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Pair Difference Cordial Labeling of Double Alternate Snake Graphs

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ABSTRACT

In this paper we investigate the pair difference cordial labeling behavior of double alternate triangular snake and double alternate quadrilatral snake graphs.

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1 Introduction

We consider only finite, undirected and simple graphs. Cordial labeling was introduced by cahit in the year 1987 [1]. Motivated by this concept we have introduce The pair difference cordial labeling in [4]. Also pair difference cordial labeling behavior of several graphs have been Studied in [4,5,6,7,8,9,10,11,12,13,14]. In this paper we investigate the pair difference cordial labeling behaviour of double alternate triangular snake and double alternate quadrilatral snake graphs.Terms not deined here follows from [2,3].

2 Preliminaries

Definition 1. [2]
The alternate triangular snake $A(T_n)$ is obtained from the path $u_1u_2\cdots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.Now we define the vertex set and edge set of $A(T_n)$ as follows.

Type 1. *The edge u_1u_2 lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle.In this case n is even.Let $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n}{2}\}$ and $E(A(T_n)) = \{u_{2i}u_{2i+1}, u_{2i}v_j, u_{2i-1}v_j : 1 \leq i, j \leq \frac{n}{2}\}$.There are $\frac{3n}{2}$ vertices and $2n-1$ edges.*

Type 2. *The edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ not lies on the triangle.Also n is even.Let $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n-2}{2}\}$ and $E(A(T_n)) = \{u_{2i}v_j, u_{2i+1}v_j : 1 \leq i, j \leq \frac{n-2}{2}\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$.There are $\frac{3n-2}{2}$ vertices and $2n-3$ edges.*

Type 3. *The edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle.In this type n is odd.Let $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n-1}{2}\}$ and $E(A(T_n)) = \{u_{2i}v_j, u_{2i+1}v_j : 1 \leq i, j \leq \frac{n-1}{2}\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$.There are $\frac{3n-1}{2}$ vertices and $2n-2$ edges.*

Definition 2. [2] *The double alternate triangular snake $A(T_n)$ is obtained from the path $u_1u_2\cdots u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i and w_i.Now we define the vertex set and edge set of $DA(T_n)$ as follows.*

Type 1. *The edge u_1u_2 lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle. In this case n is even. Let $V(DA(T_n)) = \{u_i, v_j, w_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n}{2}\}$ and $E(DA(T_n)) = \{u_{2i}u_{2i+1}, u_{2i}v_j, u_{2i-1}v_j, u_{2i}w_j, u_{2i-1}w_j : 1 \leq i, j \leq \frac{n}{2}\}$.There are $2n$ vertices and $3n-1$ edges.*

Type 2. *The edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ not lies on the triangle. Here clearly n is even. Let $V(DA(T_n)) = \{u_i, v_j, w_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n-2}{2}\}$*

and $E(DA(T_n)) = \{u_{2i}v_j, u_{2i+1}v_j, u_{2i}w_j, u_{2i+1}w_j : 1 \leq i, j \leq \frac{n-2}{2}\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$*.There are* $2n-2$ *vertices and* $3n-5$ *edges.*

Type 3. *The edge* u_1u_2 *not lies on the triangle and the edge* $u_{n-1}u_n$ *lies on the triangle. In this type* n *is odd. Let* $V(A(T_n)) = \{u_i, v_j, w_j : 1 \leq i \leq n, 1 \leq j \leq \frac{n-1}{2}\}$ *and* $E(A(T_n)) = \{u_{2i}v_j, u_{2i+1}v_j, u_{2i}w_j, u_{2i+1}w_j : 1 \leq i, j \leq \frac{n-1}{2}\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\}$*.There are* $2n-1$ *vertices and* $3n-3$ *edges.*

Definition 3. [2] *The alternate quadrilateral snake* $A(Q_n)$ *is obtained from the path* $u_1u_2 \cdots u_n$ *by joining* u_i *and* u_{i+1} *(alternatively) to new vertices* v_i, w_i *respectively and then joining* v_i *and* w_i*. That is every alternate edge of a path is replaced by a cycle* C_4*.Now we define the vertex set and edge set of* $A(Q_n)$ *as follows.*

Type 1. *The edge* u_1u_2 *lies on the quadrilateral and the edge* $u_{n-1}u_n$ *lies on the quadrilateral.In this type* n *is even. Let* $V(A(Q_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_j : 1 \leq j \leq \frac{n}{2}\}$ *and* $E(A(Q_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i-1}v_i, v_iw_i, w_iu_{2i} : 1 \leq i \leq \frac{n}{2}\}$*.There are* $2n$ *vertices and* $\frac{5n-2}{2}$ *edges.*

