1
5000	v_2
2000	h
0.2	γ

5.3.1 Decentralized

Table 5: results of solving the decentralized model

The optimal order quantity for the first period	71
The optimal order quantity for the second period	94
Retailer's optimal profit	2188400
Supplier's optimal profit	1485000
Whole System's profit	3673400

5.3.2 Centralized

Table 6: results of solving the centralized model

The optimal order quantity for the first period	71
The optimal order quantity for the second period	121
Whole System's profit	3763700

The difference between profits of the whole supply chain in both centralized and decentralized states indicates the lost profits due to the lack of coordination for the supply chain's components. As can be seen in this chain, the difference between these two profits is equal to 90300 and nearly 2.4% of the total profit of the chain (in the centralized mode). Therefore, by using various methods of chain coordination, including contracts, part of this lost profit can be compensated.

5.3.3 Insured

Now that we know the endpoints of our range, we solve the insured model for every β between 0 and 1, and the result is as follows:

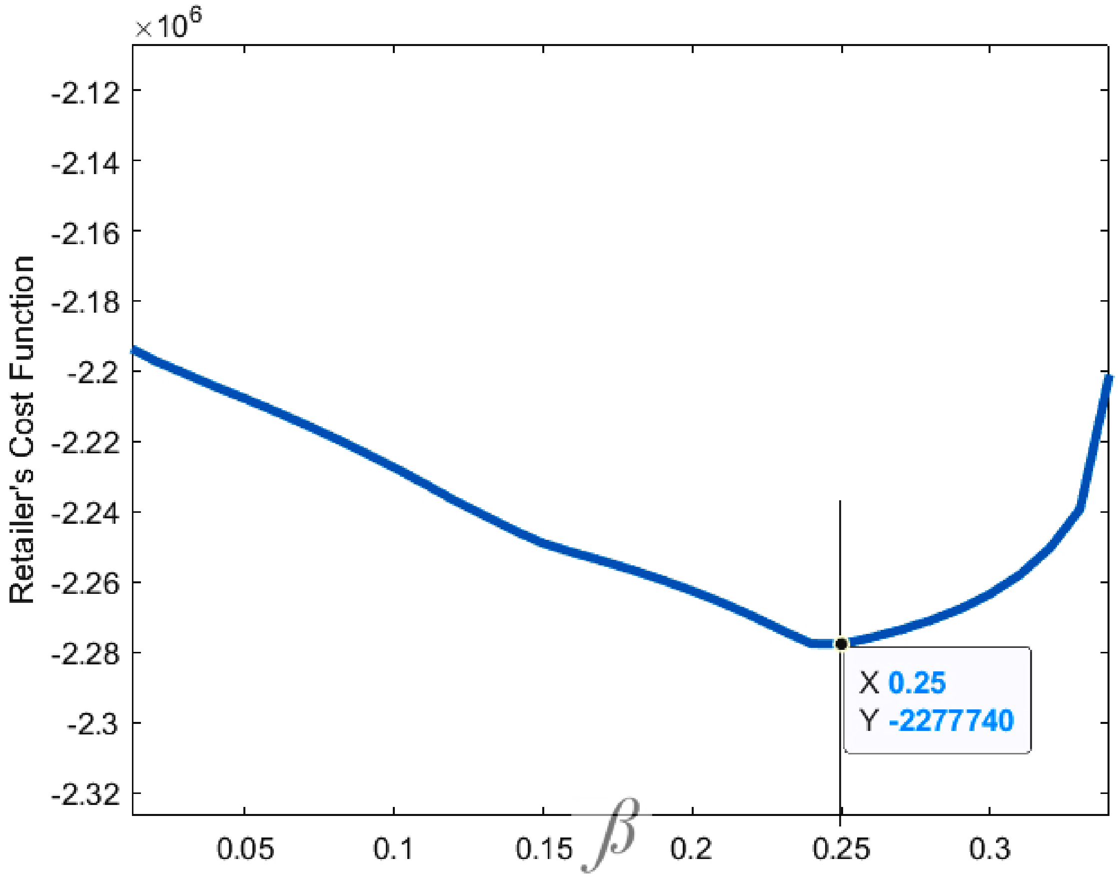

Figure 7: Effects of β on Retailer's cost function

This figure shows the main result of this research, which is optimizing β for the insurance contract. As you can see, it came out to nearly .25, which means that two-period supply chains work better and with more gain for everyone when the upstream share .25 % of predictable loss of downstream.

