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A Note on : The Effect of Lying in a Negotiation Game

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ABSTRACT

Lying creates some problems. It can be cognitively depleting, it can increase the risk that agents and systems will be punished, it can threaten systems self-worth by preventing them from seeing themselves as good system, and it can generally erode trust in society. Lying may be considered as a game. This paper is concerned with the effect of lying in a system containing two agents using the game theory. From a repeated measurement model, two agents (which constitute a system) play a global game and it is seen that during a repeated game, the system will be destroyed by laying an agent. The probability of failing negotiation is derived and its behavior is studied under different scenarios. Simulation results are proposed to support theoretical results. Finally, concluding remarks are given.

Keyword: Agents, Game theory, Global game, Lying, Probability of failure in negotiation, Repeated game, Repeated measurement model

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1 Introduction

It's easy to see why lying might had some damaging effects on your life, since lies are sneaky, malicious, and often hurt others. Lying destroys you with its vicious cycle. When one continues to lie; it's like putting a giant rock on his back and having to carry it around everywhere him. There are as many advantages for truthfulness as there are disadvantages for lying. Truthfulness is one of the most beautiful traits and lying one of the ugliest. The tongue translates man's internal feelings to the outside, therefore if lying stems from envy and or enmity it is one of the dangerous signs of anger; and if it stems from stinginess or habit, it is from the effects of the burning lusts of man, (Mahon, 2008).

Further, dishonesty may be grounds for a range of discipline. An employer may decide to demote, temporarily suspend, or take work or clients away from an employee as discipline. However, more serious consequences like getting fired, sued, losing your license, or facing criminal charges may be possible. Even when you have concrete evidence of lying, it's difficult to take action. We're taught as children that lying is wrong and devious. We may feel hurt that the other person didn't trust us, or angry that they were able to manipulate and take advantage of us, see Druzin (2011) and references there in.

But once you've gone through the normal reactions of hurt and anger, instead of losing faith in all your team members and your ability to manage them. Some employees are afraid that you or others will have a negative reaction to the truth. They feel personally involved and are fearful of creating a bad outcome for themselves or you. They want to make sure that you're not disappointed in them, or that you won't have any need to discipline them, see Brashier and Marsh (2020).

In economics and game theory, global games are games of incomplete information where players receive possibly correlated signals of the underlying state of the world. The most important practical application of global games has been the study of crises in financial markets such as bank runs, currency crises, and bubbles. However, they have other relevant applications such as investments with payoff complementarities, beauty contests, political riots and revolutions, and economic situation which displays strategic complementarily. In the current paper, the effect of lying between two agents, throughout the global game, under the repeated measurement models is studied; see Morris and Shin (2001).

Consider two agents A and B which communicate about the common parameter θ. For example, in a duopoly market in the presence of two producers, the negotiate about the price of a specific product. Each agent (player) sends a different signal about θ and this game is repeated many times, i.e.,

$$\begin{cases} x_i = \theta + \varepsilon_i, \\ y_i = \theta + \xi_i, \end{cases}$$

$i \geq 1$.This type of formulation is applied in the context of global game, see Morris and Shin (1998) and references therein. Assume that ε_i's and ξ_i's are independent

sequences of independent random variables with zero means and variances σ_ε^2 and σ_ξ^2, respectively.
The rest of paper is organized as follows. Next section studies the agent's behavior in the present of lying form his/her opponent. Concluding remarks are given in section 3.
2 Lying game. Consider a fixed $t > 0$. Notice that, given θ, the sample mean $\overline{x}_t = \frac{1}{t}\sum_{j=1}^{t} x_j$ has normal distribution with mean θ and variance σ_ε^2/t, i.e., $N(\theta, \sigma_\varepsilon^2/t)$. Assume that θ has an improper uniform distribution on real line, that is, $U(-\infty,\infty)$. Using the Bayes theorem, it is seen that the posterior θ given $\overline{x}_t$ has $N(\overline{x}_t, \sigma_\varepsilon^2/t)$. Hence, as soon as a new observation y_{t+1} is observed, then y_{t+1} given θ has $N(\theta, \sigma_\xi^2)$ distribution. Hence, the y_{t+1} given $\overline{x}_t$ has $N(\overline{x}_t, \sigma_\xi^2 + \sigma_\varepsilon^2/t)$ distribution. Suppose that as soon as y_{t+1} (or x_{t+1}) is larger than threshold L, then the negotiation is failed. Thus, the related probability is given by $p_t = P\left(y_{t+1} > L \,\middle|\, \overline{x}_t\right) =$