Type 2. *The edge* u_1u_2 *not lies on the quadrilateral and the edge* $u_{n-1}u_n$ *lies on the quadrilateral.In this case* n *is odd. Let* $V(A(Q_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_j : 1 \leq j \leq \frac{n-1}{2}\}$ *and* $E(A(Q_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i}v_i, v_iw_i, w_iu_{2i+1} : 1 \leq i \leq \frac{n-1}{2}\}$*.There are* $2n-1$ *vertices and* $\frac{5n-5}{2}$ *edges.*

Type 3. *The edge* u_1u_2 *not lies on the quadrilateral and the edge* $u_{n-1}u_n$ *not lies on the quadrilateral.Here clearly* n *is even. Let* $V(A(Q_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_j : 1 \leq j \leq \frac{n-2}{2}\}$ *and* $E(A(Q_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i}v_i, v_iw_i, w_iu_{2i+1} : 1 \leq i \leq \frac{n-2}{2}\}$*.There are* $2n-2$ *vertices and* $\frac{5n-8}{2}$ *edges.*

Definition 4. [2] *The double alternate quadrilateral snake* $DA(Q_n)$ *is obtained from the path* $u_1u_2 \cdots u_n$ *by joining* u_i *and* u_{i+1} *(alternatively) to new vertices* v_i, w_i *and* x_i, y_i *respectively and then joining* v_i *and* w_i *and also joining* x_i *and* y_i*.Now we define the vertex set and edge set of* $dA(Q_n)$ *as follows.*

Type 1. *The edge* u_1u_2 *lies on the quadrilateral and the edge* $u_{n-1}u_n$ *lies on the quadrilateral.In this type* n *is even. Let* $V(DA(Q_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_j, w_j, x_j, y_j : 1 \leq j \leq \frac{n}{2}\}$ *and* $E(DA(Q_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i-1}v_i, v_iw_i, w_iu_{2i}, u_{2i-1}x_i, x_iy_i, y_iu_{2i} : 1 \leq i \leq \frac{n}{2}\}$*.There are* $3n$ *vertices and* $4n-1$ *edges.*

Type 2. *The edge* u_1u_2 *not lies on the quadrilateral and the edge* $u_{n-1}u_n$ *lies on the quadrilateral.In this case* n *is odd. Let* $V(DA(Q_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_j, w_j, x_j, y_j : 1 \leq j \leq \frac{n-1}{2}\}$ *and* $E(DA(Q_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i}v_i, v_iw_i, w_iu_{2i+1}, u_{2i}x_i, x_iy_i, y_iu_{2i+1} :$

$1 \leq i \leq \frac{n-1}{2}\}$. *There are* $2n-1$ *vertices and* $4n-4$ *edges.*

Type 3. *The edge* u_1u_2 *not lies on the quadrilateral and the edge* $u_{n-1}u_n$ *not lies on the quadrilateral.Here clearly* n *is even. Let* $V(DA(Q_n)) = \{u_i : 1 \leq i \leq n\} \cup \{v_j, w_j, x_j, y_j : 1 \leq j \leq \frac{n-2}{2}\}$ *and* $E(DA(Q_n)) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i}v_i, v_iw_i, w_iu_{2i+1}, u_{2i}x_i, x_iy_i, y_iu_{2i+1} : 1 \leq i \leq \frac{n-2}{2}\}$. *There are* $2n-2$ *vertices and* $4n-7$ *edges.*

3 Main results

Theorem 5. *The double alternate triangular snake* $DA(T_n)$ *is pair difference cordial if the edge* u_1u_2 *lies on the triangle and the edge* $u_{n-1}u_n$ *lies on the triangle for all even* $n \geq 4$.

Proof. The vertex set and edge set are taken from the definition 2. There are two cases arises.

Case 1. $n \equiv 0 \pmod 4$.
Assign the labels $1, 5, 9, \cdots, n-3$ to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n-2}{2}}$ respectively and assign the labels $3, 7, 11, \cdots, n-1$ respectively to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n}{2}}$. Now we assign the labels $-1, -5, -9, \cdots, -(n-3)$ respectively to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, \cdots, u_{n-1}$ and assign the labels $-3, -7, -11, \cdots, -(n-1)$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}}, \cdots, u_n$ respectively.