The problem was not feasible out of this range (0to.35). And it was usually for not satisfying the warranty content restriction.

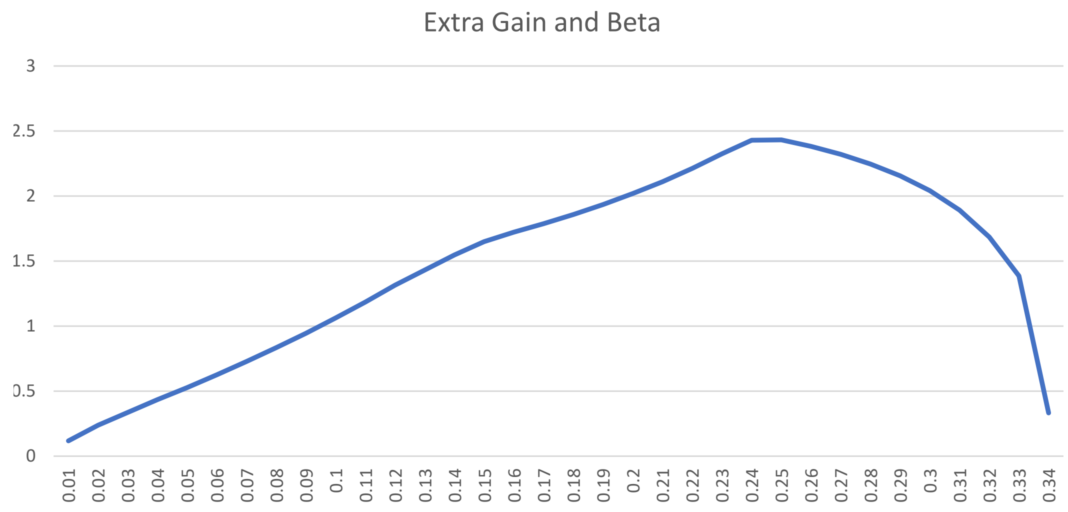

Figure 8: Effects of β on Extra profit percentage in the whole

Here again, you can see at $\beta = .25$ we have our maximum, and the extra gain which is the contract brings is maximum there, and it means that two-period supply chains work better and with more gain for everyone when the upstream share .25 % of predictable loss of downstream