$$\Phi\left(\frac{\overline{x}_t - L}{\sqrt{\sigma_\xi^2 + \frac{\sigma_\varepsilon^2}{t}}}\right).$$

Here, Φ is the distribution function of standard normal distribution. Let $q_t = \Phi^{-1}(p_t)$. Therefore, $q_t = \frac{\overline{x}_t - L}{\sqrt{\sigma_\xi^2 + \frac{\sigma_\varepsilon^2}{t}}}$. Notice that $\overline{x}_t - L = \left(1 - \frac{1}{t}\right)(\overline{x}_{t-1} - L) + \left(\frac{1}{t}\right)(x_t - L)$. Then,

$$q_t = a_t\left(1 - \frac{1}{t}\right)q_{t-1} + b_t\left(\frac{1}{t}\right)(x_t - L),$$

where $a_t = \frac{\sqrt{\frac{1}{t-1} + k^{-2}}}{\sqrt{\frac{1}{t} + k^{-2}}}$ and $b_t = \frac{1}{\sqrt{\frac{1}{t} + k^{-2}}}$, where $k = \frac{\sigma_\varepsilon}{\sigma_\zeta}$.
The above formula can be studies under different scenarios. Three scenarios are observed, as follows:
(a) $k = 1$, then, $q_t = \sqrt{\frac{t-1}{t+1}}q_{t-1} + \sqrt{\frac{t}{t+1}}(x_t - L)$.
(b) $k \to \infty$, then, $q_t = \sqrt{\frac{t}{t-1}}q_{t-1} + \sqrt{\frac{1}{t}}(x_t - L)$.
(c) $k \to 0$, then, $q_t = (1 - \frac{1}{t})q_{t-1}$.
Let $t \to \infty$, then cases (a), (b) and (c) reduce to the
(a) $k = 1$, then, $q_t = q_{t-1} + (x_t - L)$.
(b) $k \to \infty$, then, $q_t = q_{t-1}$.
(c) $k \to 0$, then, $q_t = q_{t-1}$.
That is, as soon as, one agent (it does not matter) does not behave accurately, about the estimation of θ, then, the probability of failure in negotiation is constant. However, when both agents are accurate, then q_t obeys a random walk. If one of them starts to lying, then, the innovations of q_t is positive and thus it converges to infinity, too soon. Thus, p_t converges to 1, soon. The following proposition summarizes the above discussion.

Proposition 1.
(a) The probability p_t of cutting a negotiation because of lying of an agent is given by the recursive formula

$$q_t = a_t\left(1-\frac{1}{t}\right)q_{t-1} + b_t\left(\frac{1}{t}\right)(x_t - L),$$

at which $q_t = \Phi^{-1}(p_t)$. Here, Φ is the distribution function of standard normal distribution.
(b) The following scenarios are given for several values of k.

$$q_t = \begin{cases} \sqrt{\frac{t-1}{t+1}}q_{t-1} + \sqrt{\frac{t}{t+1}}(x_t - L) & k=1 \\ \sqrt{\frac{t}{t-1}}q_{t-1} + \sqrt{\frac{1}{t}}(x_t - L) & k\to\infty \\ (1-\frac{1}{t})q_{t-1} & k\to 0 \end{cases}$$

(c) The following scenarios are given for several values of k, as $t\to\infty$.

$$q_t = \begin{cases} q_t = q_{t-1} + (x_t - L) & k=1 \\ q_t = q_{t-1} & k\to\infty \\ q_t = q_{t-1} & k\to 0 \end{cases}$$