We next Assign the labels $2, 6, 10, \cdots, n-2$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n}{4}}$ respectively and assign the labels $4, 8, 12, \cdots, n$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n}{4}}$. Now we assign the labels $-2, -6, -10 \cdots, -(n-2)$ to the vertices $v_{\frac{n+4}{4}}, v_{\frac{n+8}{4}}, v_{\frac{n+12}{4}}, \cdots v_{\frac{n}{2}}$ respectively and assign the labels $-4, -8, -12, \cdots, -n$ respectively to the vertices $w_{\frac{n+4}{4}}, w_{\frac{n+8}{4}}, w_{\frac{n+12}{4}}, \cdots w_{\frac{n}{2}}$.

Case 2. $n \equiv 2 \pmod 4$.
Assign the labels $1, 5, 9, \cdots, n-5$ to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n-4}{2}}$ respectively and assign the labels $3, 7, 11, \cdots, n-3$ respectively to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-2}{2}}$. Now we assign the labels $-1, -5, -9, \cdots, -(n-5)$ respectively to the vertices $u_{\frac{n}{2}}, u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, \cdots, u_{n-3}$ and assign the labels $-3, -7, -11, \cdots, -(n-3)$ to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, \cdots, u_{n-2}$ respectively.

We next Assign the labels $2, 6, 10, \cdots, n-4$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-2}{4}}$ respectively and assign the labels $4, 8, 12, \cdots, n-2$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-2}{4}}$. Now we assign the labels $-2, -6, -10 \cdots, -(n-4)$ to the vertices $v_{\frac{n+2}{4}}, v_{\frac{n+6}{4}}, v_{\frac{n+10}{4}}, \cdots, v_{\frac{n-2}{2}}$ respectively and assign the labels $-4, -8, -12, \cdots, -(n-2)$ respectively to the vertices $w_{\frac{n+2}{4}}, w_{\frac{n+6}{4}}, w_{\frac{n+10}{4}}, \cdots, w_{\frac{n-2}{2}}$.

Finally assign the labels $n-1, n, -(n-1), -n$ respectively to the vertices $u_{n-1}, v_{\frac{n}{2}}$, $u_n, w_{\frac{n}{2}}$.
This vertex labeling gives the pair difference cordial labeling of double alternate triangular snake $DA(T_n)$.
In both cases $\Delta_{f_1} = n, \Delta_{f_1^c} = n-1$.

□

Theorem 6. *The double alternate triangular snake $DA(T_n)$ is pair difference cordial if the edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ not lies on the triangle for all even $n \geq 4$.*

Proof. The vertex set and edge set are taken from the definition 2. There are two cases arises.
Case 1. $n \equiv 0 \pmod 4$.
Assign the labels $2, 6, 10, \cdots, n-6$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-4}{2}}$ respectively and assign the labels $4, 8, 12, \cdots, n-4$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n-2}{2}}$. Now we assign the labels $-2, -6, -10 \cdots, -(n-6)$ respectively to the vertices $u_{n-1}, u_{n-3}, u_{n-5}, \cdots$, $u_{\frac{n+6}{2}}$ and assign the labels $-4, -8, -12, \cdots, -(n-4)$ to the vertices $u_{n-2}, u_{n-4}, u_{n-6}$, $\cdots, u_{\frac{n+4}{2}}$ respectively.

We next Assign the labels $3, 7, 11, \cdots, n-5$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-4}{4}}$ respectively and assign the labels $5, 9, 13, \cdots, n-3$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-4}{4}}$. Now we assign the labels $-3, -7, -11 \cdots, -(n-5)$ to the vertices $v_{\frac{n-2}{2}}, v_{\frac{n-4}{2}}, v_{\frac{n-6}{2}}, \cdots, v_{\frac{n+4}{4}}$ respectively and assign the labels $-5, -9, -13, \cdots, -(n-3)$ respectively to the vertices $w_{\frac{n-2}{2}}, w_{\frac{n-4}{2}}, w_{\frac{n-6}{2}}, \cdots, w_{\frac{n+4}{4}}$.

Now we assign the labels $(n-2), -(n-1), n-1, -(n-2)$ respectively to the vertices $u_{\frac{n}{2}}, u_{\frac{n+2}{2}}, v_{\frac{n}{4}}, w_{\frac{n}{4}}$.

Case 2. $n \equiv 2 \pmod 4$.
Assign the labels $1, 5, 9, \cdots, n-5$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-2}{2}}$ respectively and assign the labels $3, 7, 11, \cdots, n-3$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n}{2}}$. Now we assign the labels $-1, -5, -9, \cdots, -(n-5)$ respectively to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, \cdots$, u_{n-2} and assign the labels $-3, -7, -11, \cdots, -(n-3)$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}}$, $\cdots, u_{n-1}$ respectively.