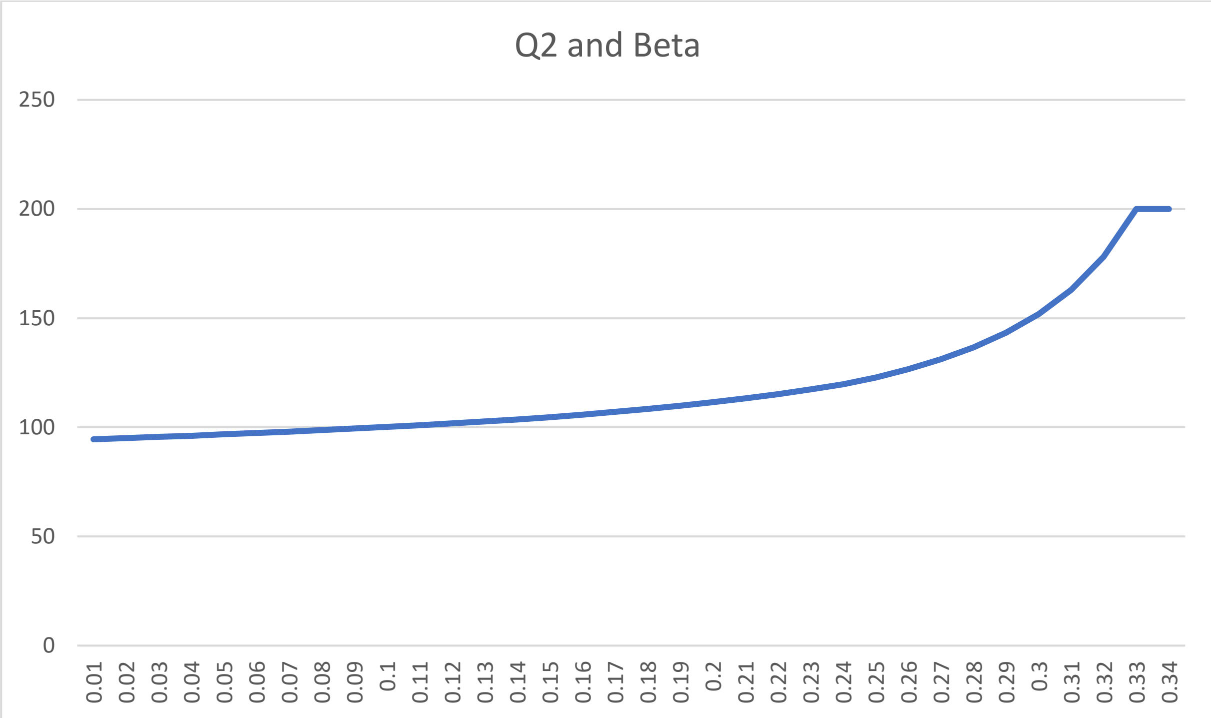

Figure 9: Effect of β on Second order's amount

This figure shows that when β increases Q_2 increases too, and it's better for upstream when the downstream order more, so in the insurance contract and in its feasible area, it's better for upstream and, of course, the downstream to sign a contract with big β.

5.4 Comparing the results

Table 7: Overall Results compared to each other

	Insured at $\beta = 0.25$	Centralized	Decentralized
The optimal order quantity for the first period	71	71	71
The optimal order quantity for the second period	123	121	94
Retailer's optimal profit	2277738		2188400
Supplier's optimal profit	1485000		1485000
Whole System's profit	3762738	3763700	3673400

After arranging the supply chain, retailer's and the whole system's profit increase by the insurance contract, these results are before splitting the extra money that the contract earns. After splitting the excess gain caused by the contract, the supplier profit rises too. It depends on how to break the extra income.

5.5 Managerial Insight

In this article, we advocate a logistic insurance agreement. Similar to various other arrangements, it organizes the supply chain by transferring the risk from the retailer to the supplier. This risk-transferring protects the retailer from dealing with ambiguous demands to a confident grade. And it advances the supply chain's efficiency. The proposed agreement maximizes the yield of all agents. Furthermore, compared with the provider, the vendor expresses the bazaar straight; thus, it is easier to gather bazaar demands. Still, bazaar demand is always uncertain, and collecting data and forecasting demand includes high expenses. If the vendor carries all the risks produced by the ambiguous request, he will try to gather as much data as possible to predict well. Usually, vendors, providers, and supply chain executives can use the outcome of an insurance contract to make a more suitable supply chain model.

6 Conclusion and Future Research

We confirmed that the insurance agreement organizes the supply chain if the parameters amounted properly. While the insurance agreement efficiently coordinates the supply chain, it also has some boundaries. The most severe extraordinary restraint is that the supplier sustains a managerial charge in monitoring the retailer's sales state. A critical hypothesis of the insurance agreement is that the vendor shares the data of bazaar demand and invention sales condition with the provider decently; so, the provider needs to monitor the vendor's sales condition to avoid the vendor from magnifying his lost sales. Based on this restriction, agreement application and monitoring instruments to perfect the insurance agreement should be considered. The second restriction is that the insurance agreement may decrease retailers' interest. Numerous articles on sales efforts (e.g.,[19]; [24]) share the joint declaration that bazaar demand is artificial by vendors' sales efforts. Under the insurance agreement, the vendor only shares a risk fraction. This situation may decrease the vendor's sales energy mirrored in data updates. This circumstance could be a likely future research path. In conclusion, when using an insurance contract, the supply chain's expected profit function is a concave function concerning α. The study shows that the supplier's expected profit increases as α increases, while the retailer's expected profit decreases as α increases. This phenomenon agrees with our insight: a higher α means a higher risk and a higher possibility of losses for the retailer. This makes the retailer order smaller and, consequently, receives a smaller expected profit.

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7 Appendix

7.1 Matrices

To solve or optimize an objective function constraint to some restrictions, you must first define seven matrices (maybe big matrices) for the solver. MATLAB solver is not an exception. For every linear programming which is done by the computer, you must design seven matrices.