When the game is repeated infinitely (*i.e.*, t is large), if any player is irrational, that is $k\to 0$ or ∞, then p_t does not change. However, when the level of rationality of both players are the same, then,

$$p_t = \Phi(q_{t-1} + (x_t - L)).$$

Here, assuming $\sigma_\varepsilon(x_t - L) = v$ and $x_t - L$ is close to zero, in probability, then,

$$p_t = \Phi\left(q_{t-1} + \frac{v}{\sigma_\varepsilon}\right).$$

Assuming $\frac{v}{\sigma_\varepsilon}$ goes to zero, then using the Taylor expansion, it is seen that

$$p_t = p_{t-1} + \emptyset(q_{t-1})\frac{v}{\sigma_\varepsilon} + \emptyset'(q_{t-1})\frac{v^2}{2\sigma_\varepsilon^2}.$$

To maximize the difference $p_t - p_{t-1}$ with respect to v, by differentiating it with respect to v, it is seen that the maximizing v is $\frac{-\emptyset(q_{t-1})}{\emptyset'(q_{t-1})}\sigma_\varepsilon$. Then, the maximized $p_t - p_{t-1}$ is $\frac{-\emptyset^2(p_{t-1})}{2\emptyset'(p_{t-1})}\sigma_\varepsilon$. Therefore, $p_t = p_{t-1} - 0.5\frac{\emptyset^2(q_{t-1})}{\emptyset'(q_{t-1})}$. Here, $\emptyset$ and $\emptyset'$ are density function and its derivative of standard normal distribution. Notice that $\emptyset'(z) = -z\emptyset(z)$. Thus, $\frac{\emptyset^2(z)}{\emptyset'(z)} = -\frac{\emptyset(z)}{z}$. Thus, $p_t = p_{t-1} + 0.5\frac{\emptyset(p_{t-1})}{p_{t-1}}$. The following proposition summarizes the above discussion.
Proposition 2. For large t's, the probability of existing is given

$$p_t = \min\left(1, p_{t-1} + 0.5\frac{\emptyset(q_{t-1})}{q_{t-1}}\right).$$

3 Simulation. Here, some simulated examples are given, to study the behavior of p_t.
Example 1: p_t series. Let $\sigma_\varepsilon = 0.1$, $\sigma_\zeta = 0.2$, $L = 0.5$, $k = 0.5$. Thus,

$$a_t = \frac{\sqrt{\frac{1}{t-1} + 4}}{\sqrt{\frac{1}{t} + 4}}, \; b_t = \frac{1}{\sqrt{\frac{1}{t} + 4}}.$$

Here, x_t comes from normal distribution $N(\theta, 0.01)$. Let the initial value for θ is 0.001. The following figure shows the time series plot of difference of logarithm of p_t. Points are high, show times both players agree to exit the negotiation and points are low indicates times both players agree to contribute.