We next Assign the labels $2, 6, 10, \cdots, n-4$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-2}{4}}$ respectively and assign the labels $4, 8, 12, \cdots, n-2$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-2}{4}}$. Now we assign the labels $-2, -6, -10 \cdots, -(n-4)$ to the vertices $v_{\frac{n+2}{4}}, v_{\frac{n+6}{4}}, v_{\frac{n+10}{4}}, \cdots v_{\frac{n-2}{2}}$ respectively and assign the labels $-4, -8, -12, \cdots, -(n-2)$ respectively to the vertices $w_{\frac{n+2}{4}}, w_{\frac{n+6}{4}}, w_{\frac{n+10}{4}}, \cdots w_{\frac{n-2}{2}}$.

Finally assign the labels $n-1, -(n-1)$ respectively to the vertices u_1, u_n.
This vertex labeling gives the pair difference cordial labeling of double alternate triangular snake $DA(T_n)$.
In both cases $\Delta_{f_1} = \frac{3n-8}{2}, \Delta_{f_1^c} = \frac{3n-2}{2}$.

□

Theorem 7. *The double alternate triangular snake $DA(T_n)$ is pair difference cordial if the edge u_1u_2 not lies on the triangle and the edge $u_{n-1}u_n$ lies on the triangle for all odd $n \geq 3$.*

Proof. Take the vertex set and edge set from the definition 4. There are two cases arises.
Case 1. $n \equiv 1 \pmod 4$.
Assign the labels $1, 5, 9, \cdots, n-4$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-1}{2}}$ respectively and assign the labels $3, 7, 11, \cdots, n-2$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n+1}{2}}$. Now we assign the labels $-1, -5, -9, \cdots, -(n-4)$ respectively to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+7}{2}}, u_{\frac{n+11}{2}}, \cdots,$ u_{n-1} and assign the labels $-3, -7, -11, \cdots, -(n-2)$ to the vertices $u_{\frac{n+5}{2}}, u_{\frac{n+9}{2}}, u_{\frac{n+13}{2}},$ $\cdots, u_n$ respectively.

We next Assign the labels $2, 6, 10, \cdots, n-3$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-1}{4}}$ respectively and assign the labels $4, 8, 12, \cdots, n-1$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-1}{4}}$. Now we assign the labels $-2, -6, -10 \cdots, -(n-3)$ to the vertices $v_{\frac{n+3}{4}}, v_{\frac{n+7}{4}}, v_{\frac{n+11}{4}}, \cdots v_{\frac{n-1}{2}}$ respectively and assign the labels $-4, -8, -12, \cdots, -(n-1)$ respectively to the vertices $w_{\frac{n+3}{4}}, w_{\frac{n+7}{4}}, w_{\frac{n+11}{4}}, \cdots w_{\frac{n-1}{2}}$ and assign the label 1 to the vertex u_1.

Case 2. $n \equiv 3 \pmod 4$.
Assign the labels $1, 5, 9, \cdots, n-6$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-3}{2}}$ respectively and assign the labels $3, 7, 11, \cdots, n-4$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n-1}{2}}$. Now we assign the labels $-1, -5, -9, \cdots, -(n-6)$ respectively to the vertices $u_{\frac{n+1}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+9}{2}}, \cdots,$ u_{n-3} and assign the labels $-3, -7, -11, \cdots, -(n-4)$ to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+7}{2}}, u_{\frac{n+11}{2}},$ $\cdots, u_{n-2}$ respectively.

We next Assign the labels $2, 6, 10, \cdots, n-5$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-3}{4}}$ respectively and assign the labels $4, 8, 12, \cdots, n-3$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-3}{4}}$. Now we assign the labels $-2, -6, -10 \cdots, -(n-5)$ to the vertices $v_{\frac{n+1}{4}}, v_{\frac{n+5}{4}}, v_{\frac{n+9}{4}}, \cdots v_{\frac{n-1}{2}}$ respectively and assign the labels $-4, -8, -12, \cdots, -(n-3)$ respectively to the vertices $w_{\frac{n+1}{4}}, w_{\frac{n+5}{4}}, w_{\frac{n+9}{4}}, \cdots w_{\frac{n-3}{2}}$ and assign the label 2 to the vertex u_1.