7.1.1 "A"

A matrix: the left side of none equal constraints In this problem, we have just one none equal constraint, and that is the limitation which warranty content of the contract is creating. This means that the supplier's profit has no reduction compared to the case in which there is no contract. The warranty that the retailer gives to the supplier. So, it's just one none equal constraint, and it is like the following:

$$\sum_{i\epsilon I}(w_i - c_i)Q_i - \beta U(Q_1, Q_2) \geq \Pi_s^*$$

7.1.2 "b"

B matrix: the right hand of none equal constraints (the limits, the boundaries) The optimal value of π_s^* can be reached as:

$$\pi_s^* = \sum_{i\epsilon I}(w_i - c_i)Q_i - \beta U(Q_1, Q_2)$$

$$= (w_1-c_1)Q_1+(w_2-c_2)Q_2-\beta\sum_{s\epsilon S}k_s(v_1 Lack_{1,s}+v_2 Lack2, s+Inv_{1,s}h+Inv_{2,s}(1-\alpha)P_{2,s}) = 1485000$$

7.1.3 “Aeq”

Aeq matrix: the left side of equal constraints

Aeq	Q1	Q2	Sell1,1	Sell1,2	Sell1,3	Sell1,4	Sell1,5	Sell1,6	Sell1,7	Sell1,8	Sell1,9	Sell1,10	Sell1,11
first Cons Scenario 1	-1	0	1										
first Cons Scenario 2	-1	0	0	1									
first Cons Scenario 3	-1	0	0		1								
first Cons Scenario 4	-1	0	0			1							
first Cons Scenario 5	-1	0	0				1						
first Cons Scenario 6	-1	0	0					1					
first Cons Scenario 7	-1	0	0						1				
first Cons Scenario 8	-1	0	0							1			
first Cons Scenario 9	-1	0	0								1		
first Cons Scenario 10	-1	0	0									1	
first Cons Scenario 11	-1	0	0										1
first Cons Scenario 12	-1	0	0										
first Cons Scenario 13	-1	0	0										
first Cons Scenario 14	-1	0	0										
first Cons Scenario 15	-1	0	0										
first Cons Scenario 16	-1	0	0										
first Cons Scenario 17	-1	0	0										
first Cons Scenario 18	-1	0	0										

Figure 10: Northwest corner of Aeq matrix

This figure shows the north west corner of the Aeq matrix.

7.1.4 “beq”

beq matrix: the right hand of equal constraints

Table 8: beq matrix: the right hand of equal constraints

first cons scenario 1	0	third cons scenario 1	71
first cons scenario 2	0	third cons scenario 2	61
first cons scenario 3	0	third cons scenario 3	67
first cons scenario 4	0	third cons scenario 4	72
first cons scenario 5	0	third cons scenario 5	62
first cons scenario 6	0	third cons scenario 6	58
first cons scenario 7	0	third cons scenario 7	72
first cons scenario 8	0	third cons scenario 8	67
first cons scenario 9	0	third cons scenario 9	70
first cons scenario 10	0	third cons scenario 10	67
first cons scenario 11	0	third cons scenario 11	70
first cons scenario 12	0	third cons scenario 12	81
first cons scenario 13	0	third cons scenario 13	60
first cons scenario 14	0	third cons scenario 14	71
first cons scenario 15	0	third cons scenario 15	66
first cons scenario 16	0	third cons scenario 16	71
first cons scenario 17	0	third cons scenario 17	75
first cons scenario 18	0	third cons scenario 18	71
first cons scenario 19	0	third cons scenario 19	64
first cons scenario 20	0	third cons scenario 20	60
second cons scenario 1	0	forth cons scenario 1	92
second cons scenario 2	0	forth cons scenario 2	73
second cons scenario 3	0	forth cons scenario 3	83
second cons scenario 4	0	forth cons scenario 4	103
second cons scenario 5	0	forth cons scenario 5	73
second cons scenario 6	0	forth cons scenario 6	78
second cons scenario 7	0	forth cons scenario 7	95
second cons scenario 8	0	forth cons scenario 8	96
second cons scenario 9	0	forth cons scenario 9	106
second cons scenario 10	0	forth cons scenario 10	81
second cons scenario 11	0	forth cons scenario 11	90
second cons scenario 12	0	forth cons scenario 12	111
second cons scenario 13	0	forth cons scenario 13	83
second cons scenario 14	0	forth cons scenario 14	119
second cons scenario 15	0	forth cons scenario 15	92
second cons scenario 16	0	forth cons scenario 16	95
second cons scenario 17	0	forth cons scenario 17	98
second cons scenario 18	0	forth cons scenario 18	94
second cons scenario 19	0	forth cons scenario 19	79
second cons scenario 20	0	forth cons scenario 20	83

7.1.5 "f"