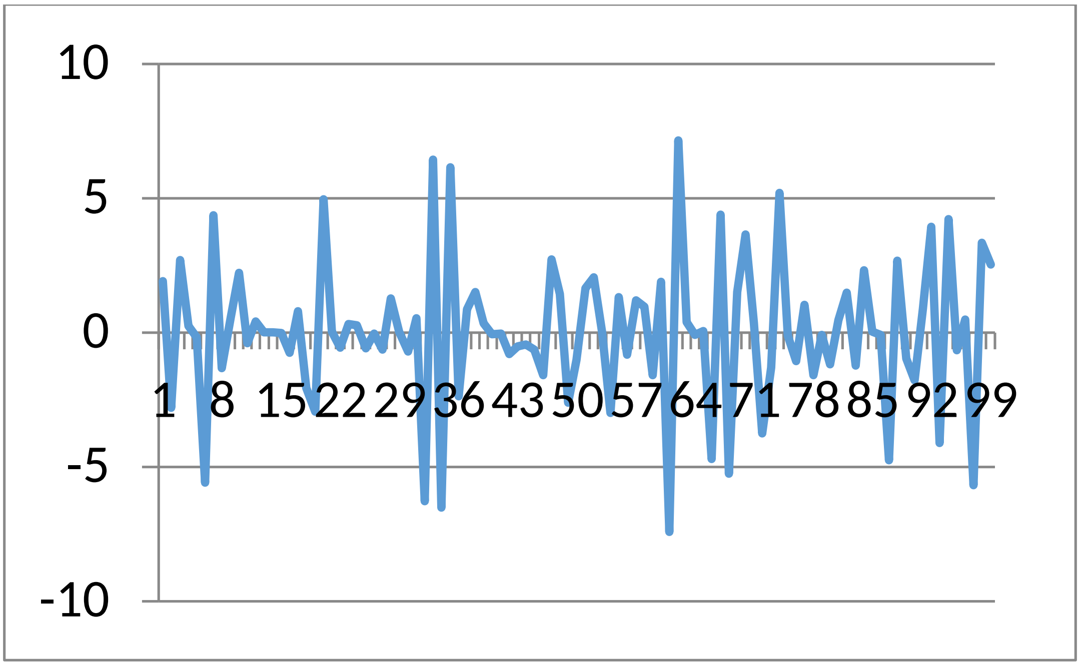

Figure 1: Plot of difference of logarithm of p_t]

Example 2: **Scenario analysis**. For large t's, and assuming the initial probability $p_0 =$ 0.5 (to be fair), then, the following figure plots p_t.
It is seen, as t's get large, then, p_t's tend to one, as it is expected.
3 Brief Concluding remarks. Unfortunately, lying is not particularly rare in the workplace. People engage in all manner of workplace dishonesty. People lie about their qualifications during job interviews, they claim sick days when they are healthy, they exaggerate their productivity, they conceal their mistakes and failures, they take credit for others' work, and they deceptively undermine others with whom they are competing for promotions or other limited resources.
This paper studies the effect of lying in a repeated negotiation game. The following features are important:

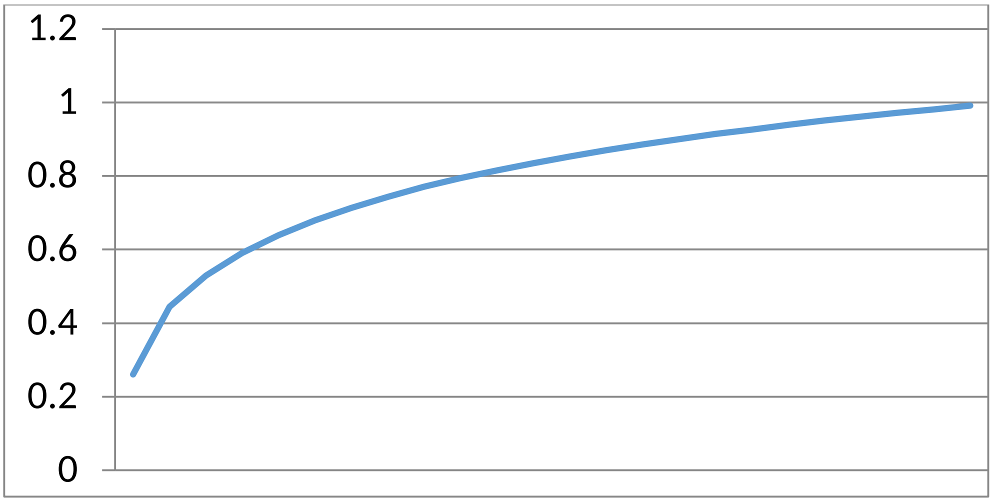

Figure 2: Plot of recursive p_t's

(i) The underlying model is repeated measurement model at which a global game is included. Indeed, there is a hidden variable which effects the actions of two agents and one of agents lies in a specific stage of game.
(ii) A recursive relation for the probability of failing in negotiation p_t is derived which shows the trend of success and failure in interaction between two agents.
(iii) Under different scenario behavior of p_t is studied.

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