Finally assign the labels $n-2, -(n-1), n-1, -(n-2)$ respectively to the vertices $u_{n-1}, u_n, v_{\frac{n-1}{2}}, w_{\frac{n-1}{2}}$.
This vertex labeling gives the pair difference cordial labeling of double alternate triangular snake $DA(T_n)$.
In both cases $\Delta_{f_1} = \frac{3n-3}{2}, \Delta_{f_1^c} = \frac{3n-3}{2}$.

□

Theorem 8. *The double alternate quadrilateral snake $DA(Q_n)$ is pair difference cordial if the edge u_1u_2 lies on the quadrilateral and the edge $u_{n-1}u_n$ lies on the quadrilateral for all even $n \geq 4$.*

Proof. The vertex set and edge set are taken from the definition 4.There are two cases arises.
Case 1. $n \equiv 0 \pmod 4$.
Assign the labels $1, 7, 13, \cdots, \frac{3n-10}{2}$ to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n-2}{2}}$ respectively and assign the labels $4, 10, 16, \cdots, \frac{3n-4}{2}$ respectively to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n}{2}}$. Now we assign the labels $-1, -7, -13, \cdots, -(\frac{3n-10}{2})$ respectively to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}}, \cdots,$ u_{n-1} and assign the labels $-4, -10, -16, \cdots, -(\frac{3n-4}{2})$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}},$ $\cdots, u_n$ respectively.

We next assign the labels $2, 8, 14, \cdots, \frac{3n-8}{2}$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n}{4}}$ respectively and assign the labels $3, 9, 15, \cdots, \frac{3n-6}{2}$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n}{4}}$. Now we assign the labels $5, 11, 17, \cdots, \frac{3n-2}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{4}}$ and assign the labels $6, 12, 18, \cdots, \frac{3n}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n}{4}}$.

Now we assign the labels $-2, -8, -14, \cdots, -(\frac{3n-8}{2})$ to the vertices $v_{\frac{n+4}{4}}, v_{\frac{n+8}{4}}, v_{\frac{n+12}{4}}, \cdots,$ $v_{\frac{n}{2}}$ respectively and and assign the labels $-3, -9, -15, \cdots, -(\frac{3n-6}{2})$ respectively to the vertices $w_{\frac{n+4}{4}}, w_{\frac{n+8}{4}}, w_{\frac{n+12}{4}}, \cdots, w_{\frac{n}{2}}$. Now assign the labels $-5, -11, -17, \cdots, -(\frac{3n-2}{2})$ respectively to the vertices $x_{\frac{n+4}{4}}, x_{\frac{n+8}{4}}, x_{\frac{n+12}{4}}, \cdots, x_{\frac{n}{2}}$ and assign the labels $-6, -12, -18, \cdots, -(\frac{3n}{2})$ respectively to the vertices $y_{\frac{n+4}{4}}, y_{\frac{n+8}{4}}, y_{\frac{n+12}{4}}, \cdots, y_{\frac{n}{2}}$.

Case 2. $n \equiv 2 \pmod 4$.
Assign the labels $1, 7, 13, \cdots, \frac{3n-16}{2}$ to the vertices $u_1, u_3, u_5, \cdots, u_{\frac{n-4}{2}}$ respectively and assign the labels $4, 10, 16, \cdots, \frac{3n-10}{2}$ respectively to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-2}{2}}$. Now we assign the labels $-1, -7, -13, \cdots, -(\frac{3n-16}{2})$ respectively to the vertices $u_{\frac{n-2}{2}}, u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}},$ $\cdots, u_{n-3}$ and assign the labels $-4, -10, -16, \cdots, -(\frac{3n-10}{2})$ to the vertices $u_{\frac{n}{2}}, u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}},$ $\cdots, u_{n-2}$ respectively.

We next assign the labels $2, 8, 14, \cdots, \frac{3n-14}{2}$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-2}{4}}$ respectively and assign the labels $3, 9, 15, \cdots, \frac{3n-12}{2}$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-2}{4}}$. Now we assign the labels $5, 11, 17, \cdots, \frac{3n-8}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-2}{4}}$ and assign the labels $6, 12, 18, \cdots, \frac{3n-6}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-2}{4}}$.