And f matrix: variables coefficients in the objective function

Variables' coefficients in the objective function of the decentralized model Table 9 demonstrates Variables coefficients' in the objective function of the decentralized model

Table 9: Variables' coefficients in the objective function of the decentralized model (fwo)

350	Lack2,1	175	Lack1,1	-518	Inv2,1	140	Inv1,1	-2590	Sell2,1	-2590	Sell1,1
270	Lack2,2	135	Lack1,2	-356.4	Inv2,2	108	Inv1,2	-1782	Sell2,2	-1728	Sell1,2
170	Lack2,3	85	Lack1,3	-163.2	Inv2,3	68	Inv1,3	-816	Sell2,3	-918	Sell1,3
260	Lack2,4	130	Lack1,4	-260	Inv2,4	104	Inv1,4	-1300	Sell2,4	-1352	Sell1,4
355	Lack2,5	177.5	Lack1,5	-369.2	Inv2,5	142	Inv1,5	-1846	Sell2,5	-1846	Sell1,5
205	Lack2,6	102.5	Lack1,6	-196.8	Inv2,6	82	Inv1,6	-984	Sell2,6	-1107	Sell1,6
190	Lack2,7	95	Lack1,7	-167.2	Inv2,7	76	Inv1,7	-836	Sell2,7	-912	Sell1,7
255	Lack2,8	127.5	Lack1,8	-316.2	Inv2,8	102	Inv1,8	-1581	Sell2,8	-1428	Sell1,8
195	Lack2,9	97.5	Lack1,9	-273	Inv2,9	78	Inv1,9	-1365	Sell2,9	-1248	Sell1,9
190	Lack2,10	95	Lack1,10	-205.2	Inv2,10	76	Inv1,10	-1026	Sell2,10	-1064	Sell1,10
265	Lack2,11	132.5	Lack1,11	-296.8	Inv2,11	106	Inv1,11	-1484	Sell2,11	-1378	Sell1,11
255	Lack2,12	127.5	Lack1,12	-265.2	Inv2,12	102	Inv1,12	-1326	Sell2,12	-1428	Sell1,12
430	Lack2,13	215	Lack1,13	-533.2	Inv2,13	172	Inv1,13	-2666	Sell2,13	-2752	Sell1,13
180	Lack2,14	90	Lack1,14	-230.4	Inv2,14	72	Inv1,14	-1152	Sell2,14	-1080	Sell1,14
270	Lack2,15	135	Lack1,15	-378	Inv2,15	108	Inv1,15	-1890	Sell2,15	-1674	Sell1,15
365	Lack2,16	182.5	Lack1,16	-394.2	Inv2,16	146	Inv1,16	-1971	Sell2,16	-2263	Sell1,16
190	Lack2,17	95	Lack1,17	-258.4	Inv2,17	76	Inv1,17	-1292	Sell2,17	-1368	Sell1,17
255	Lack2,18	127.5	Lack1,18	-265.2	Inv2,18	102	Inv1,18	-1326	Sell2,18	-1428	Sell1,18
210	Lack2,19	105	Lack1,19	-310.8	Inv2,19	84	Inv1,19	-1554	Sell2,19	-1302	Sell1,19
140	Lack2,20	70	Lack1,20	-246.4	Inv2,20	56	Inv1,20	-1232	Sell2,20	-952	Sell1,20

variables' coefficients in the objective function of the centralized model (fcenter)(the exact amount) it's a matric like Table9. the only difference between fcenter and fwo is about the Q_i coefficients and those are like Table10:

Table 10: Difference between Q_i Coefficients in fcenter and fwo

	fwo	fcenter
Q_1 Coef	15000	6000
Q_2 Coef	15500	6500

7.1.6 "ub"

The upper bounds of each variable

7.1.7 "db"

The lower bounds of each variable
Here the 7 matrices for inserting in a linear programming solver is over, in continue we describe other matrices such as answer matrices