Now we assign the labels $-2, -8, -14, \cdots, -(\frac{3n-14}{2})$ to the vertices $v_{\frac{n+2}{4}}, v_{\frac{n+6}{4}}, v_{\frac{n+10}{4}}, \cdots,$ $v_{\frac{n}{2}-1}$ respectively and and assign the labels $-3, -9, -15, \cdots, -(\frac{3n-6}{2})$ respectively to the vertices $w_{\frac{n+2}{4}}, w_{\frac{n+6}{4}}, w_{\frac{n+10}{4}}, \cdots, w_{\frac{n}{2}-1}$. Now assign the labels $-5, -11, -17, \cdots, -(\frac{3n-12}{2})$ respectively to the vertices $x_{\frac{n+2}{4}}, x_{\frac{n+6}{4}}, x_{\frac{n+10}{4}}, \cdots, x_{\frac{n}{2}-1}$ and assign the labels $-6, -12, -18,$ $\cdots, -(\frac{3n-6}{2})$ respectively to the vertices $y_{\frac{n+2}{4}},$

$y_{\frac{n+6}{4}}, y_{\frac{n+10}{4}}, \cdots, y_{\frac{n}{2}-1}$.

Finally assign the labels $\frac{3n-4}{2}, \frac{3n-2}{2}, \frac{3n}{2}$ respectively to the vertices $v_{n-1}, u_{n-1}, w_{n-1}$ and assign the labels $-(\frac{3n-4}{2}), -(\frac{3n-2}{2}), -(\frac{3n}{2})$ respectively to the vertices v_n, u_n, w_n

This vertex labeling gives the pair difference cordial labeling of double alternate quadrilateral snake $DA(Q_n)$.
In both cases $\Delta_{f_1} = 2n, \Delta_{f_1^c} = 2n-1$.

□

Theorem 9. *The double alternate quadrilateral snake $DA(Q_n)$ is pair difference cordial if the edge u_1u_2 not lies on the quadrilateral and the edge $u_{n-1}u_n$ not lies on the quadrilateral for all even $n \geq 4$.*

Proof. The vertex set and edge set taken from the definition 4.There are two cases arises.
Case 1. $n \equiv 0 \pmod 4$.
Assign the labels $1, 7, 13, \cdots, \frac{3n-22}{2}$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-4}{2}}$ respectively and assign the labels $4, 10, 16, \cdots, \frac{3n-16}{2}$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n-2}{2}}$. Now we assign the labels $-1, -7, -13, \cdots, -(\frac{3n-22}{2})$ respectively to the vertices $u_{\frac{n}{2}}, u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, \cdots,$ u_{n-3} and assign the labels $-4, -10, -16, \cdots, -(\frac{3n-16}{2})$ to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}},$ $\cdots, u_{n-1}$ respectively. Next assign the labels $\frac{3n-10}{2}, \frac{3n-8}{2}, \frac{3n-6}{2}, \frac{3n-4}{2}$ respectively to the vertices $v_{\frac{n-2}{2}}, u_{n-2}, x_{\frac{n-2}{2}}, u_1$ and assign the labels $-(\frac{3n-10}{2}), -(\frac{3n-8}{2}), -(\frac{3n-6}{2}), -(\frac{3n-4}{2})$ respectively to the vertices $w_{\frac{n}{2}}, u_{n-1}, y_{\frac{n}{2}}, u_n$.

We next assign the labels $2, 8, 14, \cdots, \frac{3n-20}{2}$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-4}{4}}$ respectively and assign the labels $3, 9, 15, \cdots, \frac{3n-18}{2}$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-4}{4}}$. Now we assign the labels $5, 11, 17, \cdots, \frac{3n-14}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-4}{4}}$ and assign the labels $6, 12, 18, \cdots, \frac{3n-12}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots,$
$y_{\frac{n-4}{4}}$.

Now we assign the labels $-2, -8, -14, \cdots, -(\frac{3n-8}{2})$ to the vertices $v_{\frac{n}{4}}, v_{\frac{n+4}{4}}, v_{\frac{n+8}{4}}, \cdots, v_{\frac{n-4}{2}}$ respectively and and assign the labels $-3, -9, -15, \cdots, -(\frac{3n-6}{2})$ respectively to the vertices $w_{\frac{n}{4}}, w_{\frac{n+4}{4}}, w_{\frac{n+8}{4}}, \cdots, w_{\frac{n-4}{2}}$. Now assign the labels $-5, -11, -17, \cdots, -(\frac{3n-2}{2})$ respectively to the vertices $x_{\frac{n}{4}}, x_{\frac{n+4}{4}}, x_{\frac{n+8}{4}}, \cdots, x_{\frac{n-4}{2}}$ and assign the labels $-6, -12, -18, \cdots, -(\frac{3n}{2})$ respectively to the vertices $y_{\frac{n}{4}}, y_{\frac{n+4}{4}}, y_{\frac{n+8}{4}}, \cdots, y_{\frac{n-4}{2}}$.

Case 2. $n \equiv 2 \pmod 4$.
Assign the labels $1, 7, 13, \cdots, \frac{3n-16}{2}$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-2}{2}}$ respectively and assign the labels $4, 10, 16, \cdots, \frac{3n-10}{2}$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n}{2}}$. Now we assign the labels $-1, -7, -13, \cdots, -(\frac{3n-16}{2})$ respectively to the vertices $u_{\frac{n+2}{2}}, u_{\frac{n+6}{2}}, u_{\frac{n+10}{2}},$ $\cdots, u_{n-2}$ and assign the labels $-4, -10, -16, \cdots, -(\frac{3n-16}{2})$ to the vertices $u_{\frac{n+4}{2}}, u_{\frac{n+8}{2}}, u_{\frac{n+12}{2}},$ $\cdots, u_{n-1}$ respectively. Next assign the labels $\frac{3n-4}{2}, -(\frac{3n-4}{2})$ respectively to the vertices

u_1, u_{n-2}, u_n .

We next assign the labels $2, 8, 14, \cdots, \frac{3n-14}{2}$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-2}{4}}$ respectively and assign the labels $3, 9, 15, \cdots, \frac{3n-12}{2}$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-2}{4}}$. Now we assign the labels $5, 11, 17, \cdots, \frac{3n-8}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-2}{4}}$ and assign the labels $6, 12, 18, \cdots, \frac{3n-6}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-2}{4}}$.

Now we assign the labels $-2, -8, -14, \cdots, -(\frac{3n-8}{2})$ to the vertices $v_{\frac{n+2}{4}}, v_{\frac{n+6}{4}}, v_{\frac{n+10}{4}}, \cdots,$ $v_{\frac{n}{2}}$ respectively and and assign the labels $-3, -9, -15, \cdots, -(\frac{3n-6}{2})$ respectively to the vertices $w_{\frac{n+2}{4}}, w_{\frac{n+6}{4}}, w_{\frac{n+10}{4}}, \cdots, w_{\frac{n}{2}}$. Now assign the labels $-5, -11, -17, \cdots, -(\frac{3n-2}{2})$ respectively to the vertices $x_{\frac{n+2}{4}}, x_{\frac{n+6}{4}}, x_{\frac{n+10}{4}}, \cdots, x_{\frac{n}{2}}$ and assign the labels $-6, -12, -18, \cdots, -(\frac{3n}{2})$ respectively to the vertices $y_{\frac{n+2}{4}}, y_{\frac{n+6}{4}}, y_{\frac{n+10}{4}}, \cdots, y_{\frac{n}{2}}$.

This vertex labeling gives the pair difference cordial labeling of double alternate quadrilateral snake $DA(Q_n)$.
In both cases $\Delta_{f_1} = 2n-4, \Delta_{f_1^c} = 2n-3$.

□

Theorem 10. *The double alternate quadrilateral snake $DA(Q_n)$ is pair difference cordial if the edge u_1u_2 not lies on the quadrilateral and the edge $u_{n-1}u_n$ lies on the quadrilateral for all odd $n \geq 3$.*

Proof. The vertex set and edge set taken from the definition 4.There are two cases arises.
Case 1. $n \equiv 1 \pmod 4$.
Assign the labels $1, 7, 13, \cdots, \frac{3n-13}{2}$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-3}{2}}$ respectively and assign the labels $4, 10, 16, \cdots, \frac{3n-7}{2}$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n-1}{2}}$. Now we assign the labels $-1, -7, -13, \cdots, -(\frac{3n-13}{2})$ respectively to the vertices $u_{\frac{n+1}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+9}{2}}, \cdots,$ u_{n-1} and assign the labels $-4, -10, -16, \cdots, -(\frac{3n-7}{2})$ to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+7}{2}}, u_{\frac{n+11}{2}},$ $\cdots, u_n$ respectively.

We next assign the labels $2, 8, 14, \cdots, \frac{3n-11}{2}$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-1}{4}}$ respectively and assign the labels $3, 9, 15, \cdots, \frac{3n-9}{2}$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-1}{4}}$. Now we assign the labels $5, 11, 17, \cdots, \frac{3n-5}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-1}{4}}$ and assign the labels $6, 12, 18, \cdots, \frac{3n-3}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-1}{4}}$.