7.2 Answer matrices

7.2.1 answers to the decentralized models

answers to the decentralized models are in table 11
Q1=71,Q2=94

Table 11: answers to the decentralized models

0	Lack2,1	0	Lack1,1	73	Inv2,1	0	Inv1,1	92	Sell2,1	71	Sell1,1
0	Lack2,2	0	Lack1,2	31	Inv2,2	10	Inv1,2	73	Sell2,2	61	Sell1,2
0	Lack2,3	0	Lack1,3	15	Inv2,3	4	Inv1,3	83	Sell2,3	67	Sell1,3
10	Lack2,4	1	Lack1,4	0	Inv2,4	0	Inv1,4	94	Sell2,4	71	Sell1,4
0	Lack2,5	0	Lack1,5	30	Inv2,5	9	Inv1,5	73	Sell2,5	62	Sell1,5
0	Lack2,6	0	Lack1,6	29	Inv2,6	13	Inv1,6	78	Sell2,6	58	Sell1,6
2	Lack2,7	1	Lack1,7	0	Inv2,7	0	Inv1,7	94	Sell2,7	71	Sell1,7
0	Lack2,8	0	Lack1,8	2	Inv2,8	4	Inv1,8	96	Sell2,8	67	Sell1,8
11	Lack2,9	0	Lack1,9	0	Inv2,9	1	Inv1,9	95	Sell2,9	70	Sell1,9
0	Lack2,10	0	Lack1,10	17	Inv2,10	4	Inv1,10	81	Sell2,10	67	Sell1,10
0	Lack2,11	0	Lack1,11	5	Inv2,11	1	Inv1,11	90	Sell2,11	70	Sell1,11
27	Lack2,12	10	Lack1,12	0	Inv2,12	0	Inv1,12	94	Sell2,12	71	Sell1,12
0	Lack2,13	0	Lack1,13	22	Inv2,13	11	Inv1,13	83	Sell2,13	60	Sell1,13
25	Lack2,14	0	Lack1,14	0	Inv2,14	0	Inv1,14	94	Sell2,14	71	Sell1,14
0	Lack2,15	0	Lack1,15	7	Inv2,15	5	Inv1,15	92	Sell2,15	66	Sell1,15
1	Lack2,16	0	Lack1,16	0	Inv2,16	0	Inv1,16	94	Sell2,16	71	Sell1,16
8	Lack2,17	4	Lack1,17	0	Inv2,17	0	Inv1,17	94	Sell2,17	71	Sell1,17
0	Lack2,18	0	Lack1,18	0	Inv2,18	0	Inv1,18	94	Sell2,18	71	Sell1,18
0	Lack2,19	64	Lack1,19	22	Inv2,19	71	Inv1,19	143	Sell2,19	0	Sell1,19
0	Lack2,20	60	Lack1,20	22	Inv2,20	71	Inv1,20	143	Sell2,20	0	Sell1,20

7.2.2 answers to the centralized models

answers to the centralized models are in table 12
Q1=71,Q2=121

Table 12: Answer to the centralized model

0	Lack2,1	0	Lack1,1	100	Inv2,1	0	Inv1,1	92	Sell2,1	71	Sell1,1
0	Lack2,2	0	Lack1,2	58	Inv2,2	10	Inv1,2	73	Sell2,2	61	Sell1,2
0	Lack2,3	0	Lack1,3	42	Inv2,3	4	Inv1,3	83	Sell2,3	67	Sell1,3
0	Lack2,4	1	Lack1,4	17	Inv2,4	0	Inv1,4	104	Sell2,4	71	Sell1,4
0	Lack2,5	0	Lack1,5	57	Inv2,5	9	Inv1,5	73	Sell2,5	62	Sell1,5
0	Lack2,6	0	Lack1,6	56	Inv2,6	13	Inv1,6	78	Sell2,6	58	Sell1,6
0	Lack2,7	1	Lack1,7	25	Inv2,7	0	Inv1,7	96	Sell2,7	71	Sell1,7
0	Lack2,8	0	Lack1,8	29	Inv2,8	4	Inv1,8	96	Sell2,8	67	Sell1,8
0	Lack2,9	0	Lack1,9	16	Inv2,9	1	Inv1,9	106	Sell2,9	70	Sell1,9
0	Lack2,10	0	Lack1,10	44	Inv2,10	4	Inv1,10	81	Sell2,10	67	Sell1,10
0	Lack2,11	0	Lack1,11	32	Inv2,11	1	Inv1,11	90	Sell2,11	70	Sell1,11
0	Lack2,12	10	Lack1,12	0	Inv2,12	0	Inv1,12	121	Sell2,12	71	Sell1,12
0	Lack2,13	0	Lack1,13	49	Inv2,13	11	Inv1,13	83	Sell2,13	60	Sell1,13
0	Lack2,14	0	Lack1,14	2	Inv2,14	0	Inv1,14	119	Sell2,14	71	Sell1,14
0	Lack2,15	0	Lack1,15	34	Inv2,15	5	Inv1,15	92	Sell2,15	66	Sell1,15
0	Lack2,16	0	Lack1,16	26	Inv2,16	0	Inv1,16	95	Sell2,16	71	Sell1,16
0	Lack2,17	4	Lack1,17	19	Inv2,17	0	Inv1,17	102	Sell2,17	71	Sell1,17
0	Lack2,18	0	Lack1,18	27	Inv2,18	0	Inv1,18	94	Sell2,18	71	Sell1,18
0	Lack2,19	64	Lack1,19	49	Inv2,19	71	Inv1,19	143	Sell2,19	0	Sell1,19
0	Lack2,20	60	Lack1,20	49	Inv2,20	71	Inv1,20	143	Sell2,20	0	Sell1,20