Now we assign the labels $-2, -8, -14, \cdots, -(\frac{3n-11}{2})$ to the vertices $v_{\frac{n+3}{4}}, v_{\frac{n+7}{4}}, v_{\frac{n+11}{4}},$ $\cdots, v_{\frac{n-1}{2}}$ respectively and and assign the labels $-3, -9, -15, \cdots, -(\frac{3n-9}{2})$ respectively to the vertices $w_{\frac{n+3}{4}}, w_{\frac{n+7}{4}}, w_{\frac{n+11}{4}}, \cdots, w_{\frac{n-1}{2}}$. Now assign the labels $-5, -11, -17, \cdots, -(\frac{3n-5}{2})$ respectively to the vertices $x_{\frac{n+3}{4}}, x_{\frac{n+7}{4}}, x_{\frac{n+11}{4}}, \cdots, x_{\frac{n-1}{2}}$ and assign the labels $-6, -12, -18,$ $\cdots, -(\frac{3n-3}{2})$ respectively to the vertices $y_{\frac{n+3}{4}}, y_{\frac{n+7}{4}}, y_{\frac{n+11}{4}}, \cdots, y_{\frac{n-1}{2}}$.

Case 2. $n \equiv 3 \pmod 4$.
Assign the labels $1, 7, 13, \cdots, \frac{3n-19}{2}$ to the vertices $u_2, u_4, u_6, \cdots, u_{\frac{n-3}{2}}$ respectively and assign the labels $4, 10, 16, \cdots, \frac{3n-13}{2}$ respectively to the vertices $u_3, u_5, u_7, \cdots, u_{\frac{n-1}{2}}$. Now we assign the labels $-1, -7, -13, \cdots, -(\frac{3n-19}{2})$ respectively to the vertices $u_{\frac{n+1}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+9}{2}}, \cdots, u_{n-3}$ and assign the labels $-4, -10, -16, \cdots, -(\frac{3n-16}{2})$ to the vertices $u_{\frac{n+3}{2}}, u_{\frac{n+7}{2}}, u_{\frac{n+11}{2}}, \cdots, u_{n-2}$ respectively. Next assign the labels $\frac{3n-7}{2}, \frac{3n-5}{2}, \frac{3n-3}{2}, 1$ respectively to the vertices $v_{\frac{n-1}{2}}, u_{n-1}, x_{\frac{n-1}{2}}, u_1$ and assign the labels $-(\frac{3n-7}{2}), -(\frac{3n-5}{2}), -(\frac{3n-3}{2})$ respectively to the vertices $w_{\frac{n-1}{2}}, u_n, y_{\frac{n-1}{2}}$.

We next assign the labels $2, 8, 14, \cdots, \frac{3n-17}{2}$ to the vertices $v_1, v_2, v_3, \cdots, v_{\frac{n-3}{4}}$ respectively and assign the labels $3, 9, 15, \cdots, \frac{3n-15}{2}$ respectively to the vertices $w_1, w_2, w_3, \cdots, w_{\frac{n-3}{4}}$. Now we assign the labels $5, 11, 17, \cdots, \frac{3n-11}{2}$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-3}{4}}$ and assign the labels $6, 12, 18, \cdots, \frac{3n-9}{2}$ respectively to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-3}{4}}$.

Now we assign the labels $-2, -8, -14, \cdots, -(\frac{3n-17}{2})$ to the vertices $v_{\frac{n+1}{4}}, v_{\frac{n+5}{4}}, v_{\frac{n+9}{4}}, \cdots, v_{\frac{n-3}{2}}$ respectively and and assign the labels $-3, -9, -15, \cdots, -(\frac{3n-15}{2})$ respectively to the vertices $w_{\frac{n+1}{4}}, w_{\frac{n+5}{4}}, w_{\frac{n+9}{4}}, \cdots, w_{\frac{n-3}{2}}$. Now assign the labels $-5, -11, -17, \cdots, -(\frac{3n-11}{2})$ respectively to the vertices $x_{\frac{n+1}{4}}, x_{\frac{n+5}{4}}, x_{\frac{n+9}{4}}, \cdots, x_{\frac{n-3}{2}}$ and assign the labels $-6, -12, -18, \cdots, -(\frac{3n-9}{2})$ respectively to the vertices $y_{\frac{n+1}{4}}, y_{\frac{n+5}{4}}, y_{\frac{n+9}{4}}, \cdots, y_{\frac{n-3}{2}}$.

This vertex labeling gives the pair difference cordial labeling of double alternate quadrilateral snake $DA(Q_n)$.
In both cases $\Delta_{f_1} = 2n - 2, \Delta_{f_1^c} = 2n - 2$.

□

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