7.3 Scenario Reduction Algorithm

The volume of calculations to solve scenario-based optimization models depends on the number of scenarios. Therefore, it is necessary to reduce the set of main scenarios so that the characteristics of the potential problem do not change harshly. The number of decreased scenarios depends on the type and nature of the optimization problem and should be less than a quarter of the generated scenarios (Heitsch and Römisch, 2000). The primary idea of reducing the scenario is to eliminate low-probability and close-up scenarios. Therefore, scenario reduction algorithms identify a subset of scenarios and calculate the probabilities for the new scenarios so that the probabilities of the reduced scenarios are added to the nearest scenario in terms of probability distance. The scenario reduction algorithm reduces the batch scenarios using the Kantrovich distance matrix. For example, each scenario consists of twenty-four hours for the next day's hour-by-hour scenarios, and deleting a scenario is equivalent to deleting a twenty-four-hour string. The Kantrovich distance is the probability distance between two different sets of scenarios, and the small space between the two scenarios indicates two identical possible processes. The Kantrovich distance ensures that the maximum probability scenarios are reduced without tolerable error. The probability of all deleted scenarios is considered zero. The new probability of the preserved scenario is equal to the sum of the prior probabilities and the probabilities of the closest deleted scenarios.

7.3.1 Probability interval for scenario reduction

If the probability distance is the measurement norm in probabilistic programming problems, a set of scenarios is reduced to a more straightforward set of scenarios close to the original state. Assume that the initial probability distribution Q is defined on the scenario set Ω. The problem of optimal reduction of the Ω set can be accurately expressed as follows: Specify a subset of the $\Omega_s(\Omega_s \Subset \Omega)$ scenario and assign a new distribution to the remaining scenarios. The reduced probability distribution Q defined on the Ω set is the closest distribution to the original Q distribution in terms of probability distance. The Kantrovich distance can be expressed as follows:
KD (S, S') $=prob_s$ d (S, S')
S is a scenario string with an H subset is in the above relation. d (S, S ') is the vector distance between the two scenarios S and S' and is expressed as follows:

$$d(S,S') = \left(\left(\sum_{i=1}^{H}(s_i - s_i')\right)^2\right)^{\frac{1}{2}}$$

We use the reversal reduction technique to reduce the demand and price scenarios.

7.3.2 Scenario reduction algorithm

In this subsection, a step-by-step method for reducing the scenario is described.

1. Collect the generated scenarios (scenarios are made with the algorithm described in the scenario-making section for random parameters). Determine the probabilities of the gotten scenarios. The sum of the probabilities of the scenarios of each stage must be one. In the first stage, the likelihood of each scenario is considered to be 1/N, where N is the total number of scenarios.

2. Calculate the Kantrovich distance matrix. Calculate the distance matrix for each pair of scenarios and the Kantrovich distance matrix by multiplying the probabilities of the scenarios.

3. Scenario selection. Find the lowest Kantrovich distance scenario and mark it in the Kantrovich distance matrix.

4. Delete Scenario. Select the scenario with the least Kantrovich distance and the scenario with the closest Kantrovich distance to it. Eliminate the scenario with the shortest Kantrovich distance due to the low probability of occurrence and proximity to another scenario, and add the probability to the closest scenario. This work allows the sum of the probabilities of the remaining scenarios to remain one. The likelihood of individual scenarios must be greater than or equal to the ratio of the aggregated scenarios to the whole production scenarios. Reducing the scenario will lead to creating a lower-order probability matrix.

5. Update the probability matrix. Update the initial probability matrix with the new matrix.

6. Go to step 2.